MOTION SYNCHRONIZATION FOR MULTI-CYLINDER ELECTRO-HYDRAULIC SYSTEM

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ABSTRACT
This paper presents a nonlinear control algorithm to address the motion synchronization problem for a multi-cylinder electro-hydraulic (EH) system. A 2-step design approach is applied such that it utilized linear MIMO robust control technique to design an outer loop motion synchronization controller. A nonlinear SISO perturbation observer based pressure/force controller is designed for each of the lift cylinder as the inner loop controller to handle the nonlinearities associated with the EH actuators. Experimental results on a 4-cylinder system are presented to verify the effectiveness of the proposed approach.

INTRODUCTION
The problem of synchronizing multiple linear hydraulic actuators [4][12] arises in hydraulic operated heavy-duty applications such as lifting equipment and rolling mills, where the synchronous operation of multiple hydraulic actuators under load has important performance implications. This issue is most pronounced in hydraulically operated multi-actuator lifting equipment. Due to uneven loading and the inherent differences in multiple hydraulic circuits and components, the lift distance among the linear actuators will be different under open-loop operation. Specifically, the overall open-loop system is unstable in the sense that the difference in lift distance will increase and will eventually results in the toppling of the load.

There are three approaches to address the issue of synchronizing multiple hydraulic actuators. The simplest approach is to design a flow-divider circuit that will maintain the same cylinder velocity by maintaining the same flow rate to the cylinders. The performance of the synchronization is dependent upon the performance of flow-divider as well as the compressibility of the working fluid and the consistency of the hydraulic components. Another approach is to mechanically connect the hydraulic actuators through either cabling or other linkage design. The drawback to mechanical synchronization is the added system weight and complexity as well as the limitation to the operation range of the equipment. Compared with pure hydraulic and mechanical approaches, electro-hydraulic synchronization provides a flexible alternative. With EH system, synchronization control strategies can be designed to handle uneven loading as well as uncertainties and external disturbances associated with the hydraulic system.

In 1994, Hogan and Burrow [4] looked at the issue of synchronizing unevenly loaded hydraulic cylinders. Their results indicated that to achieve synchronization of the unevenly loaded cylinders, individual control of each cylinder is required. However, with the added flexibility of individually controlled cylinders, Hogan and Burrow did not elaborate on the design of a control algorithm that will explicitly improve the synchronization performance. Xiong et al [12] proposed a model-reference adaptive control algorithm together with a cross-coupled controller [7] to improve synchronization performance as well as attempt to handle the parameter variation associated with the hydraulic systems. Chiu [1] formulated the synchronization of multiple motion axes in a geometrical framework and proposed three different approaches to explicitly address the motion synchronization issue. Sun and Chiu [10][11] proposed both linear and nonlinear approaches to achieve synchronization on electro-hydraulic lift.

The research of EH synchronization control has focused on the linear approach and less research has focused on the nonlinear EH synchronization control. In this paper we will consider the synchronization of a hydraulic system with n ram-type cylinders acting to lift a common unknown load. The cylinders are controlled by n individual solenoid flow control valves. The hydraulic circuit diagram of the system is shown in Figure 1. A 2-step nonlinear motion synchronization control design is proposed to address the synchronization issue. The 2-step design approach utilized linear MIMO robust control technique to design an outer loop motion synchronization.
controller. A nonlinear SISO perturbation observer-based pressure/force controller is then designed for each of the cylinder as the inner loop controller.

![Hydraulic Circuit of an n-Post Lift](image1)

Figure 1 Hydraulic Circuit of an n-Post Lift.

The remaining of this paper is organized as follows: The system modeling is presented in the next section followed by the 2-step MIMO nonlinear synchronizing controller design. The experimental results are presented in section IV. Conclusions together with discussion of on-going investigations are presented in the last section.

SYSTEM MODELING

Consider a hydraulic system with \( n \) ram-type cylinders acting to lift a common load, as shown in Figure 2. The top view of the system is shown in Figure 3 that indicates the location of the load. The load is free to rotate around the roll axis \( r \) and the pitch axis \( p \). The roll axis \( r \) is defined to be perpendicular to the line connecting cylinder 1 to cylinder 2 and the pitch axis \( p \) is defined to be parallel to the line connecting cylinder 1 to cylinder 2. Together with one degree of freedom of moving along the gravity direction, the hydraulic lift system with 3 or more cylinders has 3 degrees of freedom (DOF) that can be controlled. The lift system with 2 cylinders has 2 DOF (without the pitch motion). In the following, we will derive the system model for the lift system with 3 or more cylinders.

Figure 2 shows the forces acting on the hydraulic cylinders. Spherical contact surfaces are assumed between the cylinders and the load. From Newton's second law and the conservation of angular momentum, the following equations can be obtained to represent the motion of the load along gravity direction and rotations about the axes \( r \) and \( p \):

\[
\begin{align*}
\sum_{i=1}^{n} f_i - M g &+ \sum_{i=1}^{n} F_{ri} \sin(\theta_r) + \sum_{i=1}^{n} F_{pi} \sin(\theta_p) = M \ddot{x}, \\
\sum_{i=1}^{n} \left( f_i + F_{ri} \sin(\theta_r) \right) l_{ri} & = J_{r} \dot{\theta}_r, \\
\sum_{i=1}^{n} \left( f_i + F_{pi} \sin(\theta_p) \right) l_{pi} & = J_{p} \dot{\theta}_p
\end{align*}
\]

where \( x_p \) represents the position of the center of gravity of the load (\( x_p = 0 \) when the load is on the ground). \( M \) is the mass of the load and \( g \) is the gravitational constant. In Eq. (1), \( f_i \) represents the reaction force acting on cylinder \( i \) \((i = 1, 2, \ldots, n)\) and \( l_{ri} \) (or \( l_{pi} \)) is the moment arm for \( f_i \) \((i = 1, 2, \ldots, n)\) with respect to rotational axis \( r \) (or axis \( p \)). \( J_{r} \) (or \( J_{p} \)) represents the rotational moment of inertia of the load about the axis \( r \) (or the axis \( p \)) and \( \theta_r \) (or \( \theta_p \)) represents the rotation angle about the axis \( r \) (or the axis \( p \)). \( F_{ri} \) (or \( F_{pi} \)) represents the friction force between the load and cylinder along the direction of the axis \( r \) (or the axis \( p \)).

