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Roughness rendering by sinusoidal friction modulation: Perceived intensity and gradient discrimination*

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Abstract— **Four experiments used a programmable ultrasonic friction-modulation device to explore parameters that might be candidates for roughness modulation and to assess whether spatially modulated texture gradients could be discriminated by their direction of change. Candidate roughness parameters included frequency, amplitude and two implementations of local friction variation (noise). Amplitude, frequency, and noise all moderated roughness. Observed interactions between parameters could reflect peripheral or attentional effects. Directional discrimination of graded frictional changes was well above chance, but did not indicate accessible and reliable differentiation that could readily be exploited in use contexts.**

I. INTRODUCTION

By means of purposive exploration, humans perceive structural and material properties of objects. Texture is a highly salient material property that is encoded by lateral motion between the object's surface and the skin or an exploring tool [1]. Texture is multi-dimensional, including, for example, bumpiness, roughness, or slipperiness [2]. Such textural features have regular variations in intensity that allow people to make judgments along absolute or relative scales. Theories of the neural mechanisms that transduce textural judgments have recognized two channels: spatial coding, using the outputs of slowly adapting mechanoreceptors, or temporal coding, based on rapidly adapting mechanoreceptors [3]. Deep receptors of the latter type, called Pacinian Corpuscles, are known to produce neural signals that entrain to the vibration of the skin during stroking and are proposed to provide a frequency code analogous to that of the cochlea in hearing [4].

Haptic technologies have been developed in various forms with the goal of rendering textural variations like those found in real objects and surfaces. Spatially based approaches have made use of pin arrays. Force-feedback devices model textures by delivering model-determined interaction forces and torques to a tool held by the user. Another approach is to modulate the friction on a glass surface by ultrasonic [5] or electrostatic effects [6], [7], [8], which has the advantage of stimulating the skin directly rather than by pins or through a tool. The magnitude of the friction effect, and the restriction that it requires the user to be in motion, suggest that the resulting textures are temporal rather than spatial. Vardar et al. [9] found direct evidence for the contribution of the PC channel to the electrovibration signal.

Evaluations of texture rendering suggest that many approaches succeed in delivering intensive variation. Sinusoidal textures have been used in a number of studies, because of the parametric manipulations in frequency and amplitude that they afford. For example, Unger et al. [10] used a magnetic-levitation device to simulate sinusoidal textures explored with a frictionless probe over a broad range of frequencies and found that roughness ratings were highly correlated with the power of the force signal, consistent with the idea that roughness is essentially an intensive dimension. Whereas the model in [10] included only forces normal to the texture plane; another magnetic-levitation device was used to render interaction torques, which further modulated roughness [11].

Frequency manipulations were used exclusively to vary textures in [10, 11]. Less attention has been directed to the effects of amplitude on perception of virtual textures. Hwang et al. [12] investigated the effects of frequency and amplitude on perceived intensity of vibration delivered through a mobile device. Both variables were found to affect intensity, with the ultimate report being predicted by the physical power of vibration absorbed by the hand (cf. [10]). Strohmeier and Hornbaek simulated textures by vibrating a slider and also found that amplitude modulated ratings of roughness, bumpiness, and sharpness [13].

Local variation in the signal is also a candidate for roughness modulation with virtual textures. Vardar et al. [9] compared wave-form shapes generated through electrovibration and found that square waves were perceived as rougher than pure sine, triangle, or saw-tooth shapes. Their results indicated that rate of change of the normal and tangential forces, rather than peak force magnitude, was the predictor of roughness. A detection advantage for square waves over sinusoids was also found with another haptic rendering device [14], again indicating sensitivity to local intensity variation.

