

# ON DIMENSIONALITY

by Hong Z. Tan

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# 1

## Introduction

Dimensionality is an important concept in psychophysical experiments. The word *dimension* and its derivatives appear in almost every paper that deals with the absolute identification paradigm. However, it's not clear that a complete definition for dimensionality exists. In this paper, we will attempt to discuss the issues involved in developing a definition for the concept. Although no claim is made that we will give such a definition at the end of the paper, we do hope that this paper will inspire more thoughts on this issue and hopefully lead to a better understanding of the concept.

First of all, reviews of two very important classical papers, Miller's "The magical number seven, plus or minus two" and Pollack & Ficks' "Information of elementary multidimensional auditory displays", will be given. Discussion of how these authors talked about dimensionality in their papers will follow each review. Then, our current thinking on the topic will be presented.

## 2

# Miller's paper

## 2.1 A Review

In his classical paper “The magical number seven, plus or minus two: some limits on our capacity for processing information”, Miller reviewed numerous psychophysical experiments that “tested how accurately people can assign numbers to the magnitudes of various aspects of a stimulus” (p.81) — the so-called absolute identification (AI) experiments. Through many experimental results, Miller showed that the number of equally likely unidimensional stimuli we can absolutely identify without error is limited to  $7\pm 2$ . To get around this bottleneck, Miller discussed two strategies: (1) “to increase the number of dimensions along which the stimuli can differ”; and (2) “to arrange the task in such a way that we make a sequence of several absolute judgments in a row” (p.90).

### 2.1.1 Unidimensional stimuli

First of all, let's look at the experiments with unidimensional stimuli. Table 2.1 is a brief summary of the experiments Miller cited. For each of the first 4 experiments, Miller showed data on a plot of transmitted information *vs.* input information. The same pattern was found in each case: as input information increases, transmitted information initially equaled input information and increased accordingly, until it eventually leveled off at some asymptotic value. This asymptotic value is called *channel capacity* and is listed for each experiment in Table 2.1. As we can see from the table, when stimuli “differ from one another in only one respect” (p.87), information transmission is limited to 1.6 – 3.9 bits (3 – 15 items); thus the magical number  $7\pm 2$ . As Miller pointed out, this range is remarkably narrow considering the wide variety of different variables that had been studied. These results were

also robust with respect to the number of responses available to the observer (equal to or greater than the number of stimulus categories), the parameter range, and different parameter groupings. Although Miller stated that channel capacity for judgments of visual position seemed to be significantly larger compared to that of other modalities, it is interesting to note that both the highest (3.9 *bits*) and the lowest (1.6 *bits*) channel capacities were obtained from visual studies.

	Author	Stimuli/varying parameter	Channel Capacity
#1	Pollack	tones/frequency	2.5 <i>bits</i>
#2	Garner	tones/intensity	2.3 <i>bits</i>
#3	Beebe-Center <i>et al.</i>	salt solutions/concentration	1.9 <i>bits</i>
#4	Hake & Garner	two scale markers/dot on line	3.25 <i>bits</i>
#5	Coonan & Klemmer	same as above	3.2 - 3.9 <sup>†</sup> <i>bits</i>
#6	Eriksen & Hake	squares/size	2.2 <i>bits</i>
#7	Eriksen	visual/size	2.8 <i>bits</i>
		visual/hue	3.1 <i>bits</i>
		visual/brightness	2.3 <i>bits</i>
#8	Geldard	vibrators on chest/intensity	2 <i>bits</i>
		/duration	2.3 <i>bits</i>
		/location	2.8 <i>bits</i>
#9	Pollack	visual display/area	2.6 - 2.7 <sup>†</sup> <i>bits</i>
		/length of line	2.6 - 3.0 <sup>†</sup> <i>bits</i>
		/direction of line	2.8 - 3.3 <sup>†</sup> <i>bits</i>
		/curvature (fixed arc length)	2.2 <sup>‡</sup> <i>bits</i>
		/curvature (fixed chord length)	1.6 <i>bits</i>

<sup>†</sup> short to long exposure.

