Adaptive tuning of an electrodynamically driven thermoacoustic cooler

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The commercial development of thermoacoustic coolers has been hampered in part by their low efficiencies compared to vapor compression systems. A key component of electrodynamically driven coolers is the electromechanical transducer, or driver. The driver’s electroacoustic transduction efficiency, defined as the ratio of the acoustic power delivered to the working gas by the moving piston and the electrical power supplied, must be maintained near its maximum value if a high overall system efficiency is to be achieved. Modeling and experiments have shown that the electroacoustic efficiency peaks sharply near the resonance frequency of the electro-mechano-acoustic system. The optimal operating frequency changes as the loading condition changes, and as the properties of the working gas vary. The driver efficiency may thus drop significantly during continuous operation at a fixed frequency. In this study, an on-line driver efficiency measurement scheme was implemented. It was found that the frequency for maximum electroacoustic efficiency does not precisely match any particular resonance frequency, and that the efficiency at resonance can be significantly lower than the highest achievable efficiency. Therefore, a direct efficiency measurement scheme was implemented and validated using a functional thermoacoustic cooler. An adaptive frequency-tuning scheme was then implemented. Experiments were performed to investigate the effectiveness of the control scheme to maintain the maximum achievable driver efficiency for varying operating conditions.


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I. INTRODUCTION

Research efforts have been made over the past 20 years to develop thermoacoustic heat pumping systems for air-conditioning, refrigeration, and other cooling applications. These systems do not require refrigerants. They use environmentally benign, inert gases as the working fluid. They are mechanically simple, requiring no sliding seals or lubrication. At this stage of their development, the performance of thermoacoustic systems has been poor relative to traditional vapor compression systems. However, existing models based on linear theory suggest that better performance is achievable. One key to improving performance relates to the maximization of efficiency.

The overall efficiency of an electrodynamically driven thermoacoustic heat pumping device is a function of heat pumping effectiveness and electroacoustic transduction efficiency. In this study, methods to maintain optimal electroacoustic efficiency for varying operating conditions were investigated. This efficiency is defined as the ratio between the acoustic power delivered and the electrical power supplied. The acoustic power is defined as the time-averaged product of the acoustic pressure over the driving piston and the piston volume velocity. The electrical input power is the time-averaged product of the voltage fed to the coil and the corresponding coil current. Both the acoustic and the electric power are very sensitive to the phase difference between the pressure and velocity signals, or between the voltage and current signals. The acoustic power, and thus the electroacoustic transduction efficiency, is highly frequency dependent. In practical applications as well as in the laboratory, it is imperative to maintain peak efficiency in order to minimize power consumption, and thus operating costs. In operation, variations in refrigeration loads cause the temperature of the working gas and the heat exchangers to vary, which alters the acoustic load impedance and thus the input electrical impedance. This in turn changes the frequency of optimal electroacoustic efficiency of the driver, referred to here as the “tuned frequency.”

The design of thermoacoustic coolers has been the object of many previous studies. Minner et al. performed systematic design optimization for a few different applications. Grant investigated the use of a passive mass element to replace the gas-filled half-wavelength resonance tube. The selection of electrodynamically driven thermoacoustic refrigerators was discussed recently in a study by Wakeland, which investigated the effects of key parameters and frequency on driver efficiency. The optimization processes utilized in these and other studies may be designated as a priori or “off-line” tuning since they do not require the knowledge (in real-time) of measured quantities. In practice, thermoacoustic phenomena often involve nonlinear, time-varying processes affected by many operating and design factors. Even carefully designed and fabricated devices are subject to uncertainty, and variability which cannot be accounted for using existing linear models. From a control system viewpoint, the system needs to adapt to transient variations of the plant. It must be able to tune itself without user intervention, a posteriori or after the initial tuning pro-
cess, utilizing dynamic signals fed to and processed by a digital processor. This is referred to as “on-line” tuning.

