One page of notes, front and back. Closed book. 50 minutes.

Score ____________________________
1. True or false. (2 points if correct, 1 point if left blank.)
   (If you wish, write an explanation of your thinking.)
   (a) T  F  In the inverse cdf transformation to generate a random variate from the
       distribution \( F_X \), if \( P(X = x) \) has positive probability, then there is exactly one random
       number \( u \) that yields the value \( X = x \).
   (b) T  F  If \( U \) has a uniform distribution on the interval \([0, 1]\) and if \( X \) has an
       exponential distribution with mean 2, then the fractional part of \( U + X \) has a uniform
       distribution on the interval \([0, 1]\).
   (c) T  F  Micro-macro replications can be an effective method for reducing the
       variance of the point estimator \( \theta \).
   (d) T  F  For independent sampling, obtaining one more meaningful digit in the point
       estimator requires increasing the sample size by a factor of 10.
   (e) T  F  The performance measure to be estimated is a property of the distribution of
       the output data.
   (g) T  F  When common random numbers are used to compare two systems, the two
       input models remain unchanged (compared to using independent sampling).
   (h) T  F  If random numbers are truly random in three dimensions, they are also truly
       random in two dimensions.
   (i) T  F  If random numbers are truly random in three dimensions, they are also truly
       random in four dimensions.
   (j) T  F  Floyd’s algorithm can be used even if the functional form of the random-
       number generator is unknown.

2. Consider the sentences
   "When using common random numbers to compare two systems, say \( A \) and
   \( B \), we obtain point estimators \( \hat{\theta}_A \) and \( \hat{\theta}_B \). Let \( \rho = \text{corr}(\hat{\theta}_A, \hat{\theta}_B) \). Positive
   values of \( \rho \) are good in the sense that \( \text{V}(\hat{\theta}_A - \hat{\theta}_B) \) is smaller than when the
   two point estimators are independent.

   Suppose that you rerun a simulation experiment, keeping everything the same except the
   random-number seeds. Using the above three sentences for context, circle "constant",
   "random", and "undefined" for each part below.

   (a) \( \hat{\theta}_A \)  constant  random  undefined
   (b) \( \rho \)  constant  random  undefined
   (c) \( (\hat{\theta}_A)^2 \)  constant  random  undefined
   (d) \( \text{V}(\hat{\theta}_A) \)  constant  random  undefined
   (e) \( P(A - B) \)  constant  random  undefined
   (f) \( \text{corr}(A - B) \)  constant  random  undefined
3. Monte Carlo simulation can be used to evaluate an integral by interpreting it as an expected value

\[ E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x) \, dx , \]

where \( f_X \) is the density function of \( X \).

In class, we considered the integral

\[ \int_0^\infty \int_0^\infty \min\{x_1, x_2\} f_{X_1}(x_1) f_{X_2}(x_2) \, dx_1 \, dx_2 , \]

where \( f_{X_i} \) is the density function of the time until failure of component \( i \).

(a) In this integral, what is the input model?

(b) In this integral, what is the logic model?

(c) Write pseudo-code to estimate the value of the integral using Monte Carlo simulation. (You do not need to specify how to generate random variates; just say what you want to generate.)

(d) In your pseudo-code, circle your point estimator.
4. Consider using micro/macro replications to estimate the standard error of \( \bar{Y} \), the sample average of \( n = 100,000 \) independent random variables \( Y_i \). Let \( \bar{Y}_j \) denote the \( j \)th macro-replication point estimator, \( j = 1, 2, ..., k \), and assume that \( k = 100 \). Let \( \bar{Y} \) denote the average of the \( \bar{Y}_j \)s.

(a) What is the performance measure?

(b) What is the relationship between \( \bar{Y} \) and \( \bar{Y} \)?

(c) Suppose that \( Y_i \) are binary, one if an event \( A \) occurs and zero otherwise. Explain an alternative to micro/macro replications for estimating \( \text{ste}(\bar{Y}) \).

(d) If \( \bar{Y} = 123.456789 \) and \( \text{ste}(\bar{Y}) = 0.0111111111 \), which digits of \( \bar{Y} \) are meaningful?
5. Suppose that the random variable $X$ has density function $f_X(x) = (x - 1) / 50$ for $1 \leq x \leq 11$ and zero elsewhere. The cdf is then $F_X(x) = (x - 1)^2 / 100$ over the interesting values of $x$. If we use the inverse transformation to generate random variates from this distribution, for every random number $u$ in $[0, 1]$, what is the corresponding random variate $x$?

6. Suppose that a particular linear congruential random-number generator

$$X_{i+1} = (aX_i + c) \text{ (mod } m)$$

produces the values $u_1, u_2, \ldots$

(a) If $u_{32} = u_1$, then what do you know about $u_{34}$?

(b) If $a = 1$ and $c = 3$, how many Marsaglia hyperplanes will occur for $k = 2$ dimensions?

(c) If we discuss 3-dimensional uniformity, what is it that is 3 dimensional?