In the present experiments, we used a programmable ultrasonic friction-modulation device [8] to explore parameters that might be candidates for roughness modulation. These included frequency, amplitude and two implementations of local friction variation, constituting noise. Given indications that perceived roughness magnitude could be modulated by these variables, two subsequent experiments investigated whether parametric variations in the form of spatially modulated texture gradients could be discriminated. Specifically, participants were asked to report the direction of

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change as increasing or decreasing in roughness. This question was previously addressed with the Senseg electrostatic device; gradient direction discrimination was found in that study to be particularly effective at the low end of the device's intensity range [15]. However, the limited programmability of the Senseg did not allow systematic exploration of potential parameters that could deliver graded changes over a spatial region. The current device allowed us to render sinusoidal gratings with increasing or decreasing amplitudes at different frequencies, with and without local friction variation.

II. TEXTURE RENDERING DEVICE

The apparatus used in all experiments was a variable friction device that reduces friction, as described in [15], shown in Figure 1. The active display is a glass panel 104 mm in length that renders a 1-D friction variable by reducing the friction from the resident level of the glass. The position sensing acuity of the display is 5.3 μm, and commanded friction based on finger position is refreshed at 8333 Hz. Figure 2 shows the device response to a test swipe of the finger over 10 mm of a commanded sine-modulated friction gradient with a spatial frequency of 1 cycle / mm. To characterize the response, lateral force and finger position were measured over time. After low-pass filtering at 500 Hz, measured force was related to the corresponding commanded friction reduction by position. As shown, the device response is nearly linear.

III. EXPERIMENT 1: ROUGHNESS OF SINUSOIDAL FRICTION MODULATED BY AMPLITUDE AND "ZERO-DROPS"

In this experiment, we explored the effect of the amplitude of sine-modulated friction on perceived roughness. In addition to varying the amplitude of the wave, we introduced a manipulation intended to increase noise in the sine contour, which gives rise to a subjective impression of jumpiness or buzz. Noise essentially constituted signal loss: it was introduced by randomly deleting the profile of the sinusoid at some percentage of points along the wave and substituting the midpoint. Of interest was whether the noise factor would impact on roughness judgments, and if so, whether it would further moderate amplitude effects.

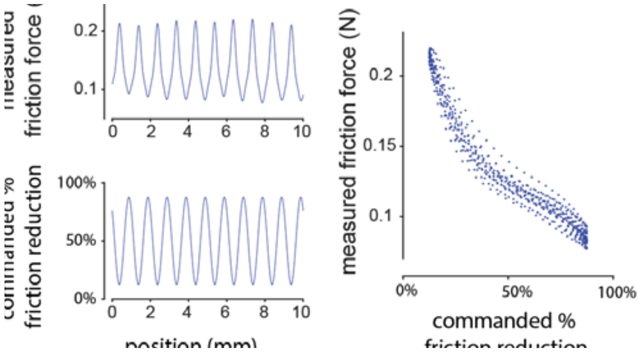


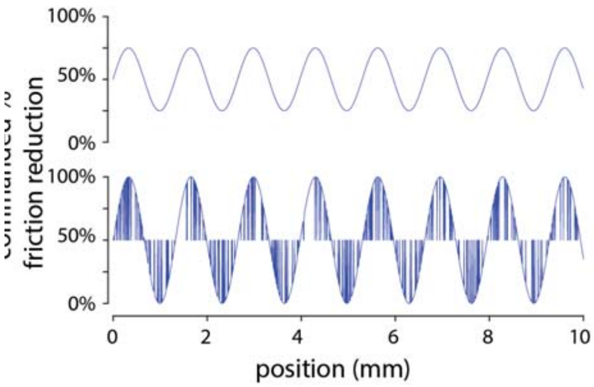
Figure 2. Device characterization. Left bottom: commanded % friction reduction; top: corresponding measured friction force. Right: Measured friction in relation to commanded friction. See text for details.

A. Method

The participants were 12 students at Carnegie Mellon University who received credit for a Psychology Department requirement. All gave informed consent under a University protocol. Participants wore sound-abating headphones.

The stimuli were 40 different waveforms resulting from the combinations of three variables: two amplitudes (50% and 100% of the device's full friction range), five spatial frequencies in cycles per mm (0.38/mm, 0.75/mm, 1.13/mm, 1.51/mm, and 1.89/mm; i.e., reciprocal of spatial period in mm), and four percentages of zero-drops, defined as randomly selected points along the waveform where the amplitude dropped to the midpoint of the range (0%, 16%, 32% and 50%). Figure 3 shows commanded friction levels and measured device-generated lateral forces for pure versus perturbed sines.