<sup>‡</sup> short exposure.

Table 2.1: Summary of AI experiments with unidimensional stimuli.

### 2.1.2 Multidimensional stimuli

Second, let's look at experiments where "the number of independently variable attributes of the stimuli" (p.87) is more than one. Table 2.2 summarizes these experiments. If one compares the results of Pollack's two-dimensional experiment (#3 in Table 2.2,  $IT_{frequency,intensity} = 3.1 \text{ bits}$ ) with those of Pollack's one-dimensional experiment (#1 in Table 2.1,  $IT_{frequency} = 2.5 \text{ bits}$ ) and Garner's one-dimensional

experiment (#2 in Table 2.1,  $IT_{intensity} = 2.3 \text{ bits}$ ), two things can be noted: (1)  $IT_{frequency,intensity} > IT_{frequency}$ ,  $IT_{frequency,intensity} > IT_{intensity}$ ; and (2)  $IT_{frequency,intensity} < IT_{frequency} + IT_{intensity}$ . Similar relations can be found by comparing other corresponding experiments in Tables 2.1 and 2.2. Therefore, greater information transmission can be obtained from multidimensional stimuli, although at the cost of lower information transmission per dimension. Notice that in Eriksen's experiment (#6 in Table 2.2), the three variables size, brightness, and hue of a visual target varied in perfect correlation. In other words, the amount of input information was the same as in Eriksen's unidimensional experiments (#7 in Table 2.1). Thus Eriksen demonstrated that greater information transmission can result from redundancy in input coding without increasing the amount of input information.

	Author	stimuli/varying parameter	total IT <sup>a</sup>
#1	Klemmer & Frick	position of dot in a square	4.6 bits
#2	Beebe <i>et al</i>	solutions/saltiness,sweetness	2.3 bits
#3	Pollack	tones/frequency, intensity	3.1 bits
#4	Halsey & Chapanis	colors/hue, saturation	3.6 bits
#5	Pollack & Ficks	acoustic signal/6 variables <sup>b</sup>	7.2 bits
#6	Eriksen	size, hue, brightness <sup>c</sup>	4.1 bits
#7	Pollack & Ficks	acoustic signal/8 variables <sup>d</sup>	6.9 bits

<sup>a</sup> IT stands for information transmission.

<sup>b</sup> Each variable had 5 values.

<sup>c</sup> The three variables varied in perfect correlation.

<sup>d</sup> Each variable had 2 values.

Table 2.2: Summary of AI experiments with multidimensional stimuli.

Note that the concept of channel capacity is also valid here as was shown by Klemmer & Frick's data: Information transmission initially increased as information in stimuli increased; however, it eventually leveled off at an asymptotic value.

### 2.1.3 Sequential stimuli

Third, let's look at "the span of immediate memory". In the classical AI paradigm, the subject is presented a single stimulus (could be multi-dimensional, the key is that all attributes are presented *simultaneously*), and asked to name it immediately

thereafter. In an experiment on immediate memory, however, the subject is told to withhold his response until several stimuli in succession are given. Two experiments were cited. The first was performed by Hayes on our ability to recall binary digits (1 bit of information per digit), decimal digits (3.3 bits/digit), letters of the alphabet (4.7 bits/letter), letters plus decimal digits (5.2 bits/item), and 1000 monosyllabic words (10 bits/word). The result showed that regardless of the information content per item, subjects could only recall about 5-9 items. The second experiment was performed by Pollack using letters and digits. He showed that the total amount of information transmission increased almost linearly as the amount of information per item is increased. The slope of the linear relationship is roughly 7 (estimated from Figure 8 in Miller's paper). In other words, subjects could only recall about 7 items presented in a sequence, regardless of the amount of information per item.

Clearly, it's possible to transmit more information by arranging items in a sequence. Although we are limited by the number of items we can recall, this limitation is nevertheless independent of the amount of information per item. At this point, Miller cautioned that although the number of items we can absolutely identify and the number of items we can recall are both about 7, the phenomena are in fact quite different. In the former case, total information transmission is also limited to 2-3 *bits*; in the latter, total information transmission increases as information per item increases.

