One page of notes, front and back. Closed book. 50 minutes.

Score ________________
1. True or false. (If you wish, write an explanation of your thinking.)

(a) F Acceptance/rejection can be used to generate random variates only from continuous distributions in one dimension.

(b) F Composition can be used to generate random variates only from continuous distributions in one dimension.

(c) F Special properties can be used to generate random variates only from continuous distributions in one dimension.

(d) F The inverse transformation can be used to generate random variates only from continuous distributions in one dimension.

(e) F The purpose of analyzing simulation output data using micro/macro replications is to reduce the variance of the alternate point estimator, \( \hat{\theta} \).

(g) T The purpose of antithetic variates is to reduce the variance of the point estimator.

(h) F Micro/macro replications is applicable only when the point estimator \( \hat{\theta} \) is a sample mean.

(i) F The instructor suggested using between 10 and 30 microreplications within each macroreplication, regardless of the total sample size \( n \).

(j) F Let \( Y = \sum_{i=1}^{n} Y_i / n \). If \( Y_1, Y_2, \ldots, Y_n \) are identically distributed, then \( \text{var}(Y) = \text{var}(Y_1) / n \).

(k) T Let \( Y = \sum_{i=1}^{n} Y_i / n \). If \( Y_1, Y_2, \ldots, Y_n \) are identically distributed and independent, then \( \text{var}(Y) = \text{var}(Y_1) / n \).

(l) F If arrivals to a system are generated from a Poisson process with rate \( \lambda(t) \), then the times between the arrivals are independent and exponentially distributed.

2. In next-event simulation, an event can perform the following actions: (a) Update statistics, (b) Schedule future events, (c) Update the state of the system.

(a) Can one event do all three? yes ← no

(b) At any time \( t \), what is the maximum number of states that the system can be in? one ← infinite

(c) For any statistic that is a mean, "updating statistics" involves adding an observation to an accumulator and incrementing the number of observations by one. True ← False

(d) Each event is executed at a point in time. True ← False
3. Statistics collected in a dynamic simulation fall into two categories. "Time" statistics involve an output variable that is defined for every time \( t \). "Observational" statistics involve an output variable that is defined only occasionally by dynamic behavior of the model.

Classify the following:

(a) Estimate \( E(\text{"number in system"}) \) time observational both

(b) Estimate \( V(\text{"number in system"}) \) time observational both

(c) Estimate \( E(\text{"time in system"}) \) time observational both

(d) Estimate \( P(\text{"time in system"} > 10 \text{ minutes}) \) time observational both

4. Consider the sentences

In the proof that the acceptance/rejection algorithm is valid for generating a random variates from the random variable \( X \) with density function \( f \), we used the notation that \( A = \text{"Step 2 results in acceptance"} \). Suppose that within the proof, we have

\[
P(A \mid X = x) = f(x) / [c \ r(x)].
\]

Using the above information for context, circle "constant", "random variable", "event", and "undefined" for each part below.

(a) \( x \) constant random variable event undefined

(b) \( A \) constant random variable event undefined

(c) \( X = x \) constant random variable event undefined

(d) \( c \) constant random variable event undefined

(e) \( P(A \mid X = x) \) constant random variable event undefined
5. Suppose that \( n \) independent and identically distributed observations \( Y_1, Y_2, \ldots, Y_n \) are obtained in one replication of a simulation experiment. In performing micro/macroreplications, we use \( k \) macroreplications, each composed of \( m \) microreplications, where \( n = mk \). The \( j \)th macro replication results in \( \theta_j \). These are averaged to obtain \( \bar{\theta} \).

Suppose that we wish to estimate the mean \( \mu_Y \) and standard deviation \( \sigma_Y \) using

\[
\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} Y_i \\
\hat{\sigma} = \left[ \frac{\sum_{i=1}^{n} Y_i^2 - n \hat{\mu}^2}{n-1} \right]^{1/2}
\]

(a) Suppose that \( k = 3 \) and \( m = 100 \). Write \( \hat{\sigma}_1 \) in terms of \( Y_1, Y_2, \ldots, Y_n \).

We answer for general values of \( m \). (Setting \( m = 100 \) is trivial.)

Let \( \hat{\mu}_1 = \frac{1}{m} \sum_{i=1}^{m} Y_i \). Then

\[
\hat{\sigma}_1 = \left[ \frac{\sum_{i=1}^{m} Y_i^2 - m \hat{\mu}_1^2}{m-1} \right]^{1/2}
\]

(b) Write \( \text{std}(\hat{\sigma}_j) \) in terms of \( \text{std}(\bar{\sigma}) \).

\[
\text{std}(\hat{\sigma}_j) = \sqrt{k} \ \text{std}(\bar{\sigma})
\]

(because \( \bar{\sigma} = \sum_{i=1}^{k} \hat{\sigma}_j / k \) and the \( k \) macro-replications are iid)

(c) The distribution of each of the following items depend upon none, one or more than one of the experiment parameters \( n, m, \) and \( k \). Specify which. (There can be more than one answer.)

(i) \( \mu \) \hspace{1cm} (none)
(ii) \( \hat{\mu} \) \hspace{1cm} (none)
(iii) \( \hat{\mu}_j \) \hspace{1cm} (none)
(iv) \( Y_1 \) \hspace{1cm} (none)
(v) \( \bar{\sigma} \) \hspace{1cm} (none)
(vi) \( \bar{\sigma} \) \hspace{1cm} (none)
6. Consider a Poisson process with rate function \( \lambda(t) = 2t \) for all non-negative times \( t \). The cumulative rate function is then \( \Lambda(t) = t^2 \). At time \( t_{i-1} \), the cdf of the next time \( T_i \) is \( F(t) = 1 - \exp[-\Lambda(t) + \Lambda(t_{i-1})] \). (That is, the area under \( \lambda(t) \) between \( t_{i-1} \) and \( T_i \) is exponentially distributed with mean one.)

What is the logic to transform a random number \( u \) into the next arrival time \( t \)?

Set \( u = F(t) \) and solve for \( t \).

In particular, \( u = 1 - \exp[-\Lambda(t) + \Lambda(t_{i-1})] \),
which implies \( \exp[-\Lambda(t) + \Lambda(t_{i-1})] = 1 - u \),
which implies \( \Lambda(t) - \Lambda(t_{i-1}) = -\ln(1 - u) \),
which implies \( \Lambda(t) = \Lambda(t_{i-1}) - \ln(1 - u) \),
which implies \( t^2 = t_{i-1}^2 - \ln(1 - u) \),
which implies \( t = [t_{i-1}^2 - \ln(1 - u)]^{1/2} \).

7. We discussed the "three-step" method for generating a random vector \( (X_1, X_2, \ldots, X_k) \).
Multivariate normal random variates are transformed to a multivariate uniform random vector, which is then transformed into the target random vector.

(a) The method does not guarantee a particular joint distribution for the random vector. What model information does the method require?

The correlation matrix \( R_X \), which contains all target correlations \( \rho_{ij} = \text{corr}(X_i, X_j) \), for \( i = 1, 2, \ldots, k-1 \) and \( j = i+1, \ldots, k \).

The \( k \) marginal distributions, specified via the inverse cdf \( F_i^{-1} \) for \( i = 1, 2, \ldots, k \).

(b) Why not assume that the components of the target random vector are independent? (Check all that are true.)

(i) Modeling error would increase.
(ii) Computation time would increase.
(iii) Sampling error would increase.
(iv) Verification error would increase.