On-line monitoring and control of electroacoustic efficiency to maintain peak efficiency for varying operating conditions is possible. Garrett developed a passive method of frequency compensation for changes in the temperature of the working gas by placing a gas mixture and an adsorbent within the resonator. Hofler used a method for real-time acoustic power measurements. In Hofler’s work, a voltage-controlled-frequency (VCF) signal generator was used along with a proportional-plus-integral (PI) controller to tune the driving frequency for acoustic resonance, i.e., for pressure and velocity to be in phase on the piston. Another PI controller was used to maintain a constant output pressure amplitude. Issues such as filtering and overload protection were considered. The system was implemented using analog circuits.

In the present study, a method for acoustic power measurement similar to that in Ref. 9 was used. Real-time efficiency monitoring and control schemes were implemented using a DSP (digital signal processor). The electroacoustic efficiency is not a directly measurable quantity. It is usually calculated “off-line” from measured pressure, piston velocity, coil voltage, and current data. Either time or frequency domain methods can be used to calculate the acoustical and electrical power. The approach here utilized digital signal processing techniques to calculate the instantaneous acoustic and electrical power of the driver for a prototype thermoacoustic cooler on-line. The instantaneous driver efficiency was supplied to a real-time adaptive frequency tuning algorithm to ensure maximum driver efficiency for changing and uncertain operating conditions.

The remainder of the paper is organized as follows. A brief description of the prototype thermoacoustic cooler is given in the next section, followed by the description and the analysis of the on-line driver efficiency measurement technique. It is then shown that there can be a significant efficiency loss if the system is operated at resonance, and that a direct measurement of efficiency is required. The real-time adaptive algorithm for tuning the electromechanical driver is finally described. Some implementation details are included in the hope that they may be of some assistance to other workers in the field.

II. DESCRIPTION OF THE THERMAOACUSTIC COOLER

The thermoacoustic cooler is shown schematically in Fig. 1. The system was driven by a moving-magnet CFIC Model B-300 electromechanical transducer. The driver was designed to deliver 300 W acoustic power at 33 Hz, with an electroacoustic transduction efficiency of 70% and a maximum displacement of 6 mm. The alternator was comprised of radially extending neodymium magnet arms mounted to a central plunger (the moving element) and copper wound laminated iron pole pieces (the stationary element), which extend inward from the outer periphery. The outward-directed magnet fingers (each comprised of two magnets in contact) were thus separated by gaps, which were filled by the inward-directed laminated iron fingers. The magnetic flux pathlines in such devices go circumferentially through a magnet finger and a pole piece, radially up through the coil to the outer periphery, circumferentially to the other pole piece adjacent to the magnet finger, radially down through the coil and back to the magnet finger. Each pole piece then supports two flux paths, one for each adjacent magnet finger. The alternating current causes a fluctuated force that moves the magnet-mounted plunger axially back and forth. The plunger-mounted piston was mounted within a bore, with extremely tight clearance seals extending over a length many times the piston displacement. This minimized “blow down,” or gas leakage between the rear and front cavities. The plunger was balanced and self-centering, and supported by a single flexure. Leaf springs were added to increase the resonance frequency of the driver in-vacuo to 154 Hz. The driver was fully enclosed, and the static pressure equalized across the piston.

The measured specifications of the driver are listed in Table I. The equivalent linear parameters of the driver, described for example in Beranek’s text, were measured. They were found to depend on amplitude and frequency. The values listed in Table I are thus only approximations over a narrow range of piston displacements. More information on the driver characteristics is available in Refs. 11 and 12.

The other components of the system included (1) a stack, made of rolled up polyester film, (2) finned-tube copper heat exchangers, and (3) a pressure vessel. The working...
fluid was a 55% helium—argon mixture with a mean pressure up to 2 MPa. A single-frequency sinusoidal signal was fed to a power amplifier (TECHRON Model 5530) using a signal generator. Since the power amplifier itself could not supply the required current (up to 20 A-rms), a transformer was installed between the amplifier and the coil. Two TALEMA UR0500 500-VA transformers connected in parallel were used, each with a 1:5 winding ratio. The resulting maximum loading condition was 21 A-rms at 24 V-rms. A capacitor was installed between the amplifier and the coil. Two TALEMA UR0500 500-V A transformers connected in parallel were used, each with a 1:5 winding ratio. The resulting maximum loading condition was 21 A-rms at 24 V-rms. A capacitor was used, each with a 1:5 winding ratio. The resulting maximum loading condition was 21 A-rms at 24 V-rms.