Miller then went on to discuss the problem of recoding — i.e., squeezing as much information as possible into one item so as to increase the total amount of information one can recall. This process can be repetitive. For instance, radiotelegraphic codes *dit* and *dah* can be first organized into letters, then from letters into words, and then from words into phrases. So, as the operator learns to increase the *bits* per chunk, he can remember an increasingly larger portion of a message. In an extreme case Miller had cited, Smith trained himself to group every 5 binary digits into a number between 0 and 31. That's to say, he recoded from a base-2 arithmetic to a base-32 arithmetic, so 10100 became 20, and 11111 became 31, etc. Using this 5:1 recoding scheme, he was able to recall up to 40 (or 8 chunks of) binary digits! Miller ended his paper by stating that the process of recoding deserved much more

research attention than it had received.

## 2.2 What did Miller mean by dimension?

Let's quote some of Miller's writing which reflected his thinking on the definition of dimensionality.

- In order to understand the discrepancy between laboratorily obtained  $7 \pm 2$  and our everyday experience (i.e., we can identify hundreds of faces correctly), Miller suggested that:

*"A possible explanation lies in the number of independently variable attributes of the stimuli." (p.87)*

This seems to be an intuitive definition for dimensionality.

- *"The position of a dot in a square is clearly a two-dimensional proposition. Both its horizontal and its vertical position must be identified." (p.87)*

This statement is in accordance with the above definition.

- *"Since these colors (of equal luminance) varied in both hue and saturation, it is probably correct to regard this as a two-dimensional judgment." (p.88)*

This is still in accordance with the intuitive definition.

- When referring to Eriksen's (# 6 in Table 2.2) experiment where the size, hue, and brightness of a visual target all varied together in perfect correlation, Miller wrote:

*"It is interesting to note that the channel capacity is increased even when the several variables are not independent." (p.88)*

Miller apparently regarded the stimuli of Eriksen's experiment as three-dimensional as he noted:



*“Eriksen increased the dimensionality of the input without increasing the amount of input information; ...” (p.88)*

However, since the three attributes were completely correlated, there was only one “independently variable attribute of the stimuli”. Thus according to the intuitive definition, the input was only one-dimensional.

- When discussing an experiment by Kaufman *et al.* on subject’s ability to report how many dots were in a random pattern of dots, Miller wrote:

*“It is, as a matter of fact, very much like a two-dimensional display. Although the dimensionality of the random dot patterns is not entirely clear. ... Perhaps the two dimensions of numerosness are area and density.” (p.90)*

This is getting very vague.

- In the following paragraph, Miller discussed the concept of *chunk*, as defined in the statement “the number of *bits* of information is constant for absolute judgment and the number of *chunks* of information is constant for immediate memory”.

*“The contrast of the terms bit and chunk also serves to highlight the fact that we are not very definite about what constitutes a chunk of information. For example, the memory span of five words that Hayes obtained when each word was drawn at random from a set of 1000 English monosyllables might just as appropriately have been called a memory span of 15 phonemes, since each word had about three phonemes in it. Intuitively, it is clear that the subjects were recalling five words, not 15 phonemes, but the logical distinction is not immediately apparent. We are dealing here with a process of organizing or grouping the input into familiar units or chunks, and a great deal of learning has gone into the formation of these familiar units.” (p.93)*

Exactly the same can be said about the human perception process. After all, all stimuli to human body are transmitted to the brain by nerve pulses. A lot of organization effort truly takes place before a sensation of, say pitch, is formed.

To summarize, dimension is a property of input stimuli and it can probably be defined as the “number of independently variable attributes of the stimuli”. However, it is not clear whether several perfectly correlated attributes make the stimuli multi-dimensional (Miller seemed to think so, we disagree). There are cases where the number of independent attributes are not easy to define, as in the case of random patterns of dots. On the surface, it seems that Miller had talked about the *physical* attributes of stimuli, i.e., the horizontal and vertical position of a dot, the hue, brightness, and saturation of a color, etc. However, when discussing auditory identification experiments, Miller talked about the absolute judgments of auditory *pitch* and *loudness* instead of tonal frequency and intensity. It’s only natural to think that the definition of dimensionality has to do with not only the physical, but also the *perceptual* attributes of stimuli. However, since one can always subdivide any perceptual event into neural firings, it is not intuitively clear how one can define the *number* of independent perceptual attributes of a set of stimuli.

