

IMECE2007-41504

A PSYCHOPHYSICAL MODEL OF MOTORCYCLE HANDLEBAR VIBRATIONS

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ABSTRACT

In this study, we developed a perception-based quantitative model to relate broadband vibrations transmitted through a motorcycle handlebar to a rider's hands. The test apparatus consisted of the handlebar of a motorcycle rig assembly driven by a computer-controlled actuator. Participants were instructed to hold the handlebar and maintain a sitting posture as they would while riding a motorcycle. In Exp. 1, psychophysical detection thresholds for 10 participants were estimated at ten test frequencies between 20-300 Hz using a two-interval one-up two-down adaptive procedure. The interpolated threshold vs. frequency function specified the minimum acceleration required before a user could perceive the vibration at a particular frequency. In Exp. 2, participants were asked to rate 15 representative handlebar vibrations using a magnitude estimation procedure. The vibration patterns were measured on an actual motorcycle handlebar while the motorcycle traveled at speeds ranging from 25 to 75 mph. Several weighting functions, including the ISO-5349 standards, were applied to the broadband vibration signal in the frequency domain to estimate the total vibration energy by summing up all weighted components. The best weighting function, in the sense that the estimated total energy correlated linearly with the subjective magnitude ratings obtained in Exp. 2, were based on the detection threshold data obtained in Exp. 1. Specifically, the strength of each vibration component was calculated relative to the human detection threshold at the same frequency, thereby taking into account human sensitivity to vibration signals at different frequencies. The resulting weighting function can be applied to other recorded vibration signals to predict user rating of perceived vibration intensities.

I. INTRODUCTION

Currently, ISO standards are widely used to estimate the total acceleration (and therefore, the energy) of a broadband

mechanical stimulus presented on the skin (see: ISO 5349-1:2001(E)). The associated ISO weighting factors reflect the assumed importance of different frequencies in causing injury to the hand and are based on the mechanical and absorption characteristics of the skin [1]. Recently, Giacomini *et al.* measured weighting functions for mechanical vibrations at several amplitude levels on a steering wheel by asking participants to maintain equal-sensation levels of band-limited vibrations [2]. Giacomini *et al.* also measured the annoyance thresholds for the corresponding steering wheel setup (see also [3]). The weighting functions of Giacomini *et al.* and ISO 5349 are slightly different, mainly due to the different experimental conditions such as holding posture, contact area and direction of vibrations. Similarities in the two functions indicate that the absorbed energy is perhaps correlated with the energy perceived by human users.

Psychophysical studies on tactile perception have shown that the perceived intensity of a vibration is determined by summing up energy components above the human detection threshold or the Pacinian weighted function [4-6]. The detection threshold serves as a baseline of the human tactile perception above which vibration is detectable by a participant. Many psychophysical studies have measured position detection-thresholds of single-tone vibrations with and without an interfering stimuli (e.g., maskers) and have shown that the detection-threshold function varies with experimental conditions, such as frequency, age, body site, contact area, direction of vibrations, etc., [7-11].

Thus, in order to quantify the relationship between the subjective amplitudes (or the perceived intensity) and the physical vibration energy, it is necessary to first determine the threshold levels of the holding posture. The subjective amplitudes are then determined by assuming the critical band model defined in [5] that implies that the total perceived intensity is equal to the sum of the energy of individual frequency components above the detection threshold curve. Numerically, this can be written as

$$\frac{A_s^2 \cdot f_s^2}{T_{f_s}^2 \cdot f_s^2} = \sum_i \frac{A_i^2 \cdot f_i^2}{T_{f_i}^2 \cdot f_i^2} \quad (1)$$

where A is the physical amplitude of the vibration at frequency f . T is the amplitude of the vibration at the threshold level for frequency f . The ratio $A_s^2/T_{f_s}^2$ is the perceived intensity of the vibration. The ratio $A_i^2/T_{f_i}^2$ is the energy of the individual component above the detection threshold. The formulation of perceived intensity in Eq. (1) is very similar to that of the total acceleration presented in ISO-5349 as the frequency-weighted function (see Equation (A.1) in ISO 5349-1:2001(E)), i.e.

$$a_{hw} = \sqrt{\sum_i (W_{hr} a_{hr})^2} \quad (2)$$

where W_{hr} is the weighted factor at frequency f_i . a_{hr} is the acceleration of each frequency component and a_{hw} is the frequency weighted acceleration of the total acceleration. Comparing Eqs. (1) and (2), the weighted factors of Eq. (2) should be the normalized inverse of the sensitivity function defined as the threshold amplitude function, i.e., $W_{hr} = \text{norm}(1/T_{fi})$, see [5] and [2].

