Force Constancy and Its Effect on Haptic Perception of Virtual Surfaces

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The force-constancy hypothesis states that the user of a force-feedback device maintains a constant penetration force when stroking virtual surfaces in order to perceive their topography. The hypothesis was developed to address a real-world data perceptualization problem where the perception of surface topography was distorted when the surface stiffness was nonuniform. Two experiments were conducted. In Experiment I, we recorded the penetration depths of the probe tip while the user stroked two surfaces with equal height but different stiffness values. We found that the data could be quantitatively modeled by the force-constancy hypothesis when the virtual surfaces were neither too soft nor too hard. In Experiment II, we demonstrated that given two adjacent surfaces, their perceived height difference depended on both the surface stiffness values as well as the relative heights of the surfaces. Specifically, we showed that the higher but softer surface could be perceived to be lower, at the same height, or higher than the other surface, depending on how much higher it was than the other surface. The results were consistent with the predictions of the force-constancy hypothesis. Our findings underscore the importance of understanding the interplay of haptic rendering parameters.

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1. INTRODUCTION

This paper is concerned with how users interact with a haptic virtual surface and how the interaction strategy affects the perception of surface properties. Given that most force-feedback haptic interfaces are

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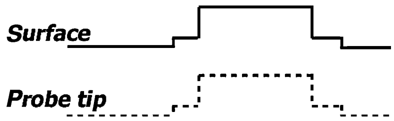

Fig. 1. An illustration of a surface profile (solid line) and the trajectory followed by the probe tip (dashed line) when the surface is explored with a haptic device.

based on discrete contact point(s), we restrict our study to probe-mediated haptic surface exploration. We observe that the way people interact with a virtual surface is fundamentally different from that with a real surface when the surfaces are reasonably hard to the touch. During exploration of a real surface, the interface probe remains in contact with the surface but never penetrates it. When interacting with a virtual surface, however, most haptic rendering methods require the user to push the probe into the virtual surface before a feedback force can be generated. This resistance to penetration is then attributed to the existence of a surface by the user. In both the real and virtual environments, the user forms a mental image of the surface topography based on the perceived trajectory of the probe tip. It follows that the trajectory of the probe tip must be parallel to that of the virtual surface in order for the user to perceive the surface topography correctly, albeit at an offset in space (see Figure 1). When the trajectory of the probe tip ceases to be parallel to the virtual surface, the user's perception of the surface topography is distorted. This can be a serious problem when the virtual surface is being rendered for scientific data visualization or perceptualization.

When the surfaces are very soft (e.g., soft foams, cotton balls), however, their surface topography cannot be accurately perceived by a probe in either a real or virtual environment. The lack of initial contact cues or sufficient resistance forces makes it difficult to discern where the probe is relative to the object surfaces. As will become clear later, some experimental results obtained with very soft surfaces were not consistent with those obtained with harder surfaces.

In the rest of the Introduction, we first describe a real-world application that inspired the current study. A system for perceptualizing multiattribute data collected with a scanning probe microscope (SPM) at the nanometer scale is presented. We describe in some detail a phenomenon involving an inversion in perceived relative height of two adjacent surfaces. We then introduce the concept of a *force-constancy hypothesis*. It states that users maintain a constant penetration force during haptic exploration of virtual surfaces in order to perceive the surface topography. We show that based on the force-constancy hypothesis, the trajectory of the probe tip always follows the surface topography as shown in Figure 1, *provided* that the virtual surfaces are rendered with the same stiffness values using a penalty-based rendering method.[1] When the virtual surfaces are rendered with different stiffness values (as was the case with our SPM data perceptualization system), distortions in perceived surface topography can occur.

1.1 Perceptualization of Multiattribute SPM Data

This study was motivated by a recent collaborative project on the perceptualization of multiattribute SPM data between the Haptic Interface Research Lab and the Nanophysics Lab at Purdue University. The SPM is a state-of-the-art measurement device that utilizes a sharp tip (10–30 nm radius) fabricated onto a cantilever to measure nanometer-scale features on a flat substrate [Sarid 1991]. By measuring the rise and fall of the tip as the substrate is rastered beneath it in a controlled and vibrationless way, three-dimensional images of the topography of a surface can be measured with nanometer-scale

[1]The penalty-based rendering method refers to a most commonly used class of rendering algorithms where the force delivered by a force-feedback device is based on the penetration depth (i.e., a penalty) of the probe into an object surface (i.e., a constraint).

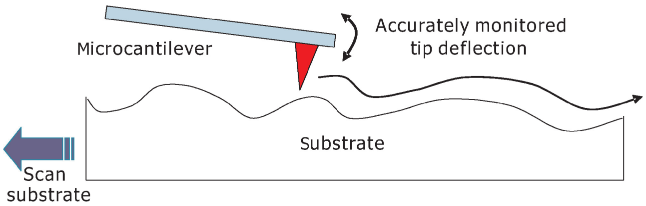

Fig. 2. A schematic representation of the operation of an SPM.

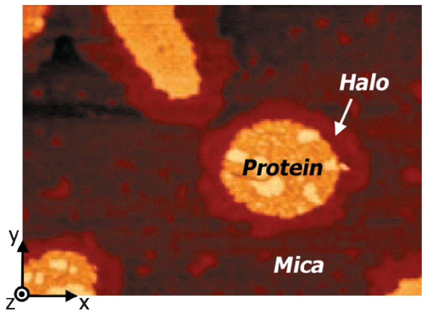

Fig. 3. A pseudocolor representation of the surface height map of the "protein-on-mica" data taken from an SPM. See Section 1.1 for details.

precision (see Figure 2). One of the advantages of the SPM is that various kinds of local information about the substrate can be inferred by measuring the tip–substrate interaction via the cantilever's deflection. For example, SPMs can provide topographic information, as well as many useful correlated quantities such as adhesion, lateral friction, stiffness, electrostatic potential, capacitance, conductivity, and energy dissipation—all with nanometer resolution. In terms of data presentation, it is customary to analyze multiple data sets from SPM experiments as multiple images lined up side by side. The experimenter is then asked to correlate features using visual information alone. While this visual approach is easy to implement, the advantages of using nonvisual representations of these multiple data sets, such as haptic rendering of local stiffness, are intriguing since they may result in a more intuitive understanding of the data (e.g., Taylor II et al. [1993] and Sitti and Hashimoto [2003]).