## 3

### Pollack & Ficks' paper

As mentioned in Miller's paper, Pollack & Ficks (1954) managed to get eight different acoustic variables that they could change and obtained a total transmitted information of 6.9 *bits* (see Table 2.2). The main purpose of Pollack & Ficks' paper on "Information of elementary multidimensional auditory displays" was to explore the potential for high information transmission by using elementary auditory displays with high dimensionality.

#### 3.1 A Review

Two classes of acoustical displays were used. One of them was an interrupted tone which varied in (1) the frequency of the tone; (2) the loudness level of the tone; (3) the rate of interruption; (4) the "on"-time fraction of the tone; (5) the total duration of presentation; and (6) the direction within the testing room from which the sound originated. The other class was an alternately presented tone and noise. It differed from the previous class in that the silent portion was filled with noise. This led to two additional aspects of the stimuli: (7) the frequency range of the noise; and (8) the loudness level of the noise.

The eight-dimensional (8D) stimuli were binary-coded (i.e., each variable had only two steps). With the six-dimensional (6D) stimuli, each of the six variables had either two (binary), three (ternary), or five (quinary) steps. The ternary case used the 1st, the 3rd, and the 5th steps of the quinary stimuli. And the binary case used the 1st and the 5th steps of the quinary stimuli. Therefore, the total ranges of all variables were kept constant in binary, ternary, and quinary cases.

The experimental results were arranged in the form of confusion matrices. Information transmission (IT) was estimated from confusion matrices for each dimension

and each subject. IT from different subjects were averaged, and IT from different dimensions summed to yield the total IT for multidimensional display.

The information content of stimulus, the average total multi-D information transmission, and the average 1-D IT is tabulated in Table 3.1. Pollack & Ficks pointed out that information transmission from 1D acoustical displays under similar conditions is around 2.0-2.3 *bits*. Thus overall, multidimensional display increases total information transmission. However, the average 1-D ITs were in the range of 0.83-1.15 *bits*, which were lower than those obtained from 1D displays.

Dimensions	Stimulus Steps	Information in Stimulus	Total multi-D IT	Average 1-D IT
8	binary	8.0 <i>bits</i>	6.9 <i>bits</i>	0.86 <i>bits</i>
6	binary	6.0 <i>bits</i>	5.2 <i>bits</i>	0.83 <i>bits</i>
6	trinary	9.5 <i>bits</i>	6.6 <i>bits</i>	1.10 <i>bits</i>
6	quinary	13.9 <i>bits</i>	7.2 <i>bits</i>	1.15 <i>bits</i>

Table 3.1: Summary of results from Pollack and Ficks' paper.

Note that with the 6D display, when stimuli steps increased from binary to quinary, total IT increased by only 2 *bits* whereas the information in stimuli increased by about 8 *bits* (from 6 to 13.9 *bits* per stimulus presentation). Pollack & Ficks also noted that subjects who had higher 6D binary IT scores benefited more from the quinary case. In other words, more proficient subjects were able to take better advantage of the finer subdivision of each dimension.

Finally, the data showed that by increasing the number of stimulus dimensions, further gain in information transmission could be achieved. Specifically, the total 6D binary IT was improved by 1.7 *bits* by employing 8 dimensions. This was an appreciable gain since the total information in stimulus presentation only increased by 2 *bits*. In fact, the 8D binary IT was almost the same as the 6D quinary IT. This implies that by employing a large number of dimensions and using the simple binary-coding, one can very efficiently achieve a great deal of information transmission. By efficiency, we mean the ratio of *Total multi-D IT* to *Information in Stimulus* is close to 1. This point is very similar to that proposed by Jakobson *et al.* (1963) on the analysis of distinctive features of speech. Jakobson *et al.* demonstrated that

phonemes can be classified along a number of binary-coded (and sometimes trinary-coded) elementary linguistic characteristics. Presumably, listeners make binary (or trinary) choices when they listen to a speech sound. Of course, as Pollack and Ficks pointed out, the number of independent stimulus dimensions employed appears to be limited by the number which can be manipulated and/or which can be discriminated by the listener.