The task was free magnitude estimation. Participants explored each texture as desired, then reported a value in whole numbers, decimals, or fractions, representing its subjective roughness intensity. The only requirement was that no number be negative, and stronger roughness should lead to a greater number. The 40 textures were randomly presented in each of two successive blocks, and the experimenter recorded th



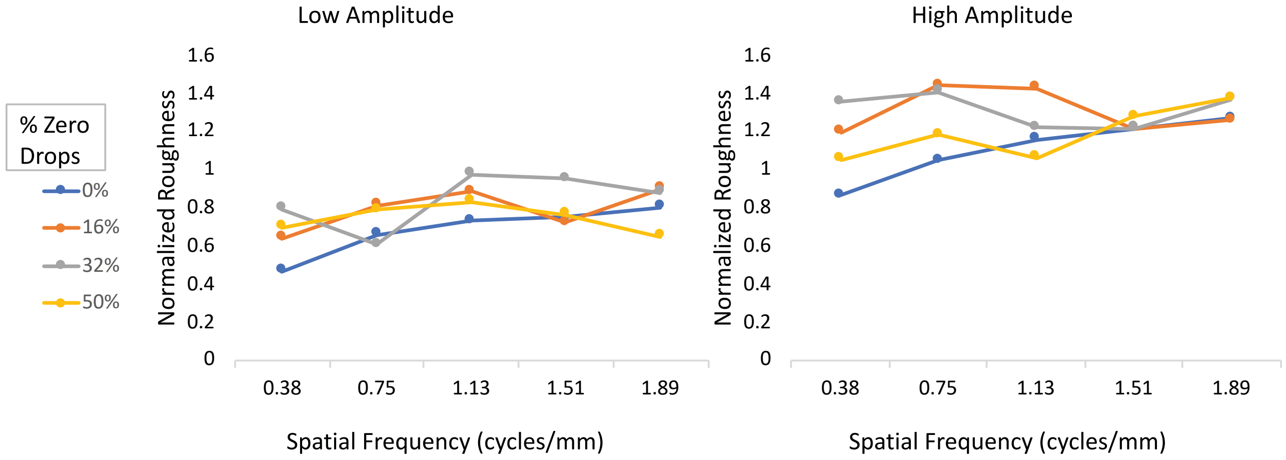


Figure 4. Normalized roughness in Experiment 1 for low and high amplitude sine waves by spatial frequency, perturbed by various percentages of "zero-drops" to the midpoint of the amplitude range.

Each texture was repeated twice in the study, over two successive blocks. The experimental trials were preceded by 5 practice trials representing the range of variations that would be tested.

B. *Results*

For purposes of equating the response range, each participant's data were normalized by dividing by that person's grand mean (equalizing subject means but not ranges). The resulting data are shown in Figure 4. Because the roughness responses tended to saturate at the high end of the participant's range very quickly with increases in amplitude, statistical analyses were performed on log-transformed normalized roughness values. An ANOVA on the factors of frequency, amplitude, and % zero-drops revealed effects of amplitude, $F(1,11) = 51.28$, $p < .001$, $\eta_p^2 = .82$, frequency, $F(4, 44) = 4.13$, $p = .01$, $\eta_p^2 = .27$, % zero-drops × frequency, $F(12, 132) = 3.94$, $p < .001$, $\eta_p^2 = .26$, and the three-way interaction, $F(12, 132) = 2.01$, $p = .03$, $\eta_p^2 = .16$. The latter reflects a tendency for the noise level to impact only the lower frequency waves. The main effect of % zero-drops approached significance ($p = .075$). An ANOVA on the lowest frequency alone, with factors of amplitude and % zero-drops, confirmed a significant effect of zero-drops, $F(3, 33) = 6.19$, $p = .002$, $\eta_p^2 = .36$, that did not interact with amplitude, $p = .62$. Amplitude also produced a significant effect, $F(1, 11) = 35.46$, $p < .001$, $\eta_p^2 = .76$.