In what follows, we investigate patches of transmembrane proteins embedded in a bi-lipid membrane measured with a SPM. Figure 3 shows the surface height map, $h(x, y)$, of patches of bi-lipid membrane with embedded proteins (bacteriorhodpsin or BR) on a mica substrate over an area of 2000 nm $\times$ 1375 nm, with lighter colors corresponding to higher surfaces. The image shows a nearly circular membrane patch surrounded by a halo (presumably of lipids that have dissociated from the membrane) resting on an atomically flat mica substrate. In this data set, the transmembrane protein embedded in a bi-lipid membrane formed circular patches that were roughly 5 nm above the mica substrate while the lipid halo regions were about 1 nm above the mica. Because the membrane patch is filled with a periodic array of the transmembrane protein BR, it should be considerably stiffer than the halo of dissociated lipids, but not as stiff as the mica substrate. For convenience, we will refer to this image of a bi-lipid membrane containing transmembrane proteins supported on a mica substrate as the "protein-on-mica" data set.

Employing a simple model to define the local stiffness $k(x, y)$ at different positions (x, y) across the image, and combining this information with the topographic data contained in Figure 3, it is possible to tap on and stroke the corresponding virtual haptic surface to gauge both the local stiffness and

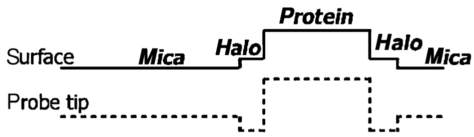

Fig. 4. A cross-section of the surface profile of the "protein on mica" data shown in Figure 3 (solid line, top trace) and the trajectory of the probe tip scanning the data set (dashed line, bottom trace).

topography. This is accomplished by rendering the height data and stiffness model by computing the feedback force during probe-tip penetration as follows:

$$f_z(x, y) = k(x, y) \cdot [h(x, y) - p_z], \tag{1}$$

where $f_z(x, y)$ was the restoring force in the z direction, $k(x, y)$ the stiffness value at location (x, y), $h(x, y)$ the surface height at (x, y), and p_z the z position of the probe tip. We used a PHANToM desktop device (SensAble Technologies, Woburn, MA) for displaying force. This penalty-based rendering method is widely used in haptic rendering (e.g., see Zilles and Salisbury [1995] and [Ruspini et al. 1997]). When users interacted with this virtual haptic environment, however, the halo regions felt consistently lower than the surrounding protein and mica regions, presumably because the halo region was modeled with a lower stiffness.

Intrigued by the perceived reversal in the relative position of the mica and halo regions, we measured the probe-tip positions during a user's lateral stroking of the "protein-on-mica" data set. The recorded position data revealed that the probe tip indeed "dipped" when it entered the halo region from the adjacent mica region, confirming the anecdotal reports. The key to an explanation of this phenomenon was that the protein patches and mica substrate were similar in stiffness values whereas the halo region (the dissociated bi-lipids) was considerably softer. We speculated that when the probe tip was moved from the mica region into the halo region, the user tried to *maintain the same penetration force.* Since the halo region was softer than the mica region, this resulted in a larger penetration depth in the halo region than in the mica. When the increase in penetration depth exceeded the height difference between halo and mica, the halo region was incorrectly perceived to be lower than the mica region (see an illustration in Figure 4).

We believe that this phenomenon is not unique to the "protein-on-mica" data set. It is likely to occur in a variety of haptic rendering systems where multiple collocated variables are rendered simultaneously (e.g., in a surgical simulation system where both the shape and the stiffness of organs are haptically rendered). In the next subsection, we develop the force-constancy hypothesis, and explain the phenomenon encountered with the "protein-on-mica" data set quantitatively.

1.2 Force Constancy

The force-constancy hypothesis states that users maintain a constant penetration force (f_p) when they stroke virtual surfaces in order to perceive the surface topography. Let us consider the simplest case of two vertical virtual planes depicted as P_1 and P_2 in Figure 5. The position of the two surfaces are denoted by h_1 and h_2 along the z-axis ($\Delta h = h_2 - h_1 > 0$ in Figure 5). The "higher" plane P_2 is closer to the user whereas the "lower" plane P_1 is farther away. As the user strokes the virtual surfaces from P_1 to P_2, the penetration depths d_1 and d_2 under P_1 and P_2 are computed as follows, respectively:

$$d_1 = h_1 - p_{z1} = \frac{f_p}{k_1} \tag{2}$$

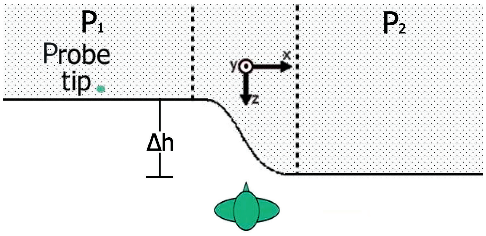

Fig. 5. Top view of a haptic rendering of two vertical planes. See Section 2.2 for details on the interpolation region between the two dashed lines. The symbol at the bottom represents a user facing the vertical surface.

and

$$d_2 = h_2 - p_{z2} = \frac{f_p}{k_2}, \tag{3}$$

where p_{z1} and p_{z2} are the probe-tip positions inside P_1 and P_2 along the z-axis, respectively, f_p is the constant penetration force maintained by the user, and k_1 and k_2 are the stiffness values of P_1 and P_2, respectively. It then follows that the surface-height difference traced by the probe is

$$\Delta p_z = p_{z2} - p_{z1} = h_2 - h_1 - f_p \left(\frac{1}{k_2} - \frac{1}{k_1} \right). \tag{4}$$

This is the surface-height difference perceived by the user. Let

$$\Delta h_d = -f_p \left(\frac{1}{k_2} - \frac{1}{k_1} \right), \tag{5}$$

then

$$\Delta p_z = \Delta h + \Delta h_d. \tag{6}$$

Therefore, the perceived surface-height difference has two components. The first term (Δh) is what we intend for the user to perceive. The second term (Δh_d), however, can introduce a distortion in perceived surface-height difference when $k_1 \neq k_2$.

There are three possible scenarios according to Eqs. (5) and (6). When $k_1 = k_2$, the probe-tip follows a trajectory that is parallel to the virtual surface topography, as depicted in Figure 1. When $k_1 > k_2$, we have $\Delta h_d < 0$ and consequently $\Delta p_z < \Delta h$. This is a case similar to that encountered with the "protein-on-mica" data set if we consider P_1 to be the stiffer and lower mica substrate and P_2 to be the softer and higher halo surface. In this case, the perceived surface-height difference is smaller than Δh. If $|\Delta h_d| > |\Delta h|$, then $\Delta p_z < 0$ and the higher surface P_2 (halo) is perceived to be lower than P_1 (mica), as depicted in Figure 4. Finally, when $k_1 < k_2$, we have $\Delta h_d > 0$ and $\Delta p_z > \Delta h$. In this case, the surface-height difference between P_1 and P_2 is perceived to be greater than what it should be.