### 3.2 Pollack & Ficks' view on dimensionality

Pollack & Ficks talked about dimensions as stimulus aspects. As they put it:

*“When a stimulus aspect can be manipulated independently, it is often called a dimension of an elementary auditory display.” (p.155)*

It's interesting to note that Pollack & Ficks mixed perceptual (i.e., loudness of noise/tone) and physical (i.e., all other aspects of stimuli) aspects of their acoustical stimuli in their description. There was apparently a good reason for doing so.

*“In both (6D and 8D) displays, it was necessary to encode the sound levels in terms of equivalent loudness levels because of serious interactions upon loudness among the variables.” (p.155)*

## 4

# Dimensionality of Stimuli

## 4.1 Effective stimulus region and its physical aspects

It takes some physical variables to form a stimulus set. The minimum number of *independent* physical variables needed to span the stimulus set is called the *physical dimension* of the set,  $N$ . With this definition, physical variables that are kept constant throughout the stimulus set do not contribute to  $N$ . This minimum set of variables can be used as axes of an  $N$ -dimensional space. The *stimulus space* is a sub-space within the  $N$ -dimensional space bounded by minimum and maximum values of these axes. The *effective stimulus space (ESS)* is a sub-space of the stimulus space that can be perceived by human beings.

The ESS can be further divided into *effective stimulus regions (ESRs)* with qualitatively distinctive perceptions (this will be clarified with examples to follow). The physical aspects of an ESR,  $n_i$ , is the minimum number of physical variables needed to span the ESR. Note that  $n \leq N$ . Usually, the number of ESR,  $N_{ESR}$ , is one.

The following examples serve to clarify the above definitions.

- **Example 1**

Stimuli are a 1  $kHz$  and a 50  $kHz$  tone with independently varying intensities. The physical dimensions of the tonal stimulus set is two: intensity of 1  $kHz$  tone, and intensity of 50  $kHz$  tone. However, since human ears can't hear the 50  $kHz$  tones, only the 1  $kHz$  tones can be perceived. Thus the ESS is one-dimensional. Here ESR is the same as ESS, since changing the intensity of the 1  $kHz$  tone changes the perception of loudness *quantitatively*,

not qualitatively. Therefore,  $N = 2$ ,  $N_{ESR} = 1$ , and  $n = 1$ . This is a one-dimensional intensity identification experiment with 1 *kHz* tones. Its information transmission is limited by Miller's  $7 \pm 2$ .

- **Example 2**

Consider an absolute identification experiment where the stimuli are pure tones varying in both intensity and frequency ( $N = 2$ ). Subjects are required to identify both physical aspects of the stimuli.

Although intensity is usually associated with the sensation of loudness and frequency with pitch, there's evidence that the two physical aspects of a pure tone can also give rise to the sensation of tonal density and tonal volume (see Stevens, 1934a; Stevens 1934b). The "volume" of a pure tone means "bigness" or its spread in space. The "density" of a pure tone means its "compactness" or "concentration" (Stevens, 1934a). Subjects can be trained to "tune in" at the tonal stimuli with any of the four sensations: loudness, pitch, volume, and density.

Note that each of the four sensations change quantitatively as either intensity or frequency varies from its minimum to its maximum level. In other words, the four distinctive sensations coexist within the same ESR. Therefore,  $N_{ESR} = 1$ .