In the present study, we compared subjective rankings of human users experiencing predefined handlebar vibrations. The predefined handlebar vibrations were measured at different riding speeds and gear levels of a commercially available motorcycle and were computer-controlled in our present setup. The objective was to form guidelines for acceptable motorcycle handlebar vibrations and to determine the relationship (function) between the physical vibration energy and the perceived vibration intensity. We measured detection thresholds of single-frequency vibrations presented through the handlebar. The subjective rankings of broadband handlebar vibrations were determined by ‘‘Jury testing’’, in which participants felt and ranked vibrations on a scale of 0-100. The rankings were normalized among participants and sessions. We compared subjective rankings with three weighted functions using: 1) ISO standards, 2) data from existing psychophysical literature, and 3) the weighted function determined in the present study. The *position* detection thresholds commonly used in the literature (see e.g., [8]) are converted to *acceleration* detection-threshold functions by multiplying $(2\pi f)^2$ at the respective frequency f Hz so that the results can be readily compared with the ISO weighting function.

The rest of the paper is organized as follows. In Sec. 2, we discuss the experimental setup and hardware used for the present study. Methods for two psychophysical experiments are presented in Sec. 3 and results of the experiments are discussed in Sec. 4. A general discussion in Sec. 5 concludes the paper.

II. EXPERIMENTAL SETUP

The test apparatus was based on a 2005 Victory Vegas motorcycle (Polaris Industries, Inc., Medina, MN, USA). A partial frame, partial fuel tank, seat, triple clamp and handlebar from

production Victory Vegas motorcycles were supplied by Polaris Industries. These components were assembled by the Ray W. Herrick Laboratories machine shop personnel, with instruction from the authors.

Test Apparatus

The motorcycle frame, seat and fuel tank were delivered in one assembly. Rails with foot posts were provided, but could not be mounted immediately, as the rear support on each rail was intended to be bolted to the motorcycle engine. Instead, an aluminum base was constructed to support the frame and rails, as shown in Fig. 1.



Figure 1. Actual motorcycle sitting posture (left) and test apparatus assembled in our laboratory (right).

The support base was used to control the elevation of the front of the frame and seat to match specifications provided by the Polaris Industries. In order to isolate the handlebar from the frame during testing, it was desirable to decouple the handlebar support and the frame support. A triple clamp and the handlebar were positioned relative to the front edge of the motorcycle frame. The base of the triple clamp was specified to be at the same angle as the steering tube at the front of the frame, with a gap of 20 mm between the underside of the triple clamp and top of the steer tube. A base was fashioned to support the triple clamp and painted black, as seen in Fig. 1. The angle between the handlebar support base and ground was adjustable, to match the adjustments available on the motorcycle seat height. The base of the handlebar support is weighted with sandbags and may be assumed to be rigid and hence not contributing to the dynamics of the handlebar and the triple clamp. The handlebar coordinate system is specified by Polaris as shown relative to the left handlebar grip in Fig. 2.

Actuation and Sensing

Actuation of the handlebar was achieved by a Motran AFX-70NS (Motran Industries Inc., Valencia, CA, USA) linear inertial force actuator (IFA). The IFA was secured to the handlebar by a custom clamp. The clamp gripped the handlebar at two points between the risers and allowed rotation about a line parallel to ground and perpendicular to the direction of the motorcycle. The angle of the IFA relative to ground was determined by analysis of data provided by Polaris. Acceleration data from production motorcycles was converted from Cartesian to Spherical

coordinates. The statistical distribution of the acceleration data was then found for each coordinate and the relative occurrences of each value of the polar and azimuth angles were calculated. The strongest contribution to the data was in the x-y plane, aligned with the x-axis. Therefore, the IFA was fixed to actuate the handlebar in the x-y plane. The IFA was periodically checked with a bubble level to maintain parallel to ground. The IFA was powered by a QSC two-channel 70 W audio amplifier (QSC Audio Products, Inc., Costa Mesa, CA, USA). In practice, the IFA exhibited many harmonics when generating tonal signals. At low amplitudes, these harmonics dominated the fundamental frequency, most likely due to friction between the moving shaft and the thrust bearings.