It is worth pointing out that our study is not about visual dominance or haptic illusion as many researchers have done (see Rock and Harris [1967], Srinivasan et al. [1996], and Robles-De-La-Torre and Hayward [2001] for representative studies of sensory illusions involving touch). We assume that a user is able to accurately perceive the trajectory of a probe tip held in the hand. We do not artificially create inconsistent visual/haptic or force/position cues. Instead, we hypothesize that a user maintains a roughly constant force during lateral exploration of surface geometry, and therefore the only useful cue available to the user is the kinesthetic perception of the probe-tip position.

One might argue that if the users had access to stiffness information of the adjacent surfaces, they might have been able to use that information in position estimation. However, this is unlikely for two reasons. Firstly, different movement patterns are required to assess stiffness and surface topography. The former requires tapping—movements vertical to the virtual surfaces, while the latter requires stroking—movements lateral to the virtual surfaces.[2] It is therefore unfeasible to gather surface stiffness and topography information simultaneously. Secondly, even if subjects had prior knowledge of the relative stiffness values of the surfaces, it is unclear how the somatosensory system can carry out the computations outlined in Eqs. (1)–(6) in order to factor out the Δh_d component from sensory information. In fact, the relatively poor resolutions associated with position and stiffness perception (see Jones and Hunter [1990] and Tan et al. [1992]) would render such computation insufficient in accuracy or resolution.

In the rest of this paper, we report two experiments conducted to examine the extent to which the force-constancy hypothesis explains how users interact with virtual surfaces. Experiment I investigated the hypothesis itself. The results showed that users indeed applied roughly constant forces when stroking virtual haptic surfaces. Experiment II explored the distortion in perceived topography when surface stiffness values were not constant. The results were consistent with those predicted by the force-constancy hypothesis.

2. GENERAL METHODS

In this section, we describe the experimental methods that were common to both experiments. Experiment-specific details are presented later when the corresponding experiment is discussed.

2.1 Apparatus

A PHANToM force-feedback device (desktop model, SensAble Technologies, Inc., Woburn, MA) was used for rendering virtual surfaces. The GHOST software development kit and the OpenGL library were used for generating the haptic stimuli and visual scenes, respectively.

2.2 Stimuli

The haptic stimuli consisted of two vertically adjoined planes facing the user as shown in Figure 5. As mentioned earlier, the height and stiffness of the two planes are denoted by h_1 and k_1 (for P_1) and h_2 and k_2 (for P_2). The surface-height difference between P_1 and P_2 is denoted by $\Delta h = h_2 - h_1$. A positive Δh corresponds to P_2 being closer to the user, and a negative Δh means that P_1 is closer. We chose to render the planes vertically in order to take advantage of the relatively larger workspace of the PHANToM in the xy (vertical) plane compared to that in the zx (horizontal) plane. By rendering forces in the zx plane, we also eliminated the effect of gravity on force perception.

One important detail of the haptic stimuli was the vertical line separating the two planes. As a probe strokes across the boundary between P_1 and P_2, any step change in the surface height results in a step change in rendered force, which in turn can induce device instability. This “glitch” can be easily perceived by a user, and can serve as a perceptual cue indicating uneven adjacent planes regardless of whether the user is able to detect Δh. To circumvent this problem, we used a Hanning window (a half-cycle sinusoidal function) to smoothly connect the two planes in both height and stiffness (see the region between the two dashed lines in Figure 5). Mathematically, the height and stiffness functions

[2]Lederman and Klatzky [1987] discussed stereotypical hand movements, called exploratory procedures, used by humans to seek specific information about an object. In the study, both tapping and stroking were found to be useful for assessing surface stiffness when subjects explored real objects with bare fingers. Although we do not have experimental evidences, it is our observation with force-feedback devices that stroking does not provide stiffness information about a *virtual* surface explored by a *probe*.

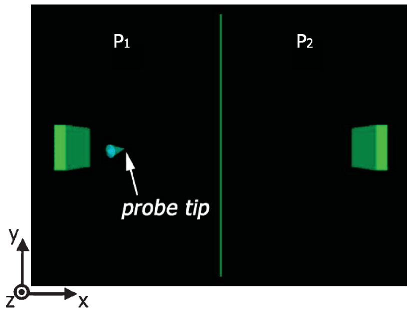

Fig. 6. The visual scene used in all experiments. The position of the probe-tip position was indicated by the (blue) cone. The (green) vertical line indicated the location of the common border between the two planes.

used in the experiments were computed as follows.

$$h(p_x) = \begin{cases} h_1, & \text{if } p_x < -\dfrac{W}{2} \\ \dfrac{1}{2}\left[(h_2 - h_1)\sin\left(\dfrac{\pi p_x}{W}\right) + h_1 + h_2\right], & \text{if } -\dfrac{W}{2} \le p_x \le \dfrac{W}{2} \\ h_2, & \text{if } p_x > \dfrac{W}{2} \end{cases} \tag{7}$$

and

$$k(p_x) = \begin{cases} k_1, & \text{if } p_x < -\dfrac{W}{2} \\ \dfrac{1}{2}\left[(k_2 - k_1)\sin\left(\dfrac{\pi p_x}{W}\right) + k_1 + k_2\right], & \text{if } -\dfrac{W}{2} \le p_x \le \dfrac{W}{2} \\ k_2, & \text{if } p_x > \dfrac{W}{2} \end{cases}, \tag{8}$$

where p_x was the x-position of the probe, and W was the width of the interpolation region. In all our experiments, W was set to 4 mm, a value that resulted in stable transitions between the surfaces used in all experimental conditions. The feedback force was rendered as

$$f_z(p_x) = k(p_x) \cdot [h(p_x) - p_z]. \tag{9}$$

The visual scene used in the experiments provided a spatial reference to the probe-tip position without revealing the nature of the haptic stimuli. As shown in Figure 6, the two blocks represented the starting and ending points for each stroke of the haptic stimuli. The subjects were instructed to move the probe tip toward the left block until the color of the block turned from red to green, indicating the beginning of a trial. Once the probe tip entered a ±5 mm band along the y-axis (centered around the line connecting the centers of the two blocks), its motion was constrained to the zx (horizontal) plane. The subject then stroked the virtual surface from left to right until the probe tip intersected the right block. The color of the right block turned from green to red to indicate the end of the current trial. The probe tip was "stuck" against the right block until a response was entered. It follows that one trial consisted of one sweep across the vertical planes. Once the probe tip was released, it could be moved to the left block again to start a new trial.