### TABLE I. Specifications of CFIC-B300 driver.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force constant</td>
<td>$Bl$</td>
<td>9</td>
<td>N/A</td>
</tr>
<tr>
<td>Stiffness</td>
<td>$K$</td>
<td>74</td>
<td>kN/m</td>
</tr>
<tr>
<td>Mech. resistance</td>
<td>$b$</td>
<td>25</td>
<td>N s/m</td>
</tr>
<tr>
<td>Mass</td>
<td>$M$</td>
<td>1.63</td>
<td>kg</td>
</tr>
<tr>
<td>Piston area</td>
<td>$S$</td>
<td>0.0068</td>
<td>m²</td>
</tr>
<tr>
<td>Coil resistance (DC)</td>
<td>$R$</td>
<td>0.11</td>
<td>Ω</td>
</tr>
<tr>
<td>Coil inductance</td>
<td>$L$</td>
<td>0.9</td>
<td>mH</td>
</tr>
</tbody>
</table>

for developing the embedded system software.

To compensate for the DC drift of the accelerometer signal, a high-pass filter was used to eliminate DC offsets in the integrated signal. High frequency noise was minimized through the application of a low-pass filter. The combined band-pass filter frequency response was

$$F_{bp}(s) = \frac{1.5786 \times 10^8 (s^2 + 8.796 \times 10^{-3} s + 3.9478 \times 10^{-5})}{(s^2 + 879.6 s + 63.165) (s^2 + 17592.9 s + 1.579 \times 10^8)}.$$  

where $s = j \omega$ is the Laplacian operator, with $\omega$ the angular frequency (rad/s). A Bode plot of the filter’s frequency response is shown in Fig. 2. The $-3$ dB criterion was used for a 10 Hz to 2000 Hz pass band. Since the thermoacoustic cooler driving frequency was between 100 and 200 Hz, the pass band response for the driving frequency range was within 0.002 dB. To compensate for the filter’s phase response, the same band-pass filter was applied to the pressure signal. Note that the band-pass filter was implemented in the Simulink program for the efficiency measurement.

Averaging a data sequence over a time interval is equivalent to a low pass FIR (Finite Impulse Response) filtering process with a very low cutoff frequency and a large roll-off. The computational requirements of averaging over a large number of data samples to obtain acceptable variance in the estimates may limit the bandwidth of the efficiency measurement. To solve this problem, an IIR (Infinite Impulse Response) filter was employed instead. It is known that

$$F_n(z^{-1}) = \frac{1 - 2r z^{-1} + r^2 z^{-2}}{1 - 2r z^{-1} + r^2 z^{-2}} (r \leq 1)$$  

is a notch filter with a notch at DC, where the discrete-time operator is $z = e^{s T_s}$, with $s$ the Laplacian operator and $T_s$ the sampling period. So the complementary filter

$$F_{cn}(z^{-1}) = 1 - F_n(z^{-1}) = \frac{2(1-r)z^{-1} + (r^2 - 1)z^{-2}}{1 - 2r z^{-1} + r^2 z^{-2}}$$  

is applied to the integrated pressure signal.
acts as a low-pass filter. When $r$ gets very close to 1, a steeper notch is obtained for $\omega$ and thus a sharper roll-off is obtained for $\omega$. The frequency response of such a filter for $r = 0.99$ and a sampling rate of 1 kHz is plotted in Fig. 3. A 40 dB roll-off is obtained at about 30 Hz. The computation time for averaging 100 samples was about 220 $\mu$s. Use of an IIR filter reduced the computational time to 15 $\mu$s. Both digital filters were implemented in the Simulink program.

A comparison was made between the on-line approach just described and an off-line approach. The off-line measurement was performed using a Bruel & Kjaer PULSE system to acquire the four sensor signals. The acoustical and electrical power were obtained from the cross spectra of the signals using 1 second long records and a 2 kHz sampling rate. The results in the vicinity of the best efficiency frequency are shown in Fig. 4. Excellent agreement was obtained, except around the frequency of maximum efficiency where the largest relative error in efficiency was about 4%, which was deemed acceptable. Note that the error on tuned frequency is very small.