In Figure 3, assuming that the distance between the cylinders does not change during the motion of the load, the points on the load that contact the cylinders vary with the rotation of the load. Its motion depends on the rotation angle \( \theta_r \) and \( \theta_p \). We will not detail the relation between them because we are only focusing on the vertical displacement and the pitch as well as roll motion of the load. Further more, in the assumption that the rotation angle is small, this motion can be ignored.

![Diagram of Acting Forces in n-Cylinder Hydraulic Lift System](image2)

Figure 2 Diagram of Acting Forces in n-Cylinder Hydraulic Lift System.

![Top View of the n-Cylinder Hydraulic Lift System](image3)

Figure 3 Top View of the n-Cylinder Hydraulic Lift System.

Consider the forces \( f_i \) \((i = 1, 2, \ldots, n)\) acting on the cylinders, the equations of motion for the cylinders can be represented by:

\[
P_i A_i - f_i - m_i g - F_{ri} - \sum_{i=1}^{n} F_{pi} \sin(\theta_p) - \sum_{i=1}^{n} F_{ri} \sin(\theta_r) = m_i \ddot{x}_i,
\]

where \( P_i \) represents the pressure in the chamber of cylinder \( i \) and \( A_i \) represents the effective piston area. \( m_i \) represents the
piston mass of cylinder $i$. $F_p$ represents stiction and $B_p$ represents the viscous friction coefficient.

If the contact between the load and the end point of the cylinder is assumed to be equivalent to a mass-spring-damper system with stiffness $k_i$ and damping ratio $b_i$, as shown in Figure 4, then the contacting effect can be modeled by the following equation:

$$f_i = k_i(x_i - x_{ia}) + b_i(x_i - x_{ia}) \quad (i = 1, 2, \ldots, n) \quad (3)$$

where $x_{ia}$ represents the relative position of the contact point between the load and cylinder $i$ to the ground and $x_i$ represents the position of the cylinder $i$ relative to the ground as shown in Figure 4.

**Figure 4: Detail of the Contacting between Load and Cylinder.**

The fluid that flows into each cylinder is controlled by a flow control valve. Consider the compressibility of the fluid in the cylinders and ignore the valve dynamics as well as leakages in the cylinders, the pressure dynamics in each cylinder can be represented by the following equation (Merritt [8]):

$$\dot{P}_i = h(x_i, \dot{x}_i) + \beta K_q \phi(x_i, P_i) u_i, \quad (i = 1, 2, \ldots, n) \quad (4)$$

where $h(x_i, \dot{x}_i) = -\frac{\beta}{V_i(x_i)} A_i \dot{x}_i$, \[\phi(x_i, P_i) = \frac{1}{V_i(x_i)} \sqrt{P_i/2 + \text{sign}(u_i)(P_i/2 - P_s)} \].

In Eq. (4), $\beta$ is the bulk modulus of the working fluid, $V_i$ represents the total fluid volume from the output port of the valve to the respective cylinder chamber and $P_s$ is the source pressure. $K_q$ is the valve flow coefficient and $u_i$ represents the control input, i.e. the valve input command.

In the condition that the rotational angles $\theta_1$ and $\theta_p$ are small, i.e. $\theta_1, \theta_p \ll 1$, then $\sin(\theta_1) \approx 0$ and $\sin(\theta_p) \approx 0$. Let $f = [f_1 \ f_2 \ \cdots \ f_n]^T$ be the resultant force acting on the load, $x_{ia} = [x_{ia} \ x_{2a} \ \cdots \ x_{na}]^T$ and $x_L = [x_1 \ x_2 \ \cdots \ x_n]^T$ be the location of the contact points and the cylinder position in the gravity direction, respectively. Equations (1), (2) and (3) can be written as:

$$\begin{bmatrix} L_t f - M_q \end{bmatrix} = M_L \ddot{x}_q \quad (5)$$

where

$$\begin{bmatrix} L_t & -M_q \end{bmatrix} = \begin{bmatrix} M_L & \ddot{x}_q \end{bmatrix} \quad (5)$$

$$f = K_i(x_i - x_{ia}) + B_i(x_i - x_{ia}) \quad (i = 1, 2, \ldots, n) \quad (5)$$