3. EXPERIMENT I: TEST OF FORCE-CONSTANCY HYPOTHESIS

The purpose of Experiment I was to investigate the extent to which the user of a force-feedback device maintains a constant penetration force while stroking haptic virtual surfaces with a probe in order to perceive surface topography. While the idea of force constancy introduced in Section 1.2 may seem intuitive, it needed to be tested empirically. We measured the trajectories of the PHANToM probe tip when subjects stroked virtual haptic surfaces of various stiffness values laterally. Based on these data, we examined how well the force-constancy hypothesis can explain the exploratory motions made by subjects to examine the topography of virtual surfaces.

3.1 Method

3.1.1 *Subject.* Ten subjects (six males and four females) participated in the experiment. All were right handed and did not report any known sensory or motor impairments with their hands or arms. Subjects S1–S3 were experienced users of the PHANToM device. Subjects S4–S8 were generally familiar with haptic interfaces but were not as experienced with the PHANToM as subjects S1–S3. Subjects S9 and S10 had not used any haptic interface before they took part in the experiment. The age of the subjects ranged from 23 to 39 years, and averaged 26.3 years.

3.1.2 *Procedure.* Throughout the experiment, the planes P_1 and P_2 always had the same surface height ($\Delta h = 0$), but different stiffness values. Five pairs of stiffness values for k_1 and k_2 were used: (0.1, 0.3), (0.2, 0.4), (0.5, 0.7), (0.6, 0.8) and (0.9, 1.1), in N/mm. Under each stiffness-pair condition, P_1 was stiffer on roughly half the trials, and softer on the remaining trials. For example, when the (0.1, 0.3) N/mm condition was tested, P_1 was rendered with a stiffness value of 0.1 N/mm (and P_2 0.3 N/mm) with a probability of 0.5. The five conditions covered a stiffness range 0.1 to 1.1 N/mm, with a constant stiffness difference of 0.2 N/mm between P_1 and P_2. The presentation order of the conditions was randomized for each subject.

Each subject completed 25 strokes per experimental condition. The subject was asked to stroke the haptic stimulus in a consistent manner from the left block to the right block (Figure 6). After each stroke, the subject was required to answer whether the right plane P_2 felt higher or lower than the left plane P_1 by pressing a key on the keyboard (“H” or “L”, respectively). Although the purpose of the experiment was not about the subjects’ ability to discriminate surface heights, we chose the task in order that the subjects used a consistent exploration strategy [Lederman and Klatzky 1987]. Of the 10 subjects tested, S4–S10 were naive regarding the purpose of the experiment. They were asked to discriminate the relative heights of the two vertical planes, and were not informed that the two planes were rendered with different stiffness values. On each trial, we recorded the probe-tip position at a sampling rate of 1 kHz over a 30 mm window along the x-axis centered on the interpolation region between P_1 and P_2 (see Figure 5) when the subject stroked the surface and the probe tip remained inside the surface. For each experimental condition, data were collected for a total of 15 s. During the experiment, the subjects did not wear sound-blocking earphones because the PHANToM did not make any noise that could be used as perceptual cues for height discrimination.

3.1.3 *Data Analysis.* Given the 15-s penetration depth data collected for each subject and experimental condition, we first separated the data for the two stiffness values. The average and the standard deviation of the penetration depths were then calculated for each of the two stiffness values. In this way, we acquired 10 averages and standard deviations for the 10 stiffness values for each subject. According to the force-constancy hypothesis, the product of surface stiffness (k) and penetration depth (d) should be a constant: $f_p = k \cdot d$. To test whether force constancy held for each subject, we fitted a function of $d = f_p/k$ with f_p as the parameter on the average d versus k data using a least square error (LSE) estimation weighted by the inverse of the variances of d. We also computed an average penetration force for each experimental condition as a product of the corresponding average penetration depth and surface stiffness values.

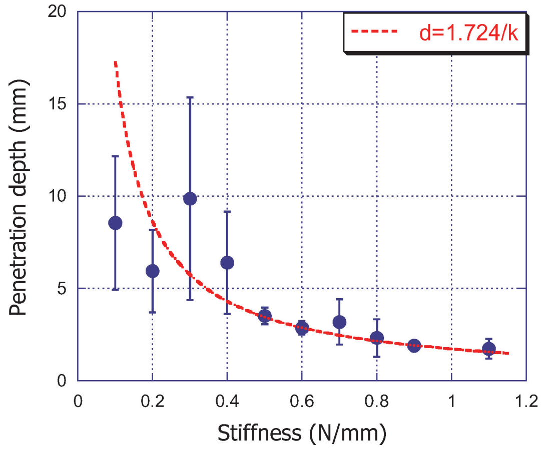

Fig. 7. Subject S1's data in Experiment I. Shown are the average penetration depths (filled circles) as a function of stiffness, along with error bars indicating ±1 standard deviations of the penetration depths. The dashed curve corresponds to the best-fitting d versus k curve with an estimated penetration force of 1.724 N.

Table I. Constant Penetration-Force Levels Estimated in Experiment I

Subject	Average f_p (N)
S1	1.72
S2	1.54
S3	1.29
S4	1.78
S5	2.49
S6	1.59
S7	2.65
S8	1.72
S9	1.99
S10	1.17
Mean	1.79
Standard deviation	0.47

3.2 Results

As an example of the collected data in Experiment I, the penetration-depth versus stiffness data for subject S1 is shown in Figure 7, along with the standard deviations of the penetration depth as error bars. The penetration depth decreased as surface stiffness increased, except for the two data points at the lowest stiffness values (0.1 and 0.2 N/mm). The standard deviations followed roughly the same decreasing trend, indicating that it was easier to maintain a consistent penetration depth when the surface was stiffer. Similar trends were exhibited by data from the other subjects. It should be noted, however, that the subjects chose to maintain different levels of penetration forces. Table I summarizes the best-fitting penetration-force levels for each subject. Overall, the penetration force levels estimated from the depth versus stiffness data ranged 0.87–2.23 N, with an average of 1.79 N ± 0.47 N.