Figure 2. Sensor and actuator mounted on the handlebar. The axes denotes coordinate systems for the handlebar.

Acceleration of the handlebar was measured by a PCB 356B18 triaxial accelerometer (PCB Piezotronics, Inc., Depew, NY, USA). The accelerometer was mounted to the left handlebar just inside of the grip, as shown in Fig. 2. A hole was drilled and tapped parallel to ground and a threaded stud screwed into the handlebar. The accelerometer was screwed onto the stud with a single brass washer between the handlebar and the accelerometer. The washer was added so that when hand tightened, the accelerometer axes were aligned with the Polaris coordinate system. The accelerometer cable was connected to three PCB 480E09 single channel ICP signal conditioners with the appropriate gain settings. For detection threshold experiments, it was necessary to set the amplifier gain to 100. For absolute magnitude estimation experiments, a gain of 1 was used. With the signal conditioner, the x-axis of the triaxial accelerometer was calibrated to have a gain of $0.0995 \text{ V}\cdot\text{sec}^2/\text{m}$ or equivalently, 0.976 V/g .

Controller Implementation Hardware

The controller hardware consisted of a desktop computer running MATLAB, a dSPACE feedback controller and a SigLab signal and system analyzer. The SigLab 40-22a 4-input, 2-output dynamic signal and system analyzer (Spectral Dynamics, Inc., San Jose, CA, USA) was a convenient tool for both system analysis and performing human testings. The SigLab input

channels were used to record the SigLab output and all three axes of the triaxial accelerometer. The function of the SigLab unit varied with the task. During analysis of the system, the SigLab unit was used for measuring the frequency response of the system and calibration of the accelerometers. In these cases, one output channel was connected directly to the audio amplifier, bypassing the dSPACE controller. During human testing the SigLab unit was used to generate a reference stimulus and record accelerometer outputs. The reference stimulus was connected to an ADC channel of the dSPACE feedback controller.

Feedback control was provided by a dSPACE ACE1104 rapid prototyping kit (dSpace Inc., Wixom, MI, USA). Unlike typical vibration control applications, control system performance for vibration perception study is significantly more stringent. During detection threshold experiments, it is important to have precise control of acceleration amplitude at the target frequency while suppressing harmonics as well as other vibration modes to at least 20 dB below the nominal detection threshold. For broadband test signals, both magnitude and phase control are essential in recreating the measured handlebar stimuli at different amplitude.

System Modeling and Controller Design

The frequency response of system was obtained by performing step sinusoidal frequency sweep. Since the human subject grips the handlebar during testing, a set of 4 frequency responses were collected from subjects of different height and weight gripping the test rig at normal gripping posture and force. The maximum and minimum frequency response envelopes were used to determine the plant model uncertainty bounds and the nominal plant model. Given the stringent performance requirement and the limit actuation, it is evident that a single controller will not be able to accomplish all of the tasks. A different controller was designed for each tonal or broadband test signal using a frequency shaping robust control approach. These controllers were compiled into C code using MATLAB and the dSPACE Control Desk software. The C code was automatically downloaded to the dSPACE controller, which was connected to both the PCB signal conditioners and the QSC audio amplifier for real time controller implementation.

Individual controllers were designed for each tonal test frequency and one controller was designed for the broadband stimulus condition. All controllers were designed as two degree-of-freedom controllers with both a feedback controller and feedforward controller. The feedback controllers were created using frequency-shaped LQR, with state weighting filters and input weighting filters varying by stimulus. For tonal stimuli, the state-weighting filter was typically chosen to have a peak at the test frequency with sharp roll-off immediately after. For the broadband stimulus, the state-weighting filter was selected as a low-pass filter. This helped to alleviate the harmonics present in the IFA. The feedforward controller was designed as a zero-phase error tracking controller, dependent on the current feedback controller in use.