$$\begin{align*}
R - B_p \dot{x}_L - f &= m \ddot{x}_L \\
M_q &= \begin{bmatrix} M_{qL} & 0 \\
0 & M_{q_p} \end{bmatrix} \quad \text{is the weight of the load matrix}, \\
M_i &= \begin{bmatrix} M_{iL} & L_{t_p} \\
L_{t_p} & M_{i_p} \end{bmatrix} \quad \text{is the constant moment arm matrix}, \\
F &= \begin{bmatrix} F_{iL} \\
F_{i_p} \end{bmatrix} \quad \text{is the resultant force matrix}, \\
K_i &= \text{diag}(\begin{bmatrix} k_1 & k_2 & \cdots & k_n \end{bmatrix}) \quad \text{is the contact stiffness matrix}, \\
B_i &= \text{diag}(\begin{bmatrix} b_1 & b_2 & \cdots & b_n \end{bmatrix}) \quad \text{is the contact viscous damping coefficient matrix}, \\
B_p &= \text{diag}(\begin{bmatrix} b_{p1} & b_{p2} & \cdots & b_{pn} \end{bmatrix}) \quad \text{is the viscous damping coefficient matrix associated with the cylinder motion and} \\
m &= \text{diag}(\begin{bmatrix} m_1 & m_2 & \cdots & m_n \end{bmatrix}) \quad \text{is the piston mass matrix}. \\
\end{align*}$$

To further simplify Eq. (5), assume that the contact between the load and the cylinders is rigid, i.e. $x_L = x_{ia}$, then the following is obtained:

$$L_t R - L_t B_p g_s \dot{x}_q - M_q = (M_L + L_t m g_c) \ddot{x}_q \quad (6)$$

where $g_s = [\begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}]^T$ with $g_s = \begin{bmatrix} 1 & l_{iL} & l_{i_p} \end{bmatrix} L_{t_p}^T$ and $L_{t_p} = \begin{bmatrix} 1 & l_{iL} & l_{i_p} \\
1 & l_{i2} & l_{i_p} \\
1 & l_{i3} & l_{i_p} \end{bmatrix}$. Note that if there are more than three cylinders in the system, i.e. $n>3$, there will be redundant actuators in the system, i.e. the matrix $L_t$ is a non-square matrix with more columns than rows. The solution for the synchronization of the lift system under such condition will need to be considered.

Let $M_{eq} = (M_L + L_t m g_c) \in \mathbb{R}^{3 \times 3}$, $B_{eq} = L_t B_p g_s \in \mathbb{R}^{3 \times 3}$, and $F = (R - L_t M_q) \in \mathbb{R}^{3 \times 1}$. In summary, if a simple rigid contact is assumed between the cylinders and the lift platform, then the EOM for an $n$ cylinder EH lift can be represented by:

$$\begin{align*}
M_{eq} \ddot{x}_q + B_{eq} \dot{x}_q = L_t F \\
\dot{P}_i = h(x_i, \dot{x}_i) + \beta K_q \phi(x_i, P_i) u_i, \quad (i = 1, 2, \ldots, n) \quad (7)
\end{align*}$$

Notice that the first part of Eq. (7) is linear EOM for rigid body dynamics, and the nonlinearity of the system comes from
the hydraulic pressure dynamics. In addition, because of the existence of the load, i.e. \( M \neq 0 \), the matrix \( M_L \) and \( I_\tau \) are non-diagonal. Hence the motion dynamic of each cylinder forms a MIMO system and is coupled through the load. In the load dynamics, potential uncertainties are the mass \( M \) and locations \( l_i \) and \( l_{ip} \) (\( i = 1, 2, \ldots, n \)). However no direct load uncertainties appear explicitly in the pressure dynamics. The pressure dynamic of each cylinder forms a SISO system, i.e. the time derivative of pressure is a function of its own states, pressure and piston location. These characteristics will be exploited in the control design.

**CONTROL STRATEGY**

A 2-step controller design approach is proposed that employs a linear time-invariant MIMO robust motion synchronization control to address the load uncertainty and motion synchronization aspects of the problem in the outer loop. A SISO perturbation observer-based nonlinear EH force/pressure control is used for each individual cylinder to address the nonlinearity and uncertainties associated with EH systems in the inner loop. In a general \( n \)-post lift system, the rigid-body motion dynamics described by Eq. (7) is a linear MIMO system with load uncertainties from the force inputs \( F \) to the position output \( x_q \). In Eq. (7), the resultant force matrix \( F \) comprises of the forces acting on the cylinders that include the forces produced by the pressures in the cylinders \( (P_{Ai}) \), the friction forces \( (F_{fr}) \) and the equivalent loads for each cylinder. The facts that the load and its location are unknown and that the frictional forces are difficult to model lead to uncertainties in the resultant force matrix \( F \). This problem can be solved since the initial pressures \( P_{i0} \) \( (i = 1, 2, \ldots, n) \) of the corresponding cylinder \( i \) can be used to estimate \( (m_i g + F_{fr} + M_{gi}) \), where \( M_{gi} \) represent the \( i^{th} \) element of \( L_i M_s \). Let

\[
F_p = \begin{bmatrix}
F_{p1} \\
F_{p2} \\
\vdots \\
F_{pn}
\end{bmatrix} = \begin{bmatrix}
A_1 (P_{10} - P_{10}) \\
A_1 (P_{20} - P_{20}) \\
\vdots \\
A_1 (P_{n0} - P_{n0})
\end{bmatrix}
\]

as the virtual control input, and

\[
d_t = \begin{bmatrix}
A_1 P_{10} - m_1 g + F_{fr1} + M_{g1} \\
A_2 P_{20} - m_2 g + F_{fr2} + M_{g2} \\
\vdots \\
A_n P_{n0} - m_n g + F_{frn} + M_{gn}
\end{bmatrix}
\]

as the input disturbance matrix. Without loss of generality, \( d_t \) is assumed to be bounded. Therefore the resultant force matrix \( F \) can be written as \( F = F_p + d_t \). Substituting \( F = F_p + d_t \) into Eq. (7), the rigid body motion dynamics can be written as:

\[
M_q \ddot{x}_q + B_q \dot{x}_q = L_t F_p + L_t d_t \quad (8)
\]

In Eq.(8), it is clear to see that there are 3 DOF, i.e. \( x_q \in R^3 \), but more than three control inputs (when \( n>3 \)), i.e. \( F_p \in R^n \). There are \( n-3 \) redundant actuations.