A two-way ANOVA (pooled penetration-force versus stiffness data, with subject and stiffness as independent variables) revealed that both subject and stiffness were statistically significant factors for penetration force [$F(9, 81) = 8.63$, $p < 0.0001$ and $F(9, 81) = 8.40$, $p < 0.0001$, respectively]. The fact that subject was a significant factor confirmed that each subject maintained a unique level of penetration force, as shown in Table I. This was not surprising considering the many factors that

Table II. Results of Linear Regression Analysis Performed on Individual Subject's Penetration-Force Versus Stiffness Data. Slopes That Were Statistically Zero Are Denoted by 0 in the Table

	Stiffness Range Used in the Analysis (N/mm)								
	0.1–1.1			0.3–1.1			0.3–0.8		
Subject	$F(1,8)$	p	Slope (mm)	$F(1,6)$	p	Slope (mm)	$F(1,4)$	p	Slope (mm)
S1	6.91	0.0303	−1.15	1.76	0.2231	0	1.62	0.2719	0
S2	18.94	0.0024	−0.54	2.47	0.1673	0	0.11	0.7554	0
S3	25.93	0.0009	−1.06	2.22	0.1868	0	1.18	0.3378	0
S4	52.33	0.0001	−1.63	10.15	0.0189	−0.95	0.01	0.9104	0
S5	4.72	0.0616	0	0.11	0.7543	0	0.01	0.9351	0
S6	4.13	0.0766	0	0.65	0.4518	0	0.32	0.6003	0
S7	27.31	0.0008	−1.84	2.84	0.1429	0	0.03	0.8636	0
S8	0.89	0.3720	0	$\simeq 0$	0.9741	0	0.51	0.5141	0
S9	10.34	0.0123	−1.45	0.31	0.5967	0	$\simeq 0$	0.9769	0
S10	1.15	0.3146	0	21.37	0.0036	2.21	5.91	0.0720	0

could have affected the preferred penetration-force level such as the mechanical impedance of hands and arms [Hajian and Howe 1997], prior experience with force-feedback devices, and the subject's interaction style. The fact that stiffness was also a significant factor was inconsistent with the force-constancy hypothesis, because it seemed to suggest that the penetration force did not remain constant over the stiffness values tested in the experiment.

To examine this inconsistency further, we applied linear regression analysis on the individual subject's data, weighted by the inverse of the variance of the estimated penetration force. The results showed that only 4 of the 10 subjects (S5, S6, S8 and S10) maintained constant penetration forces over the entire stiffness range tested (0.1–1.1 N/mm), as indicated by an estimated slope that was statistically zero (see the columns under the heading "0.1–1.1" in Table II). When the data points at the two lowest stiffness values 0.1 and 0.2 N/mm were removed from the analysis, the estimated slope for most of the subjects was statistically zero (see the columns under the heading "0.3–1.1" in Table II). In particular, data from all the experienced subjects (S1–S3) resulted in statistically zero slopes over the stiffness range "0.3–1.1" N/mm. Only data from two of the seven naive subjects (S4 and S10) showed nonzero slopes. When the stiffness range under consideration was reduced further to 0.3–0.8 N/mm, excluding both very soft (0.1 and 0.2 N/mm) and very hard (0.9 and 1.1 N/mm) surfaces, the linear regression analysis showed that all subjects maintained constant penetration-force levels while exploring the surfaces (see the columns under the heading "0.3–0.8" in Table II). In summary, the results of the linear regression analysis suggest that most subjects maintained a constant penetration force while exploring virtual surfaces that were not too soft, and all subject did so when the surfaces were neither too soft nor too hard.

A by-product of Experiment I was the subjects' response to whether the right plane felt higher or lower than the left plane. Due to the small number of trials (= 25) that prevented us from obtaining statistically significant performance estimates, we will only discuss the general trend of the data here. For the stiffness pairs (0.1, 0.3) and (0.2, 0.4) N/mm, all subjects responded that the plane rendered with the lower stiffness value was perceived to be lower than that with the higher stiffness value in all trials. Recall that the two planes were rendered at the same height but with different stiffness values. According to the force-constancy hypothesis, a subject should always perceive the plane with the lower stiffness to be lower than the other plane. Subjects commented that the planes rendered with the stiffness value of 0.1 or 0.2 N/mm felt very soft and were characteristically "mushy." As stiffness increases, the percentage of responses that the plane rendered with a lower stiffness value felt lower

Table III. Results of Linear Regression Analysis Performed on the Penetration-Force Versus Stiffness Data Collected in Our Previous Experiment (Experiment II in Walker and Tan [2004]). Slopes That Were Statistically Zero are Denoted by 0 in the Table

	Stiffness Range Used in the Analysis(N/mm)					
	0.1–1.0			0.3–1.0		
Subject	$F(1, 8)$	p	Slope (mm)	$F(1, 6)$	p	Slope (mm)
S1	0.44	0.5263	0	3.40	0.1147	0
S2	18.54	0.0026	0.56	1.83	0.2247	0
S3	8.38	0.0200	1.29	0.08	0.7828	0

generally decreased, and reached a minimum of 69.2% for the stiffness pair (0.9, 1.1) N/mm. These results are again consistent with the force constancy hypothesis. According to Eq. (5), the distortion of the perceived surface height offset caused by the stiffness difference (Δh_d) decreases when the two stiffness values (k_1 and k_2) increase with a fixed difference (0.2 N/mm for our case) and the penetration force (f_p) stays constant. Therefore, it is more difficult to discriminate the relative height of adjacent planes rendered with higher stiffness values. Note, however, that two experienced subjects (S1 and S2) reported feeling the plane rendered with the lower stiffness to be lower for all stiffness pairs and trials.

3.3 Discussion

Similar results were also obtained with linear regression analysis of data obtained in our previous study (Experiment II in Walker and Tan [2004]). The main difference between the previous and the current experiment was that the surfaces used in our previous experiment were rendered with one stiffness value (randomly chosen from ten values in the range 0.1–1.0 N/mm), instead of with a pair of stiffness values used in the current experiment. Table III summarizes the results of linear regression analysis performed on the penetration-force versus stiffness data from the previous experiment. When the whole stiffness range was used in the analysis, two of the three subjects showed statistically nonzero slopes for the penetration force (see the columns under the heading "0.1–1.0" in Table III). When the lowest two values of stiffness were excluded from the analysis, all three subjects showed a statistically zero slope (see the columns under the heading "0.3–1.0" in Table III). Therefore, we conclude that the results from the current and the previous experiments substantiate the force constancy hypothesis which states that users maintain constant penetration forces to perceive surface topography, especially when the virtual surfaces were neither too soft nor too hard to the touch.