C. *Discussion*

The experiment showed that roughness ratings increased strongly with amplitude. A doubling of the amplitude range increased the average normalized roughness by approximately 60%. The effects of the other factors were substantially weaker: A five-fold increase in frequency produced only a 20% increase in roughness, and a change from 0% to 50% zero-drops raised roughness by 8%. Overall, however, there was a marked reduction in roughness for waves that were low in frequency and noise, and the data indicate that all three factors implemented here are candidates for varying friction-induced roughness. Experiment 2 again investigated roughness magnitude, with a novel implementation of noise.

IV. EXPERIMENT 2: MAGNITUDE ESTIMATION TO SINUSOIDAL FRICTION PERTURBED BY NORMALLY DISTRIBUTED NOISE

In this study, the noise manipulation of Experiment 1, in the form of signal loss, was replaced by noise in the form of signal variability. Specifically, a random noise value, drawn from a normal distribution with a zero mean and specified standard deviation, was added to each point along the frictional sine wave.

A. *Method*

The participants were 12 students from the same population as Experiment 1. The stimuli were 40 sine wave-forms resulting from the combinations of the five frequencies used in Experiment 1 (.38/mm, .75/mm, 1.13/mm, 1.51/mm, 1.89/mm), 2 amplitudes (75% and 50% of the device's full friction modulation range) and 4 values of sigma (the standard deviation of the normal noise added to the amplitude, corresponding to percentages of the peak amplitude of the sine wave of 10%, 25%, 40%, and 55%). These values were chosen to minimize device saturation; any values drawn that exceeded machine capacity were converted to the maximum or minimum value. Figure 5 shows the commanded signal waveform for two stimuli.

Each texture was repeated twice in the study, over two successive blocks. The 80 experimental trials were preceded by 5 practice trials representing the range of variations that would be tested. The participant's task was identical to that of Experiment 1.

B. *Results*

An ANOVA on log-transformed normalized ratings with factors frequency, amplitude, and sigma showed effects of frequency, $F(4,44) = 6.85$, $p < .001$, $\eta_p^2 = .38$,and sigma, $F(3, 33) = 8.92$, $p < .001$, $\eta_p^2 = .45$, and an interaction, $F(12, 132) = 2.31$, $p = .011$, $\eta_p^2 = .17$. The effect of amplitude was not significant, $p = .23$, nor was its interaction with frequency, $p = .12$, sigma, $p = .47$, or the 3-way interaction, $p = .14$. As no effect involving amplitude reached significance, in contrast to

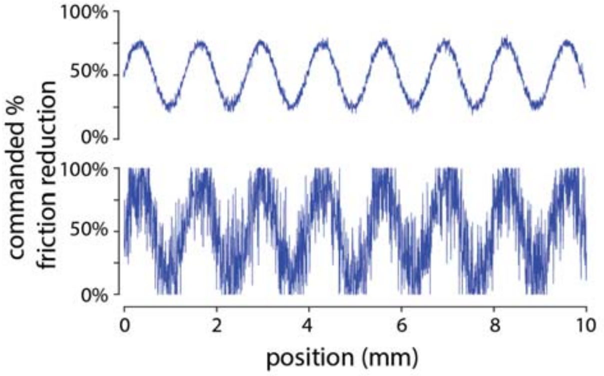


Figure 5. Upper trace: Commanded wave with frequency = 0.75/mm, amplitude = 50%of device range, sigma = 10% proportion of peak amplitude. Lower trace: Commanded wave with frequency = 0.75/mm, amplitude = 75% of device range, sigma = 55% proportion of peak amplitude. Deviations are capped beyond the device range.