III. EXPERIMENTAL METHODS

Participants

Five males and five females (age 22-44 years old, average 27.2 years old) participated in the present study. All participants were right-handed by self-report. Four of the participants had either participated in other haptic perception experiments before and/or were involved in developing the hardware/software system used in the present study. They were regarded as the “experienced” users. The rest of the participants were regarded as “inexperienced.”

Signal processing

There were total of 15 test signals provided by Polaris. The signals were measured at several riding speeds and gear levels as shown in Table 1. Also shown are the stimulus ID’s assigned to each test signal. The data file for each test signal contained three-axis acceleration data for about 30 seconds and was sampled at 65536 Hz. Each test signal was preprocessed before being sent through the controller. The data files were analyzed to identify the dominant axis of the vibration by transforming the Cartesian coordinate system data into a spherical coordinate system. It turned out that the x-axis vibrations were dominant in the data files. The resultant of the three-axis data was low-pass filtered, re-sampled, and high-pass filtered to match the operating range of the mechanical actuator in the range 20-300 Hz without distorting the data in the operating range. For each of the 15 test signals, one second long sample from the 30 second long data was used as the reference signal for the magnitude estimation experiments.

Table 1. Test signals and the corresponding speed and gear levels

Stimulus ID	Speed (MPH)	Gear
1	25	2 nd
2	25	3 rd
3	35	3 rd
4	35	4 th
5	45	4 th
6	45	5 th
7	55	4 th
8	55	5 th
9	55	6 th
10	65	4 th
11	65	5 th
12	65	6 th
13	75	4 th
14	75	5 th
15	75	6 th

Experiment 1: Detection Threshold Experiment

The participant sat comfortably on the motorcycle rig. They rested their feet on the foot-rests and held the handlebar as they would riding an actual motorcycle (see Fig. 3). A cardboard screen was placed between the participant and the handlebar to

block visual cues of hand movements. Two LEDs were attached horizontally on the cardboard in front of the participant to indicate the start and stop time of two stimulus intervals in the experimental procedure. The left LED turned on at the start of the first interval and turned off at the end of the first interval. The right LED turned on and off at the start and end of the second interval. Two foot pedals were attached to the footrests, one on each side, so the participant can enter a response without having to taking their hands off the handlebar.

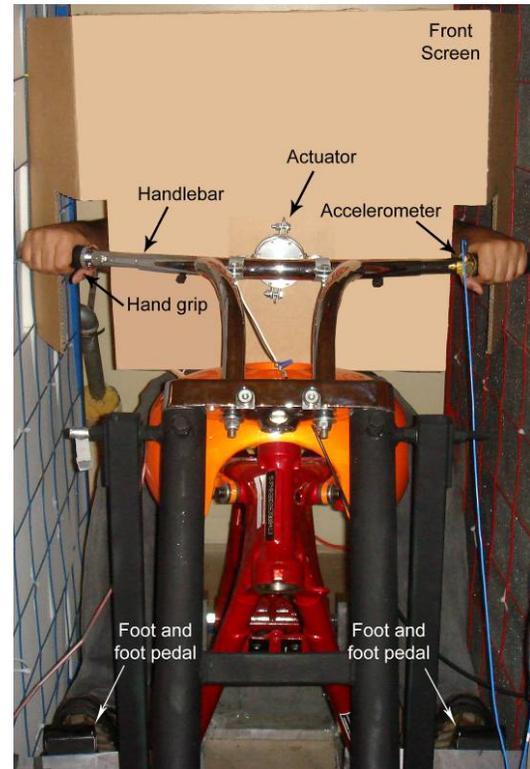


Figure 3. Experimental setup with a participant holding the handlebar.

Thresholds were obtained for ten test frequencies: 20, 30, 75, 100, 150, 170, 200, 230, 260, and 300 Hz. The order of the test frequencies was randomized for each participant. The duration of the stimulus was fixed at 1 sec with Hanning windowing (100-msec rise and fall) to reduce transient effects. Thresholds were obtained by a two-interval, forced-choice, one-up two-down adaptive method (see [12] and [13] for reviews on adaptive methods). On each trial, the participant was presented with two 1-sec long stimulus intervals with a 300-msec inter-stimulus interval. One randomly selected interval contained the test stimulus and the other one contained no signal. The participant's task was to indicate which one of the two intervals contained the stimulus by pressing the corresponding foot-pedal (left foot-pedal for the test stimulus in the first interval and right foot-pedal for the test stimulus in the second interval). In the one-up two-down adaptive method, two consecutive correct responses led to a reduction in the stimulus intensity and one incorrect response an