Since within the ESR, tonal stimuli still vary in both intensity and frequency, we have  $n = 2$ . What about the fact that there are four distinctive sensations associated with the two physical variables? Note that the four sensations are correlated in the sense that it's impossible to change the sensation of tonal volume without changing either loudness or pitch. In other words, volume and density levels are totally determined by loudness and pitch levels. On the other hand, it's possible to fix loudness and still be able to vary pitch (or volume, or density) through manipulation of tonal intensity and frequency. Therefore, these pure tones really form a 2-D display. It is generally true that the number of uncorrelated sensations do not exceed that of the physical variables needed to induce them (more on this issue in Section 4.2).

Melara and Marks (1990) used the same tonal stimuli to study perceptual primacy of dimensions. They concluded that pitch and loudness are primary perceptual dimensions in the sense that judgment of one dimension is least affected by random variations of the other.

Stevens (1934a) also regarded these pure tones as 2-D stimuli as he pointed out:

*“The fact that each type of instruction leads to a response which is a function of the two stimulus variables means that the system can be thought of as bidimensional, but the fact that there are four different types of response means that it should be possible to discover at least four distinguishing characteristics in the neural pattern emanating from the cochlea by the auditory nerve.” (p.459)*

Another similar example is color perception. When a set of color varies in both hue and saturation, both qualities can be perceived as well as the sensation of warmth/coldness of the color. Clearly, warmth/coldness is totally correlated with the perception of hue and saturation. Thus,  $N = 2$ ,  $N_{ESR} = 1$ , and  $n = 2$ . And a set of color with varying hue and saturation is a two-dimensional visual display. Higher information transmission can be achieved in this case than in Example 1.

- **Example 3**

Consider the following binaural experiment. The signal is a broad-band click. It is delivered to both ears with an interaural time delay  $\tau$ . In the vicinity of  $\tau = 0$ , there is a single fused image whose position depends on  $\tau$  (lateralization). In the region  $1 \leq \tau \leq 5 \text{ msec}$ , the image starts to split into two, one on the lead-ear side followed by one on the lag-ear side. When  $\tau$  is on the order of  $15 \text{ msec}$ , the two images become located at the two ears and there is no further interaction with respect to lateralization. (See Durlach & Colburn, 1978, for references.) The subject's task is to identify the interaural time delay  $\tau$ . Clearly,  $\tau$  is the only physical variable involved, so  $N = 1$ .



However, perception of  $\tau$  falls into three distinct categories: one image with lateralization when  $0 \leq \tau < \tau'$  (where  $1 \leq \tau' \leq 5 \text{ msec}$ ), one image splitting into two when  $\tau' \leq \tau < 15 \text{ msec}$ , and two images without lateralization when  $\tau \geq 15 \text{ msec}$ . Notice that this is different from the identification of frequency and intensity. In Example 2, the perception of loudness, pitch, volume, and density coexist for the *entire* range of intensity and frequency. Whereas here, each perception of  $\tau$  corresponds to a non-overlapping range of  $\tau$ . Therefore, there are three ESRs:  $[0, \tau')$ ,  $[\tau', 15)$ ,  $[15, +\infty)$ , that correspond to three distinctive sensations. We have  $N_{ESR} = 3$ . Within each ESR, there is only one physical aspect,  $\tau$ . Thus  $n_1 = n_2 = n_3 = 1$ . Therefore, a broadband click sent to both ears with an interaural time delay is a 3-regional (for lack of a better term) 1-D auditory display. Its information transmission is not limited by  $7 \pm 2$ , but three times that much. It's important to realize that greater information transmission can be achieved by merely increasing number of ESRs.

- **Experiment 4**

An experiment that is conceptually similar to Experiment 3 is touching a spot on a human body surface and ask the subject to report the location of the spot. Suppose that we only touch on the forearm and the upper-arm of the right hand. If the arm is kept stationary, then moving on a curved surface requires only two coordinates, thus  $N = 2$ . However, four distinctive perceptions of regions can be experienced: upper-arm dorsal, upper-arm ventral, forearm dorsal, and forearm ventral. These are the four ESRs, thus  $N_{ESR} = 4$ . Within each ESR, the number of physical aspects is two:  $n_1 = n_2 = n_3 = n_4 = 2$ . Thus this is a 4-regional 2-D tactual stimulus set. Compared to Example 3, even greater information transmission is possible in this case due to increases in both dimensionality (1-D to 2-D) and number of ESRs (3 to 4).