The design procedure for linear MIMO system will be utilized to address the complexity due to the increased DOF as well as the redundancy issue due to the increase of the number of EH actuators. The uncertainty associated with the EH system will be addressed with the perturbation observer design.

The motion synchronization control will be formulated to generate the desired force or pressure profile for each of the \( n \) actuators under a specific synchronization objective. Once the desired force or pressure profile is obtained, a set of nonlinear SISO force controllers will be employed to achieve the desired force or pressure tracking.

**ACTUATOR REDUNDANCY ISSUE**

Actuation redundancy occurs when there are more actuators than required to provide the desired motion. Having the extra support points is the primary reason for the additional actuators. Therefore, enhancing synchronization performance and maintaining contact between the load and cylinders will be the main considerations in resolving the actuator redundancy issue.

The actuator redundant system represents an under-constrained set of equations for the independent forces, \( F_{ind} \) (or torques) and resultant forces, \( F_{res} \) (or torque), e.g

\[
F_{res} = A F_{ind} \quad \text{with} \quad F_{res} \in R^n, \quad F_{ind} \in R^r \quad \text{and} \quad n > m, \quad \text{so there exists an infinite number of independent force (or torque) vectors corresponding to a given resultant force (or torque). Colbaugh and Glass [2] discussed the different resolutions for actuator redundancy and categorized them into two approaches, the "direct" inversion and "indirect" inversion. Direct inversion uses the theory of generalized inverses. The indirect approach to construct \( F_p \) from \( F_{res} \) can be utilized to obtain the \( F_{res} \) to \( F_p \) mapping which simultaneously satisfies the desired performance requirement and an additional set of \( r \) linear constraints on the elements of \( F_p \), i.e.

\[
A_p F_p = b_p, \quad (9)
\]

where \( A_p \in R^{rm} \) and \( b_p \in R^r \) define the desired evolution of the constraints. Here the rank of \( A_p \), i.e. \( r \), can be greater than \( n-3 \). Let the resultant force \( F_{res} = L_t F_p \) be the high priority task and \( A_p F_p = b_p \) represents the secondary objectives. Choose

\[
F_p = (L_t)^{-1} F_{res} + P^\mu, \quad (10)
\]

where \( P^\mu = (I_{3m} - L_t L_t) \) is a projection operator which will map any vector \( \mu \) onto the kernel space of \( L_t \), i.e. \( P^\mu \in \ker(L_t) \), and \( \mu \) is a vector to be decided to satisfy the Eq. (9). Note Eq. (10) exactly satisfies the high-priority task.

Substituting the solution \( F_p \) from Eq. (10) into the low-priority task in Eq. (9) following equation can be obtained:
The least square solution of Eq. (11) is the pseudo-inverse solution to Eq. (11),
\[ \mu = (A_p^T)^+ (b_e - A_p L^T F_{re}) . \]
Therefore,
\[ F_p = (L_t)^T F_{re} + P^T (A_p^T)^+ (b_e - A_p L^T F_{re}) . \]

Remark:
One example of choosing the constraint for a 4-post lift is to let \( A_e = [0 \ 0 \ 1 \ -1] \), and \( b_e = 0 \). This imposes a constraint that the net forces acting on the load by cylinder 3 and 4 are same.

If the rank of \( A_e \) is equal to \( n-3 \), then Eq. (9) can be exactly satisfied. Once recovering the desired force \( F_p \), it will be used as the tracking target for the pressure dynamics.

**CONTROLLER DESIGN**

**Two-Step Design Approach**

The controller will be synthesized through a two-step design process using a linear MIMO motion synchronization controller as the outer loop controller and a perturbation observer based pressure controller as the inner loop controller. Generally, a two-step design can be applied if a system has the semi-strict-feedback form. Following example illustrate the idea of the two-step design with actuator redundancy.

Given a system that is represented by the following equations:
\[
\begin{align*}
\dot{X}_2 &= f_2(X_1, X_2) + g_2(X_1, X_2)U \\
X_3 &= f_3(X_1, X_3) + g_3(X_1, X_3)U
\end{align*}
\]
where \( X_1 \in \mathbb{R}^3 \), \( X_2 \in \mathbb{R}^3 \), \( U \in \mathbb{R}^n \). \( M \) and \( B \) are matrices with the appropriate dimension. The first step is to design a controller based on the first equation of Eq. (12) in assumption that \( X_2 \) is the actual control input. \( X_2 \) is used to achieve the desirable synchronization performance described by the first equation of Eq. (12). In the next step, the actual control \( U \) is designed to achieve a desired \( Z_2 \) dynamics.

The idea of this design can be understood in another equivalent way. In the first step design, since \( X_2 \) is not the actual control input, denote \( X_{2d} \) as the desired control input which can be arbitrarily assigned and \( Z_2 = X_2 - X_{2d} \) is defined to represent the discrepancy between \( X_2 \) and \( X_{2d} \). The typical way for next step design will be based on the dynamics of \( Z_2 \) or \( X_2 \) to obtain the actual control input such that the entire system is stabilized and \( Z_2 \) will converge to zero or to a bounded neighborhood, i.e. \( \| Z_2 \| \leq \varepsilon \) for a positive scalar \( \varepsilon \). However, due to redundancy, the next step’s dynamics are represented by \( X_3 \) and \( X_{3d} \) does not have a one-to-one mapping to the state \( X_3 \). In addition, the uncertainties in the \( L \) matrix will complicate the mapping from \( X_2 \) to \( X_3 \) as well.