increase, both by a predefined step size. Thresholds obtained this way correspond to the 70.7 percentile point on the psychometric function. The initial stimulus amplitude was chosen to be well above the expected detection threshold level. The step size was initially set to 4-dB (for faster convergence) and then reduced to 1-dB (for finer resolution) after the first three reversals (a reversal occurred if the stimulus amplitude changed from increasing to decreasing, or vice versa). A test series was terminated after 12 reversals at the 1-dB step size. The last 12 reversals (six peaks and six valleys) at the 1-dB step size were used to calculate the position detection threshold (mean of the averages of the six peak-valley pairs) and its standard deviation (from the six averages) for each participant at each test frequency.

Visual and audio cues marked the start and end of each interval. The participant was required to enter a response after the end of the second interval. A new trial started right after the participant's response. Participants wore headphones that played pink noise to block auditory cues from the mechanical actuator. The participants were allowed to feel the vibrations before each test condition. At the end of the experimental session, the stimulus intensity as a function of trial number was plotted. The participant was asked to repeat the series if the data failed to converge to a threshold level upon visual inspection by the experimenter. Each series took about 4-6 minutes. Participants were asked to take a 5-minute break between test conditions. The entire experiment took about 90 minutes per participant, which was broken into several sessions of 1 to 5 test frequencies, depending on the participant's availability. No correct-answer feedback was provided during the experiment.

Experiment 2: Subjective Magnitude Experiment

The methods of subjective magnitude experiments are very similar to the ones discussed in the literature [14, 15]. The participants sat comfortably on the motorcycle rig. They rested their feet on the foot-rests and held the handlebar as they would riding an actual motorcycle (see Fig. 3). A cardboard screen was placed between the participant and the handlebar to block visual cues of hand movements. They wore ear-phones with pink noise to mask possible auditory cues emanating from the mechanical actuator.

Before the main experiment, participants were given instructions on how to rate the perceived vibration levels from the handlebar. They were asked to assign a number from 0 to 100 to each vibration segment such that 0 corresponded to "no vibration was felt" and 100 "the maximum vibration". Training was provided before the main experiment. During the training session, participants were presented with test stimulus having relatively low- and high-level vibrations. Participants were instructed to assign a relatively small number to the low-level vibration and a relatively large number to the high-level vibration. Participants were permitted to practice for as long as they wished in order to get ready for the main experiment.

The 15 one-second test stimuli were used repeatedly in the main experiment that consisted of 120 trials. The complete experiment was divided into 4 sessions of 30 trials. In each

session, each one of the test stimulus was presented twice in a random order. On each trial, the participant was presented with a test stimulus and was asked to respond with a number ranging from 0 to 100. The participants gave verbal responses and the experimenter recorded the number on a computer keyboard.

The numerical ratings were normalized by dividing the rating of each test stimulus by the average rating of each session and then multiplying the result by the overall average of all ratings in the experiment. The perceived intensity of the vibration was predicted by using Eq. (1) and the threshold levels found in the present study and from Bolanowski et al.'s study [8]. The discrete threshold levels were interpolated by a first order approximation to obtain threshold approximation at each integer frequency. The recorded sensor measurements of each test stimulus were first transformed into the frequency domain by taking the FFT of the measured samples. For 1-second long data, we obtained frequency information at discrete integer frequency. The magnitude at each frequency was divided by the interpolated threshold level at that frequency, squared and summed across the frequency range 15 to 300 Hz to obtain the total perceived intensity. The result was then converted into dB units by taking the $10 \times \log_{10}(\bullet)$ of the total perceived intensity. Similarly the total acceleration in the vibration was calculated by using Eq. (2) and converted into dB units by taking the $20 \times \log_{10}(\bullet)$ of the total acceleration.

IV. RESULTS

Absolute Detection Thresholds

The mean threshold levels for the ten participants are shown in Fig. 4. Error bars represent the standard error of the mean thresholds. For comparison, the thresholds derived from Bolanowski et al. [8] are also shown.