**IV. EFFICIENCY ANALYSIS**

Analytical expressions were derived for the electrodynamic efficiency of the driver in terms of the acoustic input impedance of the thermoacoustic cooler at the piston. The effects of feedback control and those of adding electrical capacitors for power factor improvement were evaluated.

Figure 5(a) shows a block diagram of the electrodynamic driver with piston velocity feedback. A transfer-function formulation is used in the following, which is consistent with the impedance-based circuit formulation used elsewhere. Transfer function models are convenient for feedback control. In this figure, $U(s)$ is the driving voltage, $K$ is the power amplification factor, $U_c(s)$ is the voltage on the output side of the amplifier, and $G_e(s) = 1/(Ls + R + 1/C_s)$ is the electrical impedance of the coil in series with a capacitor. The added capacitor created a second order pole in the system frequency response. This reduced the phase difference between delivered voltage and current, thereby reducing the power required by the amplifier for the same delivered power. This enabled a higher electrical power output around the driving frequency. $Bl$ is the electromechanical transduction coefficient ($N/A$). The mechanical impedance of the driver itself is $G_m(s) = s/m + b + k$, where $m$, $b$, and $k$ are the mass, the mechanical resistance, and the stiffness of the driver, respectively. The transfer function $G_{po}(s)$ is the equivalent mechanical impedance of the acoustical load at the piston, denoting the relationship between particle velocity and sound pressure at that point. $A$ is the piston area. The sound pressure exerts a force on the driver, represented in the diagram as a negative feedback at the input of

![Graph 1](image1)

**FIG. 3.** Frequency response of the IIR filter used to compute the average power.

![Graph 2](image2)

**FIG. 4.** Efficiency versus frequency.

![Graph 3](image3)

**FIG. 5.** Block diagrams of thermoacoustic driver.

![Graph 4](image4)

**FIG. 6.** Electrodynamic efficiency versus frequency.
the driver element. The driver velocity results in a “back-
emf” $E_b(s)$ fed back to the input of the electrical element.

An equivalent block transformation including an admis-
sible feedback controller, $H(s)$, is shown in Fig. 5(b). The
equation of motion for the piston is

$$m\ddot{x} + r\dot{x} + Kx = F_\text{ac}$$

where $m$ is the mass of the piston, $r$ is the damping reac-
tance of the piston element, $K$ is the stiffness of the
spring, and $F_\text{ac}$ is the acoustic force acting on the piston.

The electroacoustic efficiency is then

$$\eta = \frac{\text{Work output}}{\text{Energy input}}$$

and

$$\eta = \frac{EP(v)\cos(\Phi_{PV})}{PV(v)\cos(\Phi_{PV})}$$

The electrical power is given by $EP(v) = \frac{1}{2}A[P(j\omega)|V(j\omega)|\cos(\Phi_{PV})$, where $A$ is the piston area
and $\Phi_{PV}$ is the phase difference between the pressure and velocity. The transfer functions from input voltage to the coil current and to the coil voltage are

$$\frac{I(s)}{U(s)} = \frac{1 + AG_{PV}(s)G_m(s)}{[LC^2 + RCs + 1][1 + AG_{PV}(s)G_m(s)] + (Bl)[Bl + KH(s)]G_m(s)Cs}$$

and

$$\frac{P(s)}{U(s)} = \frac{G_m(s)G_{PV}(s)Cs}{[LC^2 + RCs + 1][1 + AG_{PV}(s)G_m(s)] + (Bl)[Bl + KH(s)]G_m(s)Cs}$$

The acoustic power delivered by the piston is given by $AP(v) = \frac{1}{2}A[P(j\omega)|V(j\omega)|\cos(\Phi_{PV})$, where $A$ is the piston area
and $\Phi_{PV}$ is the phase difference between the pressure and velocity. The transfer functions from input voltage to the coil current and to the coil voltage are

$$\frac{I(s)}{U(s)} = \frac{1 + AG_{PV}(s)G_m(s)}{[LC^2 + RCs + 1][1 + AG_{PV}(s)G_m(s)] + (Bl)[Bl + KH(s)]G_m(s)Cs}$$

and

$$\frac{P(s)}{U(s)} = \frac{G_m(s)G_{PV}(s)Cs}{[LC^2 + RCs + 1][1 + AG_{PV}(s)G_m(s)] + (Bl)[Bl + KH(s)]G_m(s)Cs}$$