The results of Experiment I showed that the force-constancy hypothesis did not hold very well for soft surfaces rendered with stiffness values 0.1 and 0.2 N/mm. As alluded to in the Introduction, these stiffness values did not produce a well-defined surface in the sense that the forces that the subjects felt upon entering the surface boundary were very small, resulting in the perception of a very soft and "mushy" object. In these cases, penetration forces were relatively small compared to those used for planes rendered with higher stiffness values (e.g., see Figure 7). It is conceivable that the penetration depth required in order for the subjects to reach a comfortable penetration-force level was quite large, and therefore the subjects opted to stay at a reasonable penetration depth that corresponded to a relatively smaller force level. Once the surface stiffness increased to be larger than or equal to 0.3 N/mm, the subjects reported that they perceived a well-defined surface. The results of Experiment I showed that all subjects used a constant penetration force for the surfaces rendered with stiffness values in the range 0.3–0.8 N/mm. It was also found that the force-constancy hypothesis did not hold well for large stiffness values in the range 0.9–1.1 N/mm. We note that this result was mainly due to data from two naive subjects (S4 and S10). Whether the finding was tainted by the lack of experience with the force-feedback device used in this study cannot be ascertained without additional data collection.

A limitation of Experiment I is that the penetration-depth data were collected during a relatively short period of time (15 s). It is unclear whether the penetration force preferred by a subject would change after prolonged usage of a haptic interface due to fatigue or change in posture. It is also unclear whether the preferred force level might change from session to session. We hasten to point out, however, that the results of Experiment I should not dependent on the force-feedback device used in our experiment, as long as the stiffness values are lower than the maximum values that the device can deliver.

4. EXPERIMENT II: MANIPULATION OF SURFACE TOPOGRAPHY PERCEPTION

The purpose of Experiment II was to demonstrate that given the force constancy hypothesis that was shown to be true for a range of stiffness values, the perceived surface height difference can be manipulated based on the value predicted in Eq. (6). A pilot study was conducted to measure the discrimination threshold of surface-height difference perception. The results were used to design the experimental conditions used in Experiment II. We then measured subjects' ability to judge the relative heights of two surfaces, and compared the results to the predictions derived from the force-constancy hypothesis.

4.1 Methods

4.1.1 *Subjects.* Three subjects (S1–S3) participated in this experiment. They were the same experienced PHANToM users who participated in Experiment I. These subjects were preferred because of the demanding requirements imposed on the experimental task (see Section 4.1.4 for details).

4.1.2 *Surface-Height Discrimination Thresholds.* In order to select the experimental parameters for Experiment II, we first measured the threshold for surface-height discrimination; that is, the smallest height difference between two adjacent planes that can be reliably perceived by a user. For the measurement, the two planes P_1 and P_2 were rendered with the same stiffness value but different heights. Two stiffness values were used: 0.4 N/mm (a relatively soft surface) and 1.0 N/mm (a relatively hard surface without perceived instability [Choi and Tan 2004]). All three subjects were tested with the 0.4 N/mm stiffness value first.

A three-interval forced choice (3IFC) one-up three-down adaptive procedure [Leek 2001] was used to estimate the surface-height discrimination thresholds. On each trial, the subject stroked the virtual surfaces from left to right three times and judged the relative heights of the two planes. During one randomly selected interval, P_2 was rendered to be lower than P_1. During the other two intervals, P_2 was rendered to be higher than P_1. The subject's task was to indicate which interval contained a lower P_2 by entering the corresponding numeric key, "1," "2," or "3," on a keyboard. The initial value of the height difference ($|\Delta h|$) was set to 3 mm. This value was found to be clearly perceivable by all three subjects. The $|\Delta h|$ value was increased every time the subject made an incorrect response, and decreased after three consecutive correct responses. The resulting threshold corresponded to the 79.4% percentile point on the psychometric function [Levitt 1971]. The value of $|\Delta h|$ was initially changed by 4 dB (approximately 1.589 times larger or smaller than the previous $|\Delta h|$), and then by 1 dB (approximately 1.120 times larger or smaller than the previous $|\Delta h|$) after the first three reversals (reversal = when the $|\Delta h|$ value changed from increasing to decreasing, or vice versa). An experimental run was terminated after 12 reversals at the 1 dB level.

An estimate of the threshold was obtained by averaging the $|\Delta h|$ values at the 12 last reversals. To estimate the standard error of the estimated threshold, six estimates of the threshold were calculated from the six pairs of the last twelve reversals, and the corresponding standard error was calculated (see Brisben et al. [1999], p. 1550, 2nd column, for details).

The surface-height discrimination thresholds measured for the three subjects are shown in Table IV along with the standard errors. These thresholds were in the range 0.47–0.90 mm for the two stiffness

Table IV. Surface-Height Discrimination Thresholds

	Threshold ± Standard Error (mm)	
Subject	$k_1 = k_2 = 0.4$ N/mm	$k_1 = k_2 = 1.0$ N/mm
S1	0.73 ± 0.06	0.61 ± 0.04
S2	0.50 ± 0.05	0.47 ± 0.08
S3	0.90 ± 0.06	0.76 ± 0.03
Average	0.71	0.61

Table V. Parameters for the Three Conditions in Experiment II

	Condition		
Parameter	C1	C2	C3
h_1 (mm)	0	0	0
h_2 (mm)	1.5	2.0	2.5
k_1 (N/mm)	0.6	0.6	0.6
k_2 (N/mm)	0.333	0.333	0.333
f_p (N)	1.5	1.5	1.5
Δp_z (mm)	−0.5	0	0.5

values tested. The average thresholds obtained with the stiffness values of 0.4 and 1.0 N/mm were 0.71 and 0.61 mm, respectively. A two-way ANOVA (pooled data, with subject and stiffness as independent variables) showed that subject was a statistically significant factor [$F(3, 32) = 6.84$, $p = 0.0034$] but stiffness was not [$F(3, 32) = 1.57$, $p = 0.2196$]. A subsequent one-way ANOVA on individual data examined the effect of surface stiffness on the thresholds. Only S3's data suggested that the surface-height discrimination thresholds were statistically different for the two stiffness values.