First we will consider the case that there is no uncertainty in \( L \). \( X_{3d} \), which represents the desired state of \( X_3 \), is calculated from \( X_{2d} \) using either the direct or indirect inversion approach mentioned in the previous section. Then the relation \( X_{2d} = L \cdot X_{3d} \) is satisfied. Denote the error between \( X_3 \) and \( X_{3d} \) as \( Z_0 \), then \( Z_2 = L \cdot Z_0 \). Eq. (15) can be written as:
\[
\begin{align*}
\dot{X}_1 &= f_1(X_1, X_2) + g_1(X_1, X_2)U \\
\dot{Z}_0 &= f_3(X_1, X_3) + g_3(X_1, X_3)U - X_{3d}
\end{align*}
\]
It is clear that a typical 2-step design approach can be implemented. First assume \( Z_0 = 0 \), the inner loop control law \( X_{2d} \) will be designed according to the first equation in Eq. (16). Then the outer loop control law, i.e. the actual control law can be design based on the second equation in Eq. (16). In other words, \( X_{2d} \), which can be treated as virtual control input, together with actual control input \( U \) are designed such that the whole system satisfies the desired performance requirements.

Now consider the situation where there are uncertainties in \( L \). The nominal value of \( L \), denoted by \( L_0 \), is used in obtaining \( X_{3d} \) from \( X_{2d} \) using any approach discussed in the last section. Therefore the relation \( X_{3d} = L \cdot X_{3d} \) will not be satisfied unless \( L = L_0 \). Suppose the way we choose \( X_{3d} \) satisfies \( X_{3d} = P_r \cdot X_{2d} \) for a specific \( P_r \). For example, \( P_r = W^{-1} (L_0)^T (L_0W^{-1} (L_0)^T)^{-1} \) for the direct method or \( P_r = (1 - P^T A_p P^{-1} A_r) L_0^{-1} \) for the indirect
method. Then the following relation between \( Z_2 \) and \( Z_3 \) can be obtained:

\[
Z_2 = LX_3 - Xzd \\
= LZ_3 + I Xzd - Xzd \\
= LZ_3 - (I - LP_\tau)Xzd
\]

By substituting \( Z_2 \) into Eq. (16), the following equation is obtained:

\[
MR_\tau + BX_1 = LP_\tau X_1 + LZ_3 (X_3) + Z_3^2 \frac{f}{f}(X_1, X_3) + g(X_1, X_3)U - X_3d
\]

Given uncertainty in the input matrix \( L_0 \), Eq. (17) will be used to implement the two-step design approach. Note that since the second step design will be based on \( Z_3 \) dynamics, \( Z_3 = 0 \) is desirable to be the assumption instead of \( Z_2 = 0 \) in the first step design. The first loop control law \( X_3d \) is designed according to the first equation in Eq. (17) under the assumption \( Z_3 = 0 \). Compared to the first equation in Eq. (16), we can see that the input uncertainties \( LP_\tau \) need to be considered in designing \( X_3d \). Note that \( LP_\tau = L \) if \( L_0 = L \). In the second step, the actual control input \( U \) will be designed based on the \( Z_3 \) dynamics. In summary, for the system described by Eq. (14) with uncertainty in \( L_0 \), the 2-step design will be proposed in this way. First the Eq. (14) is converted into Eq. (17). Then a typical 2-step design will be done based on Eq. (17). In other words, the first loop virtual control law \( X_3d \) and second loop control law \( U \) have to be designed such that the system represented by Eq. (17) is stable.

Now consider the lift dynamics described by Eq. (8). Denote \( F_{rd} \) as the desired resultant force of \( F_{res} = L_q F_p \). Then the motion synchronization design will be based on the following equation with the assumption that \( F_p \) is the actual control input:

\[
M_{\text{eq}} \ddot{x}_{\text{eq}} + B_{\text{eq}} \dot{x}_{\text{eq}} = L_{\text{eq}} F_{\text{eq}} + L_{x q} d \text{f} + L_{x q} \Lambda_{x} z
\]

where \( z = [z_1, z_2, \ldots, z_n]^T \) with \( z_i = P_i - P_i^d \) defines as the difference between the pressure \( P_i \) and the desired pressure \( P_i^d \), and \( \Lambda_{x} = \text{diag} ([A_1, A_2, \ldots, A_n]) \). To deal with the synchronization error explicitly, the motion dynamics will be transformed from the original coordinates into a new task oriented coordinates. The task-space coordinates are compacted because an \( n \)-cylinder system can have as many as \( n-1 \) independent synchronization sub-objects. Recall \( x_{eq} \in \mathbb{R}^3 \). Only three independent positions are needed to represent the motion of the load. The coordinate which represents tracking performance is required in the design process to avoid a trivial solution. The task-space can only be the frame spanned by one tangent vector \( t \) that represents the tracking direction and the other two \( n_1 \) and \( n_2 \) vectors that are orthogonal to \( t \). The other remaining secondary synchronization objectives can be imposed to account for actuator redundancy. In the task space, the synchronization error is distinguished and can be emphasized explicitly. Let the task-space coordinates to be \( x_{eq} = [t, x_{q1}, x_{q2}]^T \), where \( t = \frac{1}{n} \sum x_i \) represents the tracking direction, and \( x_{q1} = \sum a_i x_i \) and \( x_{q2} = \sum b_i x_i \) represent two directions that are orthogonal to the tracking direction. Without loss of the generality, we assume that the performance in the \( x_{q1} \) direction is less important than that in the \( x_{q2} \) direction. The physical coordinates such as \( x_{eq} \) can be transformed to the task coordinates \( x_{eq} \) through the following transformation:

\[
x_{eq} = \begin{bmatrix}
1 & \frac{1}{n} \sum a_i l_i & \frac{1}{n} \sum a_i p_i \\
0 & \sum b_i l_i & \sum b_i p_i
\end{bmatrix}
x = \begin{bmatrix}
\dot{x}_{p} \\
P_{\text{eq}}
\end{bmatrix} = L_{\text{eq}} (L_{\text{eq}})^{-1} x_{eq} = T^{-1} x_{eq}
\]

where \( T = L_{\text{eq}} (L_{\text{eq}})^{-1} \). Since the two synchronization objectives are linearly independent, the existence of \( T \) and \( T^{-1} \) is guaranteed. The motion dynamics described by Eq. (18) can be transformed into the \( x_{eq} = T^{-1} x_{eq} \) task coordinates:

\[
\begin{bmatrix}
\tilde{M}_{\text{eq}} \ddot{x}_{eq} + \tilde{B}_{eq} \dot{x}_{eq} = \tilde{L}_{eq} F_{\text{eqd}} + \tilde{L}_{eq} d_{\text{eq}} \\
y = x_{eq}
\end{bmatrix}
\]

where \( \tilde{M}_{\text{eq}} = T^{-1} M_{\text{eq}} T, \tilde{B}_{eq} = T^{-1} B_{eq} T, \tilde{L}_{eq} = T^{-1} L_{eq} P_{\text{eq}}, \tilde{L}_{eq} d_{\text{eq}} = T^{-1} L_{eq} d_{\text{eq}} \) and \( F_{\text{eqd}} = T^{-1} F_{\text{eq}}, d_{\text{eq}} = T^{-1} d_{\text{eq}} \). In Eq. (19), we have assumed that \( z = 0 \).

For convenience, we will further denote \( x_{eq} = [x_{eq}^t, x_{eq}^1, x_{eq}^2]^T \) and \( x_{eq}^* = [x_{eq}^1, x_{eq}^2]^T \). Equation (19) will be used designing the outer loop motion synchronization control. The control design will be completed in the task-space coordinates where the control law \( F_{\text{eqd}} \) will be determined and the \( F_{\text{eqd}} \) can be obtained through the inverse transform of \( T \). The actual desired force input for each cylinder will be computed through the indirect method discussed in section 3.

**Linear MIMO Design for Motion Synchronization**

Sequential loop Qualitative-Feedback-Theory (QFT) design is applied in the motion dynamics to obtain the desired cylinder pressures that are needed to maintain the synchronization of the cylinder displacement under different loading conditions. Other robust control design methodology can also be used.

For motion synchronization, in the task space, regulating \( x_{eq}^* \) to zero is more important than \( x_{eq}^t \) maintain tracking in the
The synchronization objective can be formulated as a disturbance rejection problem, see Figure 5, where the disturbance \( d \) includes the \( d_t \) term and the discrepancy between the desired force and the actual force acting on the system. The controller matrix \( G \) is designed to reject the disturbance \( d \) given the plant transfer function \( P(s) = \left[ \begin{array}{c} \text{M}_{eq} s^2 + \text{B}_{eq} \end{array} \right]^{-1} \) with \( F_{\text{rd}} \) as the input and \( X_{eq} \) as the output.

\[
\begin{aligned}
dx_{eq}(s) &= T_{rd} \quad \begin{bmatrix} q_{eq}(s) \\ \end{bmatrix} , & \text{where } T_{rd} &= \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix} \\
\text{Note that the synchronization error is also affected by the tracking reference } q_{eq} \text{ through the cross-coupling terms. To minimize the effect of the tracking reference on the synchronization error, the following constraints are imposed:} \\
&|x_{eq}(s)| = t_{11} q_{eq} \leq \epsilon_{eq}^1, \\
&|x_{eq}(s)| = t_{12} q_{eq} \leq \epsilon_{eq}^2.
\end{aligned}
\]

Combining Eqs. (21) and (22), the performance constraints (specifications) for the QFT design are summarized as follows:

\[
\begin{align}
|x_{eq}(s)| &= s_1 y^r + s_{12} y^m + s_{13} y^n \leq \lambda_1, \\
|x_{eq}(s)| &= s_{21} y^r + s_{22} y^m + s_{23} y^n \leq \lambda_2, \\
|x_{eq}(s)| &= s_{31} y^r + s_{32} y^m + s_{33} y^n \leq \lambda_3.
\end{align}
\]

where \( \lambda_1 = \epsilon_{eq}^1 / \lambda \), \( \lambda_2 = \epsilon_{eq}^2 / \lambda \) and \( \lambda_3 = \epsilon_{eq}^3 / \lambda \).

Following the standard MIMO QFT design procedure [5], a diagonal controller

\[
G(s) = \begin{bmatrix} g_1(s) & 0 \\ 0 & g_2(s) \end{bmatrix}
\]

can be obtained which satisfies the performance inequalities stated in Eq. (23) under various loading conditions. The desired input for the motion dynamics can be obtained by the following:

\[
F_{pa} = G(s) X_{eq}.
\]

Through the task space transformation \( T \) described in the previous section, the resultant force vector \( F_{rd} \) in the physical coordinates can be obtained by \( F_{rd} = T F_{pa} \). Because of the redundancy in the actuators, the mapping from \( F_{rd} \) to \( F_{pa} \) has infinite solutions. The mapping approaches discussed in section 3 can be applied to obtain the desired control input \( F_{pa} \).