The efficiency response becomes

$$\eta(\omega) = \frac{EP(\omega)}{AP(\omega)} = \frac{A[G_{PV}(j\omega)|V(j\omega)|^2\cos(\Phi_{PV})}{Uc(j\omega)}\frac{I(j\omega)}{U(j\omega)}\frac{1}{\cos(\Phi_{Uc\ell})}$$

Substituting Eqs. (8), (10), and (11) into Eq. (12) we have

$$\eta(\omega) = \frac{A[G_{PV}(j\omega)(Bl)^2]G_m(s)^2\cos(\Phi_{PV})}{[1 + AG_{PV}(s)G_m(s)][(Ls + R)[1 + AG_{PV}(s)G_m(s)] + (Bl)^2]\cos(\Phi_{Uc\ell})}$$

Equation (13) shows that the electroacoustic efficiency is re-
lated to driver parameters and to the acoustic impedance of
the tube. Note that the capacitance $C$ does not appear in Eq.
(13). This shows that the addition of a capacitor in series
with the coil does not affect efficiency.

The predicted and measured electroacoustic efficiency
responses of the thermoacoustic system operating with a
55% helium–argon mixture at 2 MPa are shown in Fig. 6.

The maximum efficiency is found within a narrow frequency
range near the overall system resonance frequency, at which
the electrical input reactance is zero. This resonance varies
for different static pressures and different refrigeration load-
ing conditions. For example, the resonance frequency in-
creases from about 173 Hz to 178 Hz for an increase in mean
pressure from 0.5 to 2 MPa.

To determine the impact of velocity feedback on effi-
ciency, the numerator and denominator of Eq. (12) may be
divided by $|Uc(j\omega)|^2$. The efficiency response becomes

$$\eta(\omega) = \frac{AP(\omega)}{EP(\omega)} = \frac{A[G_{PV}(j\omega)|V(j\omega)|^2\cos(\Phi_{PV})}{I(j\omega)}\frac{1}{\cos(\Phi_{Uc\ell})}$$

The terms on the right-hand side of Eq. (14) involve only the
quantities within the shaded block in Fig. 5(b), i.e., inside the
feedback loop. None of them is influenced by velocity feed-
back. Therefore velocity feedback does not affect the effi-
ciency for this particular problem. A similar conclusion was
also obtained for current feedback.

Hofler$^8$ used a phased-lock loop to enforce operation of
the electrodynamic driver at the acoustic resonance
frequency. This was considered an “indirect efficiency con-
trol.” The acoustic resonance may not always correspond to
the optimal operating frequency. To show this, the models described earlier were exercised for a mechanical stiffness of 300 KN/m, a mechanical resistance coefficient 25 N s/m (these values are representative of the driver), and assuming the working gas was air at mean pressure of 1 MPa. Other parameters are shown in Table I. All computation was performed over a frequency range 50–150 Hz with a frequency interval of 0.01 Hz.

Here the specific acoustic impedance is defined as the ratio of sound pressure and particle velocity at the driver. The driver mechanical impedance is the ratio of force and piston velocity for the driver in vacuo. The total mechanical impedance is the ratio of force and piston velocity in presence of the acoustic load. Referring to Fig. 5, the total impedance is the sum of the driver and the acoustic impedance, i.e.,

$$\frac{F}{V} = \frac{1}{G_m} + G_{PV}A = \left(\frac{ms^2 + bs + k}{s}\right) + G_{PV}A. \quad (15)$$

The system impedance is defined as the coil voltage (after the capacitor) over current, $U_c I$ (Fig. 5). Figure 7 shows the frequency response of the total mechanical impedance. The anti-resonance at 61.78 Hz is related to the driver response.

![FIG. 7. Frequency response of total mechanical impedance $F/V$.](image)

![FIG. 8. Frequency response of system impedance $U_c I$.](image)

![FIG. 9. Efficiency and phase of acoustic impedance versus frequency.](image)

![FIG. 10. The algorithm for adaptive frequency tuning.](image)
The resonance at 134.05 Hz and the anti-resonance at 134.82 Hz are due to the acoustic load. Figure 8 shows the frequency response of the system impedance. Similarly, the resonance at 61.75 Hz is due to the driver. The resonance at 133.98 Hz and the antiresonance at 134.87 Hz are due to the acoustic load.