4.1.3 *Experimental Conditions.* Three sets of surface parameters were used (see Table V). The values of h_1, k_1, and k_2 were kept at 0 mm, 0.6 N/mm, and 0.333 N/mm, respectively, in all three conditions. The value of h_2 was 1.5, 2.0, and 2.5 mm for conditions C1, C2, and C3, respectively. Since $\Delta h = h_2 - h_1 > 0$ in all the conditions, P_2 was always modeled to be higher (closer to the user) than P_1. However, by making P_2 softer than P_1 in all the conditions ($k_2 < k_1$), it was possible to predict a negative value for Δp_z, the difference between the probe-tip positions inside the two planes, according to the force-constancy hypothesis. In order to quantitatively calculate the value of Δp_z according to Eq. (4), it was necessary to restrict a user's penetration force to a known value. In this experiment, f_p was fixed at 1.5 N in all three conditions (see Section 4.1.4 for how this was accomplished). It now follows that in condition C1 where the modeled surface-height difference (Δh) was 1.5 mm, the perceived height difference (Δp_z) would be −0.5 mm. Therefore, subjects would perceive P_2 to be lower than P_1, resulting in a reversal in perceived relative height difference of the two surfaces. In condition C2 where $\Delta h = 2.0$ mm, Δp_z was predicted to be 0 mm. Therefore, we expected the subjects to be unable to discriminate the relative heights of the two surfaces. In condition C3 where $\Delta h = 2.5$ mm, Δp_z was 0.5 mm. The subjects were expected to correctly perceive P_2 to be higher than P_1.

4.1.4 *Procedure.* A one-interval two-alternative forced-choice paradigm was used to measure the subjects' sensitivity to the height difference between the two planes independent of their response biases. On each trial, P_1 was randomly presented on the left (with P_2 on the right) or right (with P_2 on the left) with equal probabilities. We denote these two stimulus alternatives with P_1P_2 and P_2P_1, respectively. The subject's task was to stroke the virtual surface from left to right once, and to report whether the plane on the right was perceived to be higher or lower than the one on the left, *while maintaining a constant penetration force around 1.5 N*. Three 100-trial runs were conducted per subject and per experimental condition. The order of the nine experimental runs (3 conditions × 3 100-trial runs) was randomized for each subject. Once data collection began, no feedback was available to the subject.

To help the subjects maintain the constant penetration force, visual feedback was provided to indicate the instantaneous penetration force level as belonging to the following three ranges: <1.3 N, 1.3–1.7 N, and >1.7 N. The ± 0.2 N force tolerance was based on an earlier study on force output resolution (see Table 4 in Tan et al. [1994]). Each subject went through training with trial-by-trial correct-answer feedback in order to maintain a constant penetration force within the range 1.3–1.7 N. On each trial, the subject's actual penetration force values were averaged over a 30 mm window along the x-axis centered on the interpolation region between P_1 and P_2 (see Figure 5). The trial was considered invalid if the average force and its standard deviation fell outside the range 1.3–1.7 N. Responses collected from the invalid trials were discarded. Additional trials were conducted so that each run collected responses from 100 valid trials.

4.1.5 *Data Analysis.* Experimental data were summarized by a 2-by-2 stimulus-response matrix. The rows corresponded to the two stimuli (top row for P_2P_1 and bottom row for P_1P_2), and the columns corresponded to the two responses (left column for "right plane felt lower" and right column for "right plane felt higher"). For each subject and each experimental condition, we pooled the 300 trials into one matrix and calculated the sensitivity index d' and response bias β (see Pang et al. [1991] for details on data processing). With this setup, a positive d' indicated that the subjects judged P_2 to be higher than P_1, a d' close to zero indicated that the subjects could not discriminate the relative surface height between P_1 and P_2, and a negative d' indicated that the subjects judged P_2 to be lower than P_1. The relatively large number of trials collected per condition (300 trials) was needed for estimating the standard deviation of d' [Wickens 2002].

4.2 Results

The values of the sensitivity index d' for each subject and each experimental condition from the experiment are shown in Figure 8, along with the error bars.[3] The general trend of the data was exactly as predicted by the force-constancy hypothesis as discussed in Section 4.1.3. In condition C1 where Δp_z was -0.5 mm, the average d' was -1.27. The result indicated that the subjects were able to discriminate the relative height of the two planes but that the higher plane P_2 was incorrectly judged to be lower. This occurred because P_2 was rendered to be much softer than P_1, and therefore the probe-tip penetrated P_2 more than P_1. When Δh was relatively small, it was possible that the probe tip was further away from the user inside P_2 than when the tip was inside P_1. This reversal in perceived surface height was similar to that observed with the "protein-on-mica" data set. In condition C2 where Δp_z was 0 mm, the average d' was 0.03, indicating that the subjects could hardly discern the height difference between the two planes. This result was quantitatively predicted by the force-constancy hypothesis based on the values of Δh, k_1, k_2, and f_p. In condition C3 where Δp_z was 0.5 mm, the average d' was 1.76, indicating that with a relatively large Δh, the subjects were able to correctly perceive the relative height of the two planes. However, due to the different stiffness values used to render P_1 and P_2, the perceived height difference in C3 was much smaller than that defined by Δh. Therefore, the results from all three conditions demonstrated a distortion in perceived surface topography caused by the different stiffness values used in rendering P_1 and P_2.

4.3 Discussion

The sensitivity index values shown in Figure 8 can be related to the surface-height discrimination thresholds reported in Section 4.1.2. We first average the surface-height discrimination thresholds

[3]The response biases were relatively small (average $= -0.04$). They are not reported here.

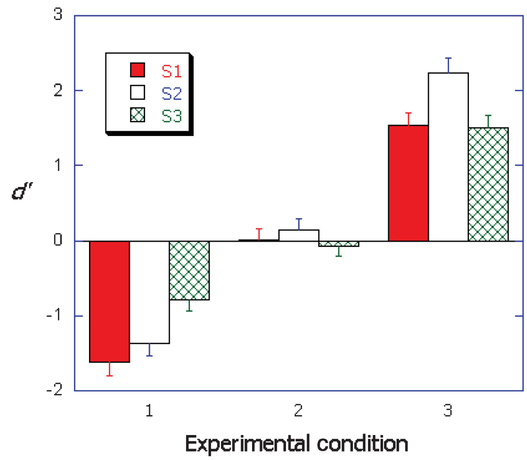

Fig. 8. Sensitivity index values measured in Experiment II.