Denoting \( P = \left[ \begin{array}{c} P_1^d \\ P_2^d \\ \cdots \\ P_n^d \end{array} \right] \) as the desired pressure required in each cylinder, then:

\[
P^d = \begin{bmatrix} P_{10}^d \\ P_{20}^d \\ \vdots \\ P_{n0}^d \end{bmatrix} = \left( \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \right),
\]

In the following, an indirect approach is applied as an example to obtain \( F_{pd} \) as well as to enhance the synchronization performance. Let the additional constraints to be written as

\[
\sum_{i=1}^{\text{\#cylinders}} d_i f_{pi}(i) = b_j.
\]
Based on the discussion in section 3, the above constraints can be written as $A_e F_p = b_e$ where

$$
A_e = \begin{bmatrix}
  \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\
  \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn}
\end{bmatrix}, \quad b_e = \begin{bmatrix}
  b_1 \\
  b_2 \\
  \vdots \\
  b_n
\end{bmatrix}.
$$

Substitute both $A_e$ and $b_e$ into Eq. (11), the desired control input for each cylinder is given by:

$$
F_{pd} = \left( L_{to}^{-1} - \sqrt{P_e} A_e^T \right) F_{da},
$$

Once the desired pressure in each cylinder is obtained, a pressure tracking controller will be designed for each cylinder to achieve the desired pressure profile.

### A. Nonlinear Force Control Design

The pressure dynamics of each cylinder forms a SISO system. The nonlinear perturbation observer based force control design technique [10] is applied to achieve the desired pressure tracking. The pressure dynamics can be rewritten as:

$$
\begin{align*}
\dot{p}_i &= \bar{h}_i(x, \beta) + \phi(x) \beta K_{oi} u_i + \hat{d}_i, \\
\dot{h}_i &= \bar{h}_i(x, \beta) - h_i(x, \beta),
\end{align*}
$$

where $\beta_0$ and $K_{oi}$ are the nominal value for the bulk modulus $\beta$ and flow coefficient $K_{oi}$, respectively. $\hat{d}_i$ represent the model uncertainties due to the uncertainties in bulk modulus. $\bar{h}_i = (K_{oi} - K_{oi0}) = \beta K_{oi} - \beta_0 K_{oi0}$ represent the input parametric uncertainties in the valve mapping model and $d_{ui}$, $i = 1, 2, \cdots, n$, represent the unmodelled uncertainties.

Following the nonlinear perturbation observer based design technique described in [10], the following control law can be obtained to achieve the pressure tracking in each of the cylinders:

$$
u_i = V_i(x, \beta) \frac{\beta_0 \dot{p}_i + \beta_0 \dot{h}_i - \dot{h}_i - K_{oi} \nu_i}{V_i(x, \beta)}, \quad i = 1, 2, \cdots, n, (27)
$$

where $K_{oi} > 0$ are the design parameters which represents the proportional feedback gain of the system. $\dot{h}_i$ are the estimated perturbation which can be obtained from the following equations:

$$
\dot{h}_i = k_p (\dot{p}_i - P_i) + k_f \int (\dot{p}_i - P_i) dt, \quad i = 1, 2, \cdots, n,
$$

where $k_p$ and $k_f$ are the observer design parameters. In the above equation, $\dot{p}_i$ ($i = 1, 2, \cdots, n$) are the solutions to the following nominal pressure dynamics:

$$
\dot{p}_i = -\frac{\beta_0}{V_i(x, \beta)} A_e \dot{\nu}_i + \frac{\beta_0}{V_i(x, \beta)} K_{oi} u_i - \frac{P_i}{2} + \text{sign}(u_i) \frac{P_i}{2} + \hat{d}_i,
$$

$\quad i = 1, 2, \cdots, n$.

The details of the perturbation observer design and the stability proof of the observer-based control design for a first order system together with the stability of the combined system with inner and outer loop control can be found in [10].

### APPLICATION TO A 4-POST LIFT SYSTEM

A 4-post lift is used as an example to illustrate the implementation of the proposed approach. The objective of the 4-post lift is to lift the load evenly under variant load conditions. However in this particular application, the synchronization between cylinder pair 1, i.e. $e_{1y} = x_1 - x_2$ and that between cylinder pair 2, i.e. $e_{2y} = x_3 - x_4$, are more important than the synchronization error between the pairs 1 and 2, i.e. $e_{3y} = (x_1 + x_2 - x_3 - x_4) / 2$. To emphasize the synchronization performance, the motion dynamics described in Eq. (8) can be transformed into a task coordinates space which is defined by:

$$
x_m = [ (x_1 + x_2 + x_3 + x_4) / 4 \quad (x_1 + x_2 - x_3 - x_4) / 2 \quad x_1 - x_2 ]^T
$$

The transformation from the physical coordinates to the task space is given by:

$$
x_m = \begin{bmatrix}
1 & -1 & -1 & 1 \\
0 & 0 & 1 & -1 \\
-1 & -1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
x_p \\
\theta_1 \\
\theta_2
\end{bmatrix}
= L_{mp} L_{mq}^{-1} x_q = T^{-1} x_q
$$

where $T = L_{mp} L_{mq}^{-1}$.