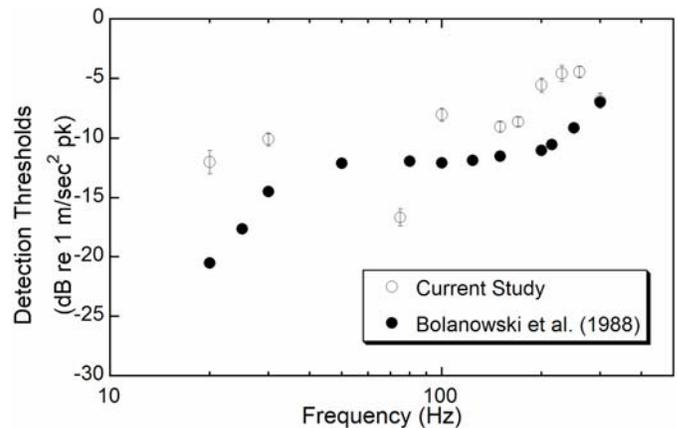


Figure 4. Comparison of Human Detection Threshold levels.

From Fig. 4, the threshold levels are slightly elevated compared to those derived from Bolanowski et al. [8], except for the datum point at 75 Hz. The difference at 75 Hz was mainly due to the nonlinear response of the handlebar vibrations at that

frequency region and can be considered an outlier. Since the threshold value obtained in this study is consistently higher than those derived from Bolanowski et al. [8], the Bolanowski's threshold curve was assumed to be the baseline of human perception for the handlebar.

Subjective Magnitude Estimation

The total perceived intensity of vibrations used in the subjective magnitude experiments were calculated by using the acceleration threshold levels derived from the position threshold levels of Bolanowski et al.'s study [8]. The sensitivity function of Bolanowski et al. [8] was evaluated and compared with the weighted functions of ISO-5349 and Giacomini et al.'s study [2] and are shown in Fig. 5. It can be seen that the ISO weighted factors are different from those in [2], which were very similar to those derived from Bolanowski et al.'s data [8]. From the results of the previous section, it appears that the Bolanowski threshold level provides a reasonable weighting factor for the perceived vibration intensity.

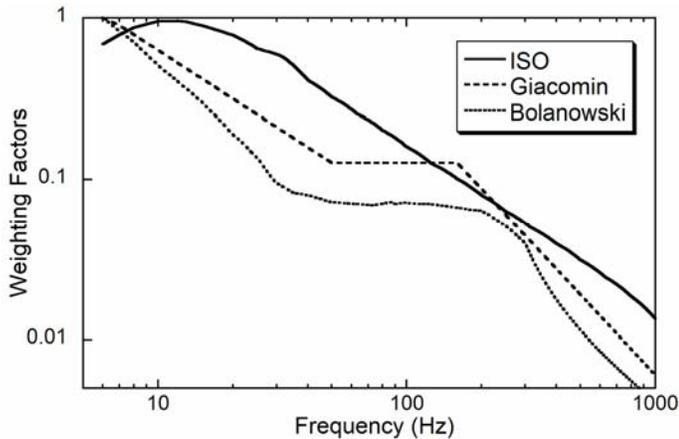


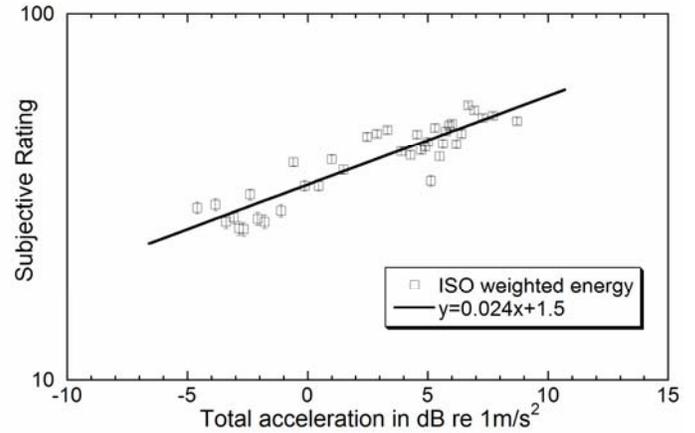
Figure 5. Comparison of Weighting Factors.