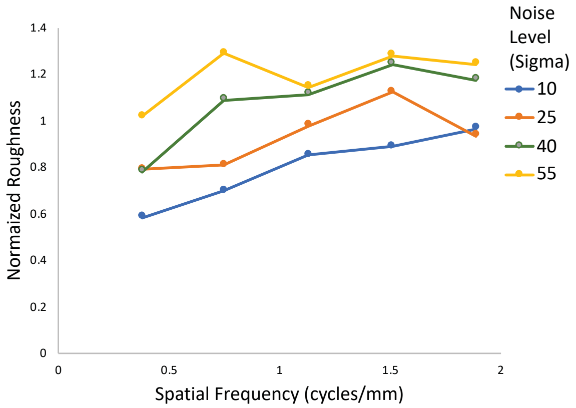


Figure 6. Normalized roughness in Experiment 2 for the manipulated levels of frequency and noise.

Experiment 1, the noise and frequency effects are shown in Figure 6 averaged across amplitude.

C. Discussion

Experiment 2 showed substantial effects on roughness magnitude of both frequency and noise. Five-fold increases in frequency and noise produced 40% and 50% increases in rated roughness, respectively. In contrast to Experiment 1, the effect of amplitude variation was not significant; a 25% increase from low to high amplitude raised roughness by 21%. Note, however, that the manipulated amplitude difference here was much smaller than the doubling of amplitude in the previous study. Comparing the two experiments does indicate that the relative effects of frequency, noise, and amplitude are contextual. In Experiment 1, amplitude dominated the roughness ratings. Here, where amplitude was held at relatively high levels, the effects of frequency and noise appear to have dominated variations in perceived roughness.

V. EXPERIMENT 3: GRADIENT DISCRIMINATION FROM AMPLITUDE MODULATION

In Experiments 3 and 4, we asked whether the variations in perceived roughness that were evidenced from manipulations of sinusoidal parameters and noise could be used to create graded friction changes that could be discriminated by the spatial direction of change. Klatzky et al. [15], using a different device with electrostatic friction, demonstrated strong discrimination of the directions of friction gradients. Here we extended the same approach to piezoelectric friction rendered as sine-wave changes. Experiment 3 used amplitude modulation alone at two frequencies; Experiment 4 added a noise perturbation.

A. Method

The participants were 11 students from the same population as previously.

Each stimulus gradient consisted of a sine wave at a given frequency, with amplitude either increasing or decreasing linearly across the full spatial width of the display, from a low amplitude spanning 25% of the device's full friction amplitude range to a high amplitude spanning 100%. Gradients were tested at two frequencies, .25/mm and .50/mm.

The task was a two-alternative forced-choice; participants judged on each trial whether a gradient became rougher or smoother in the rightward direction. Roughness was further described in terms of intensity or bumpiness. Exploration speed was freely controlled by the participant, using the dominant hand. The response was made on the computer keyboard, with the 0 button for a rightward increasing gradient and a 1 for the reverse.

Each of the 2 frequencies was tested in independent blocks, the order of which alternated between participants. Within each of the two blocks there were 10 trials with each gradient direction, randomly ordered. The trials were preceded by 2 practice trials.

B. Results

Performance was measured in terms of the d′ sensitivity statistic from signal detection theory [16]. In brief, d′ measures the distance between the means of two hypothesized normal distributions, one constituting signal and one noise, in standard-deviation normalized units. We made the arbitrary choice of defining signal as a rightward increase in amplitude; the reverse assignment does not change the d′ statistic. Accordingly, a hit (true positive) was defined as a rightward increase in amplitude that was correctly identified, and a false alarm (false positive) was a leftward increase misidentified as rightward. The d′ values were 1.10 (s.d. across individual subjects = .60) and 1.25 (s.d. = .42) for high and low spatial frequencies, respectively. The two values did not differ significantly by t-test, $p > .05$. Both d′ values had confidence intervals that excluded zero.