Through the coordinate transform $T$, the motion dynamics can be transformed into the $x_m = T^{-1} x_q$ coordinates. Two of the three synchronization objectives can be explicitly emphasized in the control design. The remaining one can be imposed by introducing the constraints

$$
F_p = 0
$$

as an example. Because $F_p$ represents the force variation around the equilibrium point, Eq. (29) implies the additional objective of the design is to maintain the same pressures in cylinders 3 and 4. Other similar constraints can be imposed. Equation (29) can be written as $A_e F_p = b_e$, where $A_e = [0 \ 0 \ 1 \ -1]$ and $b_e = 0$.

Therefore, the actual control input can be calculated out following the design procedure discussed for n-post lift with $n=4$. 

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EXPERIMENTAL RESULTS

To verify the proposed control strategy, experiments were performed on the prototype 4-post lift system at Herrick Laboratories. Solenoid flow control valves were installed to restrain the flow into cylinders and its bandwidth is about 1 to 2 Hz. Potentiometers with 0.2 in. resolution are used to measure the position of the cylinders. In the proposed approach, full state information is required. Since only the position and pressure transducers are available, velocity information is derived online from the measured position signal. To reduce the effect of noise, low pass filters are used to filter out the high frequency noise in the measured position signal. The desired trajectory for the 4 cylinders in the experiments is a ramp signal with 1.2 in./sec velocity.

Three scenarios are considered in the experiments: centered loading, 15% off-centered load along the r axis and 15% off-centered load along the p axis, see Figure 6, which are defined as:

Percentage off-center along r axis = \( \frac{2d_r}{L_r} \),

Percentage off-center along p axis = \( \frac{2d_p}{L_p} \).

Figure 6: Diagram of Off-Centered Condition for the 4-Post Lift.

The sampling rate is 100 Hz. The parameters used in experiments are \( K_{p1} = K_{p2} = K_{p3} = K_{p4} = 15 \). The perturbation observer bandwidth is set at 15 Hz (\( k_p = 42\pi \), \( k_i = (30\pi)^2 \)).

And controller \( G(s) \) is:

\[
G(s) = \text{diag} \left( \begin{bmatrix} 200 & \frac{1400 + 1400}{s} & 8000(s + 1) \end{bmatrix} \right).
\]

Figure 7 shows the synchronization error between cylinders 1 (3) and 2 (4) as well as between cylinder pair 1 (cylinder 1 and 2) and pair 2 (cylinder 3 and 4) under a 3500lb centered load. The maximum synchronization error between cylinders 1 and 2 is 0.15 in. and between cylinders 3 and 4 is 0.18 in. The synchronization error between cylinder pairs 1 and 2 is about 0.2 in., which is at resolution limit of the position sensor. Figure 8 is the control efforts for the four valves. The control input commands are somewhat noisy however it is still acceptable.

Figure 9 shows the synchronization error when the load is 15% off-centered along the r axis. The peak value of the synchronization error between cylinders 1 and 2 and that of the cylinders 3 and 4 is about 0.2 in. The synchronization between cylinder pairs 1 and 2 is about 0.27 in. Figure 10 shows the control effort for the four valves. The responses are very similar to that of the centered load responses.

Figure 11 shows the synchronization error when the load is 15% off-centered along the p axis. The peak value of the synchronization error between cylinders 1 and 2 and that of the cylinders 3 and cylinder 4 is still about 0.2 in. The synchronization between cylinder pairs 1 and 2 is also around 0.2 in. Figure 12 shows the control effort for the four valves.

The experimental scenarios verified the effectiveness of the proposed approach. Synchronization error among the cylinders can be reduced to the sensor resolution level with appropriate loading conditions. The closed loop system is not sensitive to loading variations in value and location. The proposed design approach did not consider the valve dynamics and characteristic. The valve bandwidth is one of the most significant performance limiting factor as well as the sensor SNR. This significantly limited the bandwidth of the pressure dynamics and in turn limited the achievable bandwidth of the system.

CONCLUSION

In this paper, a two-step design approach is proposed for general n-post lift systems. 4-post lifts are used as an example to illustrate the effectiveness of the proposed approach. The increase of the actuators introduced the actuator redundancy. The two-loop design approach was introduced to this general case by reformulating the motion synchronization part of the design to incorporate additional design constrain to resolve the redundancy issue.

Experimental results showed feasibility of the proposed approach on a prototype 4-post lift with low cost flow control valves and position sensors. The experimental results showed that the controlled system can reduce the synchronization error to the resolution of the position sensors and reduce system sensitivity to load variations in value and location. Higher performance can be expected with higher bandwidth valves and higher resolution position measurement.

REFERENCES


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**Figure 7:** Synchronization Error with Centered Load with Lift Speed 1.2 in/sec.

**Figure 8:** Control Effort under Centered Load.
Synchronization between Cylinder 1 and 2

Synchronization between Cylinder 3 and 4

Synchronization Error between Cylinder Pair 1 and Pair 2

Figure 9: Synchronization Error under Off-Center Load along \( r \) Axis with Lift Speed 1.2 in/sec.

Synchronization between Cylinder 1 and 2

Synchronization between Cylinder 3 and 4

Synchronization Error between Cylinder Pair 1 and Pair 2

Figure 11: Synchronization Error under Off-Center Load along \( p \) Axis with Lift Speed 1.2 in/sec.

Control Effort of Valve 1

Control Effort of Valve 2

Control Effort of Valve 3

Control Effort of Valve 4

Figure 10: Control Effort under Off-Center Load along \( r \) Axis.

Figure 12: Control Effort under Off-Center Load along \( p \) Axis.