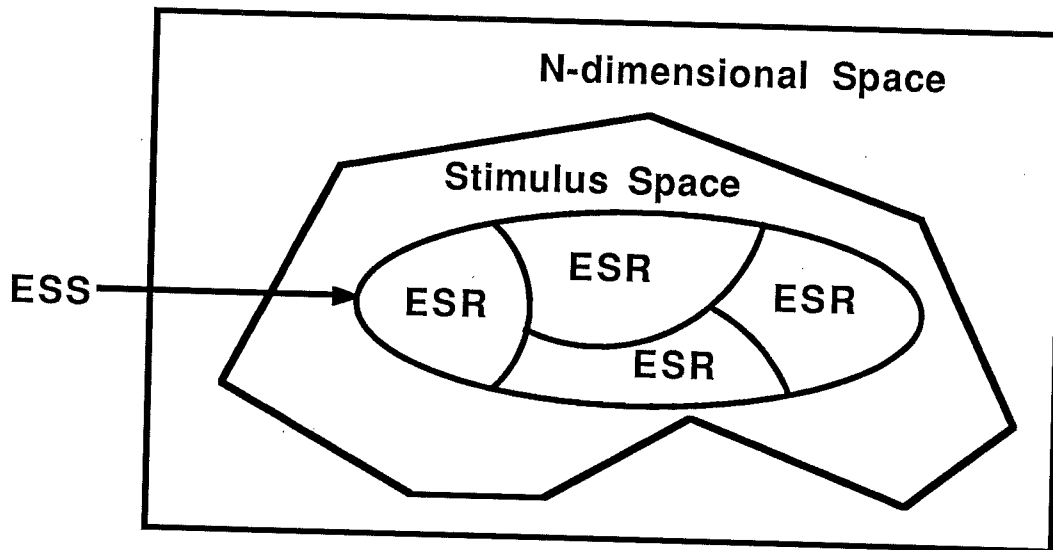


Figure 4.1: Diagram of concepts related to dimensionality.

## 4.2 Recap

Figure 4.1 is a diagram of  $N$ -dimensional space, stimulus space, ESS, ESRs, and their relationship. Notice that this diagram belongs to physical, not perceptual domain. We're talking about an  $N$ -dimensional space that is formed by  $N$  *physical* variables, and a stimulus space that is bounded by min's and max's of the same *physical* variables. However, the *boundary* of ESS is determined by whether humans can *perceive* these physical variables, and the *boundaries* of ESRs is determined by the nature of *sensations*. This reflects our view that perception is mediated solely by changes in the physical world, yet an analysis of the physical world in a psychophysical experiment is affected *qualitatively* by our perception.

Furthermore, within each ESR, we defined the concept of physical aspects but deliberately avoided perceptual aspects. We claim that physical aspects of ESR determines its dimensionality. Suppose that a stimulus set in an ESR always gets

mapped to  $M$  perceptual axes (we've learned to perceive things in certain ways), we feel that the perception of this stimulus set should collapse to an  $n$ -dimensional *perceptual* space, with  $n$  being the physical aspects of this ESR. We also feel that  $M$  should never be less than  $n$ . We realize that this idea is not very well defined at this point.

To summarize, we have three ways of increasing information transmission. At a higher level, one can increase the number of ESRs. Within each ESR, one can increase the number of physical aspects (Miller's multi-dimensional experiments all belong to this category, as well as Pollack & Ficks' experiments). A third method is chunking of sequential stimuli as discussed by Miller. In some sense, increasing ESR and chunking are both hierarchical re-organization of stimuli, whereas increasing physical aspects is more of a parallel expansion of stimuli. Thus chunking and increasing ESR seem to be more similar than increasing ESR and increasing physical aspects. However, stimuli used in chunking experiments are all highly identifiable, and well labelled (e.g., binary digits, letters, etc.). And this seems to be a prerequisite for any recoding processes.

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