C. Discussion

We achieved some success in rendering directionally recognized friction gradients by amplitude modulation. The discrimination levels are neither at chance nor error free. For purposes of evaluating the extent of discrimination, it is useful to consider the values associated with Cohen's d statistic [17] (standard-deviation-normalized difference between empirical means, in contrast to the theoretical means used to compute d′): A value of d near .02 is considered small; 0.5 medium, and .8 or above a large size. By this measure, then, substantial discrimination was achieved. Another approach is to ask what

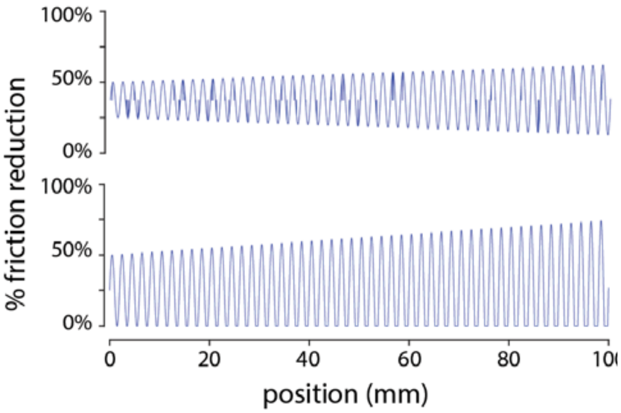


Figure 7. Upper trace: Commanded gradient with frequency 0.5/mm, amplitude range = 50%, zero drops present. Lower trace: Commanded gradient with frequency 0.5/mm, amplitude range = 100%, no zero drops present. The lower bound caps friction.

the difference means in the signal-detection model, under the assumption of normal distributions underlying the sensation of a gradient direction. Here, a d′ of 1.0 means that if we consider the perceptual sensation of a rightward increase in magnitude, 84% of the sensations from amplitude-decreasing gradients lie below the mean sensation of increasing gradients. In short, by either approach to evaluating effect size, the average effects show good discrimination, but the value of d′ near 1 allows room for error that could have impact in a task context.

Beyond the reported standard deviations, it is also worth noting that there were strong variations across individuals. Of the 11 participants, ten had a d′ of at least .5, and nine achieved an average d′ of .95 or above (with a maximum of 1.64), but one performed near chance (d′ = .19). The correlation between d′ for the two frequency conditions was .62, which was significant, t(9) = 2.37, p = .04, indicating that performance on one frequency modestly predicted the other. We discuss individual variations further below.

VI. EXPERIMENT 4. GRADIENT DETECTION WITH REDUCED AMPLITUDE AND ZERO-DROPS

Experiment 4 explored other directions to increase directional sensitivity of gradients. To assess whether saturation of the intensity sensation might mask more subtle variations, we tested amplitude variation over a relatively small range of device capability. This was compared to a gradient that spanned the full range subject to a cutoff that precluded extreme friction levels. We also tested whether zero drops, or sudden returns to the midpoint of amplitude, might be used as a binary cue to help discriminate direction. Different percentages of drops occurred on the two halves of the gradient. These percentages were small relative to the noise manipulation used in Experiment 2, again to preclude roughness saturation. (Indeed, a preliminary experiment testing a gradient of zero-drops that co-varied with a gradient of amplitude suggested that participants could not perceive directional changes at all under the given noise.)

A. *Method*

The participants were 15 students at Carnegie Mellon University who received credit for a Psychology Department requirement. All gave informed consent under a University protocol.

The stimuli were again amplitude-modulated sine-wave gradients, with frequency held constant at .5/mm. The amplitude increased or decreased from left to right linearly across the full spatial width of the display, within a sub-range of the full friction modulation capacity of the device and centered on the mean of that range. Large-range gradients spanned the full range of the device, subject to a cutoff at the high-friction value (low friction reduction), as shown in Figure 7. Small-range gradients spanned half of the total amplitude range. In addition to the range variable, the gradients varied in whether there were sudden zero-drops to the mean amplitude. If zero-drops were present, they occurred at 50 locations in the lower-amplitude half of the gradient, and at 20 locations in the higher-amplitude half (corresponding to proportions of .005 and .002 of the spatial data points in that half, respectively). This has the effect that the low-amplitude half of the display also has more variability or “buzz.” With 2 range levels and zero-drops present or absent, there were 4 possible stimuli in the experiment, each tested in two directions. Sample commanded waveforms are shown in Figure 5.