Figure 9 shows the driver efficiency and the phase response of the acoustic impedance of the tube. There are two peaks (local maxima) for efficiency, related to the antiresonance of the mechanical driver and one acoustic duct resonance. This is an agreement with the results in Ref. 6. In this example, the overall maximum efficiency is achieved at 134.8 Hz, close to the acoustic resonance at 133.98 Hz. The other local maximum efficiency at 60.47 Hz is close to the antiresonance of the mechanical driver at 61.78 Hz. Increasing stiffness or reducing the damping of the driver may change the relative amplitudes of these maxima.

The global maximum efficiency at 134.8 Hz is close to but not the same as the acoustic resonance at 133.98 Hz, and also different from the neighboring resonance/antiresonances in Figs. 7 and 8. Similarly, the local maximum efficiency at 60.47 Hz is close to the anti-resonance at 61.78 Hz in Fig. 7 and the resonance at 61.75 Hz in Fig. 8. This discrepancy is due to the complexity of the interplay among different parameters of the system, as evidenced in Eqs. (12), (13), and (14). Peak efficiencies (either local or global) do not perfectly coincide with any resonance or antiresonance condition. For example, in Fig. 9, the efficiency at the acoustic resonance frequency (133.98 Hz) is 7.4% which is about 4 times less than the optimal value of 27.4%. This occurs for a mere 0.8 Hz change in frequency. Similar observations were reported by Hunt.14 "While it is customary to assume that the maximum efficiency always occurs at 'resonance', a closer examination of Eqs (4.19) and (4.20) reveals that a still higher efficiency can be obtained for some other value of frequency or reactance than the one for which \( p = 0 = X_M + X_L \)."

V. ADAPTIVE FREQUENCY TUNING PROCEDURES

The adaptive frequency tuning scheme was implemented using a dSPACE DS1102 controller board (Refs. 15 and 16). Table II lists the variables of interest. A flow diagram for the frequency tuning algorithm is shown in Fig. 10. The frequency, frequency shift, and efficiency are first initialized. After reading the efficiency value, a new target efficiency is calculated. If the efficiency change does not exceed a predefined threshold, the frequency change is kept the same as for the previous step. If the efficiency change is significant, a decision on the next frequency adjustment is made depending on if the change is positive or negative. If the efficiency is increasing, the sign of the adjustment is kept; otherwise, the sign is reversed.
To verify the effectiveness of this adaptive tuning algorithm, a test was designed to expose the thermoacoustic cooler to a “sudden” change in operating conditions (not necessarily representative of typical transients during normal operation). The mean pressure of the cooler was increased from 0.89 MPa to 1.67 MPa over a time period of 20 seconds. The measured electrodynamic efficiency and the corresponding driving frequency are shown as functions of time in Fig. 11(a). The test started at the instant $t = 120$ s. Initially, the driver efficiency dropped from 40% to about 21% due to the change in resonance frequency. The performance degraded dramatically while the driving frequency was kept constant. The adaptive tuning, within about 45 seconds, allowed the system performance to reach a new peak efficiency of 55% at a new driving frequency of 176.2 Hz. During the pressurization process, the temperature within the tube also experienced a transient evolution. The temperature profile near the center of the heat exchanger on the hot side is shown in Fig. 11(b). A temperature fluctuation around 4°F (2°C) was observed.

VI. CONCLUSIONS

An on-line efficiency measurement scheme was implemented for a functional electrodynamically driven thermoacoustic cooler. It was found by analysis that regular feedback should not change the optimal electrodynamic efficiency of the driver. The acoustic resonance frequency does not match the frequency for optimal efficiency, and a significant performance drop can result from operation at acoustic resonance. An adaptive frequency tuning algorithm was proposed and implemented to maintain the driver at its maximum efficiency for varying operating conditions. A “pressure jump” test was performed to illustrate the effectiveness of the controller in dealing with transients in the tuned frequency. The approach can deal with most changes in operating conditions during normal operation.