Prediction of Perceived Vibration Levels

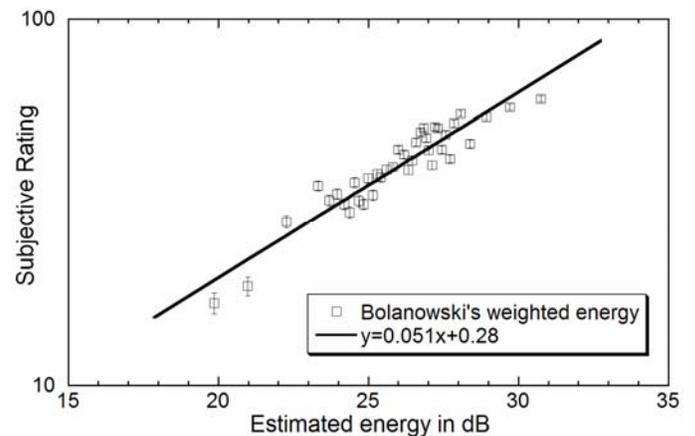
The normalized subjective ratings are plotted against the energy evaluated by both ISO (Eq. 2) and Pacinian weighted function (Eq. 1) in Fig. 6(a) and Fig. 6(b), respectively.

The energy data for each trial (i.e. 1200 trials = 30 trials/sessions \times 4 sessions/subject \times 10 subjects) was first sorted by increasing value of dB and grouped into bins of 30 trials. The average and standard error of normalized rating of each bin was calculated and plotted against the mean dB value of the bin. With the ISO weighted factors (Figure 6(a)), the data points fit well along a straight line. Two data point on the lower extreme energy were left out due to data temporal failure of the data acquisition equipment. Eliminating these two data points, the rest of the data was regressed by a straight-line model as shown in Figure 6(a). The model fits ($p < 0.001$, $r^2 = 0.82$) with slope 0.024 and intercept 1.5. Similar analysis was done with the energy evaluated by Bolanowski's Pacinian weighted function (i.e., detection threshold levels). Eliminating two extreme left data points, the rest of the data was regressed by a straight-line model in Figure

6(b). The model fits the data very accurately ($p < 0.001$, $r^2 = 0.88$) with a slope of 0.051 and an intercept of 0.28. It should be noted that the coefficient of determination in the Pacinian function is larger and closer to 1 than that of the model deduced from ISO weighted factors.



(a)



(b)

Figure 6. Normalized subjective rating vs. total perceived acceleration in dB by (a) ISO weighted function and (b) Bolanowski's detection thresholds.

V. DISCUSSION

The goal of the present study was to develop a model for predicting the perceived vibration level associated with a motorcycle handlebar. With such a model, physically measured vibration levels can be processed and used to predict human response without further psychophysical experiments. In this section, we present two models, one from the ISO weighted function and the other from the Bolanowski et al. (1988) threshold. The models are then correlated with the subjective magnitude estimation obtained in Experiment 2. Results presented in Fig. 6 showed that the straight line fit by the Pacinian weighted function had less variability than that by the ISO weighted function. Thus, the weighted functions defined in ISO

must be revised in order to include the perceived intensity of physical vibrations. With the psychophysical model estimated in this study, a new handlebar mount can be designed without future testing with human participants. The estimated perceived intensity of the vibrations can be estimated by using the Bolanowski's detection threshold levels and converted into theoretical subjective rankings by using the function,

$$\text{ranking} = 10^{(0.051 \times \text{dB} + 0.28)}$$

Acceleration threshold levels estimated in Experiment 1 with the present setup were similar to the ones reported in Bolanowski et al. [8] but were consistently shifted upward. This was due to the inability of the controllers to stimulate the vibrations only in the dominant x-axis direction. The present study controlled the handlebar vibration in one direction when the actual translational vibration during testing was two dimensional. We did not study the potential impact of coupling on the threshold and subjective magnitude result. Future work will consider the impact of two dimensional vibrations and use two actuators to have a better control of the handlebar vibration amplitudes and directions.

ACKNOWLEDGMENTS

This project was supported in part by Polaris Industries under award No. 2006-13-4265. The authors wish to thank Fritz Peacock and Bob Brown at the Herrick Laboratories for their assistance with the hardware assembly of the experimental apparatus.

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