The procedure was as in Experiment 1. Each of the four grating types was tested in independent blocks, the order of which varied between participants by a Latin Square. Within each of the two blocks there were 10 trials with each gradient direction, randomly ordered. The trials in each block were preceded by 2 practice trials.

B. *Results*

One subject who reported not being able to feel the gradients was eliminated from the data set a priori. As in Experiment 1, performance was measured in terms of the d′ sensitivity statistic. The mean d′ values were 1.11 (s.d. = .82) and 1.07 (s.d. = .92) for the small and large ranges with no zero-drops, and .96 (s.d. = .87) and.87 (s.d. = .99) for the corresponding capacities with zero-drops. These values did not differ significantly, and all d′ values had confidence intervals that excluded zero.

As in Experiment 3, there was a tendency for participants to perform consistently across the conditions. Specifically, four of the six inter-participant correlations between pairs of conditions showed values of r > .40, and principal components analysis indicated that a single factor accounted for 60% of the variability in the data attributable to conditions.

While single participants tended to be consistent across conditions, there were substantial individual differences. Considering the d′ on data pooled across conditions, 2 of the 14 participants performed essentially at chance. The remainder showed values at least at the level of .5 (a medium level from [17]), and the best two participants performed at least a standard deviation above the mean of the group as a whole with d′ levels of 1.88 and 2.39.

C. *Discussion*

The results were consistent with those of Experiment 3, which had used a different range of amplitudes. Specifically, the aggregate values indicate strong discrimination of gradient direction, but by no means error-free performance. The principal cue appears to have been the graded amplitude, as adding differential variability in the two halves of the display did not increase discrimination performance. From the two studies together, we conclude that amplitude variations constitute a moderately successful gradient cue. Additional correlated cues, or different manipulations, might enhance its effects. Another similarity to Experiment 3 is the tendency for reliable individual differences, evidenced by correlations between conditions but striking variations across the participant population.

VII. General Discussion

These exploratory studies have pointed to variables that affect perceived roughness magnitude from frictional variations induced piezoelectrically on a plate of glass. Amplitude, frequency, and noise all moderated roughness. However, the effects of these variables depended in part on the context of others. This could result from peripheral effects like saturation of the roughness sensation, or from central effects like attention to one parameter at the expense of another.

Given our initial demonstrations that perceived roughness varies with parametric frictional manipulations, we tested whether the variations permit discrimination of the spatial direction of graded frictional changes. Here the answer is mixed: Discrimination was well above chance, and by generally adopted guidelines, quite strong. However, if the goal in a user context is to provide rapid and reliable discrimination, the results are less encouraging. We also note that we did not constrain the amount of time participants spent exploring, and our more cautious participants tended to explore on the order of seconds.

These studies open several questions to be considered in future work. An obvious direction is to further explore the stimulus dimensions and parametric variations along them that will moderate perceived roughness. Another direction is to determine whether training might substantially enhance sensitivity to frictional roughness variations, and in this way improve discrimination. A related issue is whether there are optimal levels of speed and force for stimuli of this type, and whether they can be induced by training. A challenging but important further problem is to better characterize what sensations underlie the present roughness reports. One can ask how friction-based roughness compares to that of real surfaces, and whether reports are based on some general intensive dimension or reflect other properties such as stickiness or buzz, which users sometimes mention as perceptual features.

We also point to issues that are inherent to friction-based roughness, only some of which can be addressed by current technology. The strong individual variations that we observed might reflect factors out of the control of the device designer, such as hand moisture or intrinsic perceptual sensitivity. Another issue that is more amenable to technological solution arises because friction becomes evident only under motion, which means that the perceiver's hand changes its location in space. If location is not well tracked by the device, or if the lag in tracking is long, then the direction of a gradient is particularly compromised. Consider, for example, the case where position tracking lags well behind friction rendering. A user might arrive at the end of a sweep over the plate and, due to long lag, only then receive the friction levels intended for the start! The result would be a reversal of the perceived gradient direction. Despite these limitations and reservations, the present research, we believe, offer promise and suggestions for roughness rendering by friction.

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