obtained using stiffness values of 0.4 and 1.0 N/mm to arrive at an average threshold of 0.66 mm (see Table IV). In order to relate this threshold to sensitivity index values, we recall that this threshold corresponded to the 79.4% percentile point on the psychometric function. When a discrimination experiment is conducted with a one-interval two-alternative forced-choice paradigm as was the case in the main Experiment II, a d' value of 1.0 corresponds to a percent-correct score of 69%, assuming no response bias [Macmillan and Creelman 1994]. Although it is always difficult to directly compare results from two experiments conducted with different methods, we can nevertheless estimate that a surface-height difference of 0.66 mm should result in a d' value in the range 1.0–2.0.[4] Secondly, we assume that sensitivity index d' is linearly related to the proximal surface-height difference Δp_z. We can then estimate the slope of the d' versus Δp_z function to be in the range 1.0/0.66–2.0/0.66 mm. It then follows that for the three conditions tested in Experiment II (see Table V), the expected values of d' would be in the range $(-1.52, -0.75)$ for C1 and $(0.75, 1.52)$ for C3. Thirdly, we average the d' values across the three subjects tested in the main Experiment II and obtain a d' of -1.26 for C1 and 1.76 for C3. The average d' in absolute values is roughly consistent with those predicted by the average surface-height detection threshold of 0.66 mm. Fourthly, we note that the d' values predicted from Δh (without taking into account the difference in k_1 and k_2) would have been in $(2.27, 4.55)$ for C1 and $(3.79, 7.58)$ for C3, which are clearly inconsistent with the d' range of $\pm(0.75, 1.52)$ predicted from the average surface-height discrimination threshold of 0.66 mm. The d' values predicted from Δh are much higher than the actual d' values obtained in Experiment II, lending further evidence that subjects did not detect surface height differences based on Δh alone. Therefore, the observed d' values in Figure 8 were *quantitatively* consistent with the predictions of the force constancy hypothesis.

The results in Figure 8 can also be compared to those obtained in an earlier study (Experiment III in Walker and Tan [2004]) where we used two surfaces with a constant height offset but varied the stiffness of one surface. Our earlier study demonstrated a reversal in perceived surface height difference by varying the surface stiffness alone. In the current Experiment II, we used two surfaces with fixed stiffness values but varied the height of one surface. The results of both experiments clearly demonstrated

[4]The lower-bound 1.0 means that we expect the d' value corresponding to the average threshold of 0.66 mm to be higher than 1.0 because the threshold corresponds to a point on the psychometric function that is higher than 69%. The upper-bound 2.0 is a ballpark estimate that conveys the idea that although we expect the d' value corresponding to the 0.66 mm threshold to be higher than 1.0, it should not be too much higher than 1.0.

that the perceived relative height of two adjacent planes was determined by the surface topography as well as the surface stiffness values, thereby providing further support of the force-constancy hypothesis. They also suggest the feasibility of a compensation rule based on the manipulation of surface height and/or stiffness values in order to achieve a target percept of surface topography.

5. GENERAL DISCUSSION

In this paper, we presented a real-world application where multiple parameters (surface topography and stiffness maps of "protein-on-mica" data obtained with an SPM) were rendered visually and haptically for data perceptualization. With a widely used penalty-based haptic rendering method, we presented feedback forces that were proportional to the product of the local stiffness and the penetration depth of the probe tip of a haptic interface. We observed that the perception of the surface topography was sometimes distorted. We proposed a force-constancy hypothesis to explain the phenomenon. Our force-constancy hypothesis stated that when interacting with virtual surfaces, the user of a force-feedback device maintained a constant penetration force. The trajectory of the probe tip was then regarded as representing the surface topography of the haptic virtual environment. Experiments were conducted to investigate the force-constancy hypothesis. In Experiment I, we recorded the penetration depths of the probe tip while the user stroked two surfaces with equal height but different stiffness values. We found that the user penetrated the softer surface more, and the data could be quantitatively explained by the force-constancy hypothesis, especially when the virtual surfaces were neither too soft nor too hard to the touch (i.e., within 0.3–0.8 N/mm). In Experiment II, we demonstrated that given two adjacent surfaces, the higher but softer surface could be perceived to be lower, at the same height, or higher than the other surface depending on how much higher it was than the other surface. The experiment was designed based on the surface-height discrimination thresholds of 0.71 and 0.61 mm, measured with surfaces rendered with the stiffness values of 0.4 and 1.0 N/mm, respectively. The results of Experiment II were again consistent with the predictions of the force-constancy hypothesis. Therefore, both experiments supported the force-constancy hypothesis. The experiments reported in this paper were modified from our earlier study on the same topic [Walker and Tan 2004]. The current experiments were designed to be more relevant to real-world applications such as the scientific perceptualization of the "protein-on-mica" data set.

The significance of our work can be appreciated in the context of developing complex haptic (and visual) virtual environments. Many applications, such as scientific data perceptualization, surgical simulation, and teleoperation, demand increasingly complex virtual environments. It is important for the designer of the haptic virtual environment to be aware of the possible interplay among rendering parameters that can result in distorted perception of object properties. Our work also underscores the important role played by a user's interaction style on the perception of the virtual environment. In their pioneering work, Lederman and Klatzky first coined the term "exploratory procedures" to classify the stereotypical actions users often take in order to optimize the sensing of real-object properties [Lederman and Klatzky 1987]. Our work can be viewed as an extension of Lederman and Klatzky [1987] toward a more perceptually accurate haptic representation of virtual-object properties by taking into account the user's exploratory actions.

Our future work will focus on incorporating appropriate compensation rules into haptic rendering of virtual objects. For example, in the simple case of simultaneously rendering surface height and stiffness maps, we can adjust the surface height map using the force-constancy hypothesis. Specifically, assuming that the constant penetration force and the rendering parameters are known ahead of time, we can compensate for the nonzero height perception distortion term (Δh_d) by pre-warping the modeled height (Δh) (See Eqs. (5) and (6)). Instead of rendering Δh, we can render $\Delta h - \Delta h_d$. Then, the proximal height difference $\Delta p_z = \Delta h$, and the user will perceive the correct relative height difference between two adjacent surfaces. The alternative approach of adjusting the stiffness map is less desirable because accurate surface hardness perception will no longer be preserved when the user taps the virtual surfaces. The challenge lies in the fact that different users tend to use different penetration forces when interacting with a virtual surface, and it is conceivable that the same user might adopt different force levels from time to time. A successful compensation rule will therefore have to take into account the penetration force employed by a user for a given application at a given time. This can be accomplished by monitoring the penetration-force level exerted by the user in real time. We will explore ways to incorporate compensation rules into haptic rendering systems for scientific data perceptualization.

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