Abstract — This paper presents a nonlinear control algorithm to address the motion synchronization problem for a 2-cylinder electro-hydraulic (EH) system. A 2-step design approach is applied such that it utilized linear MIMO robust control technique to design an outer loop motion synchronization controller. A nonlinear SISO perturbation observer based pressure/force controller is designed for each of the lift cylinder as the inner loop controller to handle the nonlinearities associated with the EH actuators. Experimental results on a 2-cylinder system are presented to verify the effectiveness of the proposed approach.

I. INTRODUCTION

The problem of synchronizing multiple linear hydraulic actuators [3][9] arises in hydraulic operated heavy-duty applications such as lifting equipment and rolling mills, where the synchronous operation of multiple hydraulic actuators under load has important performance implications. This issue is most pronounced in hydraulically operated multi-actuator lifting equipment. Due to uneven loading and the inherent differences in multiple hydraulic circuits and components, the lift distance among the linear actuators will be different under open-loop operation. Specifically, the overall open-loop system is unstable in the sense that the difference in lift distance will increase and will eventually results in toppling the load.

There are three approaches to address the issue of synchronizing multiple hydraulic actuators. The simplest approach is to design a flow-divider circuit that will maintain the same cylinder velocity by maintaining the same flow rate to the cylinders. The performance of the synchronization is dependent upon the performance of flow-divider as well as the compressibility of the working fluid and the consistency of the hydraulic components. Another approach is to mechanically connect the hydraulic actuators through either cabling or other linkage design. The drawback to mechanical synchronisation is the added system weight and complexity as well as the limitation to the operation range of the equipment. Compared with pure hydraulic and mechanical approaches, electro-hydraulic synchronization provides a flexible alternative. With EH system, synchronization control strategies can be designed to handle uneven loading as well as uncertainties and external disturbances associated with the hydraulic system.

In 1994, Hogan and Burrow [3] looked at the issue of synchronizing unevenly loaded cylinders, individual control of each cylinder is required. However, with the added flexibility of individually controlled cylinders, Hogan and Burrow did not elaborate on the design of a control algorithm that will explicitly improve the synchronization performance. Xiong et al [9] proposed a model-reference adaptive control algorithm together with a cross-coupled controller [6] to improve synchronization performance as well as attempt to handle the parameter variation associated with the hydraulic systems. Chiu [1] formulated the synchronization of multiple motion axes in a geometrical framework and proposed three different approaches to explicitly address the motion synchronization issue.

In this paper we will consider the synchronization of two hydraulic ram type cylinders that are coupled through a linkage to an unknown load between 0 to 9000 lbs. The two cylinders are controlled by two individual servo valves. The hydraulic circuit diagram of the system is shown in Figure 1. A 2-step nonlinear motion synchronization control design is proposed to address the synchronization issue. The 2-step design approach utilized linear MIMO robust control technique to design an outer loop motion synchronization controller. A nonlinear SISO perturbation observer-based pressure/force controller is then designed for each of the cylinder as the inner loop controller.

II. SYSTEM MODELING

The remaining of this paper is organized as follows: The system modeling is presented in the next section followed by the 2-step MIMO nonlinear synchronizing controller design. The experimental results are presented in section IV. Conclusions together with discussion of ongoing investigations are presented in the last section.

Figure 1 Dual-Acting Hydraulic Circuit

The following equations can be obtained:
f_1 + f_2 - Mg = M\dot{\theta}_p, \tag{1}
-f_1L_1 + f_2L_2 = J\dot{\theta}

where \( x_p \) represents the position of the center of gravity of the load. \( M \) is the mass of the load and \( g \) is the gravitational constant. In Eq. (1), \( f_i \) represents the reaction force acting on cylinder \( i \) \((i = 1, 2)\) and \( L_i \) is the moment arm from the contacting point of \( f_i \) \((i = 1, 2)\) to the center of gravity of the system, which is assumed to be an unknown constant. \( J \) represents the rotational moment of inertia of the load about the rotational axis and \( \theta \) represents the rotation angle about the axis.

The fluid that flows into each cylinder is controlled by a servo valve. Consider the compressibility of the fluid in the cylinders and ignore the valve dynamics, the pressure dynamics in each cylinder can be represented by the following equation (Merritt [7]):

\[
P_i = \frac{\beta}{V_i(x_i)} \left( K_u u_i \sqrt{P_i f/2 + \text{sign}(u_i)(P_i f/2 - P)} - A_i x_i \right), \quad i = 1, 2, \tag{3}
\]

where \( \beta \) is the bulk modulus of the working fluid, \( V_i \) represents the total fluid volume from the output port of valve to the respective cylinder chamber and \( P_i \) is the source pressure. \( K_u \) is the valve flow coefficient and \( u_i \) represents the control input, i.e. the valve input command.

Assuming that the distance between the cylinders is much larger compared to the position difference between each cylinder, the angle \( \theta \) is small, i.e. \( \theta \ll 1 \). For small \( \theta \), we have \( \theta = \tan(\theta) = (x_2 - x_1)/L \).

Since \( x_i = (L_i/L)x_i + (L/L)x_i \), define the state vector \( x = [x_1 \ x_2 \ \dot{x}_1 \ \dot{x}_2 \ P \ P'] = [x_i'] \), \( i = 1, 2, ..., 6 \), and the input vector \( u = [u_1 \ u_2'] \). The state space representation of the system is:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{bmatrix} =
\begin{bmatrix}
0 & I & 0 & 0 \\
0 & 0 & M^{-1}B_p & 0 \\
0 & 0 & 0 & I
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
M^{-1} \tau_1
\end{bmatrix},
\tag{4}
\]

where

\[
\begin{align*}
\dot{x}_5 &= h_1(x) + g_1(x)u_1 \\
\dot{x}_6 &= h_2(x) + g_2(x)u_2
\end{align*}
\]

and

\[
M =
\begin{bmatrix}
c_1 & c_2 \\
c_3 & c_4
\end{bmatrix},
\tau =
\begin{bmatrix}
A_1 x_5 - F_{R_1} - R_1 \\
A_2 x_6 - F_{R_2} - R_2
\end{bmatrix},
\tag{6}
\]

The effective loads \( R_1 \) and \( R_2 \) are defined to be \( R_i = m_i g + (L_i/L)Mg \) and \( R_i = m_i g + (L_i/L)Mg \), respectively.

Eq. (4) describes a nonlinear 2-input 2-output system where the two cylinders are coupled through the \( c_2 \) and \( c_3 \) terms. Note that the coupling terms \( c_2 \) and \( c_3 \) will be zero if there is no load, i.e. \( M = 0 \). The amount of coupling thus depends on the loading of the system, i.e. the mass and location of the load. Note in Eq. (4), the first 4 states represent a 2-input 2-output linear invariant system with input \( [\tau_1 \ \tau_2]' \) and output \( [x_1 \ x_2]' \). The remaining pressure dynamics are nonlinear. In the load dynamics, potential uncertainties are the mass \( M \) and locations \( L_i \) \((i = 1, 2)\). However no direct load uncertainties appear explicitly in the pressure dynamics. This characteristic will be further exploited in the control design. In the pressure dynamics, both parameter uncertainty and uncertain nonlinearity exist due to variation in components and flow characteristics of the valves.

III. CONTROLLER DESIGN

Note that the rigid-body motion dynamics as described by Eq. (4),

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{bmatrix} =
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix},
\tag{5}
\]

where

\[
M =
\begin{bmatrix}
c_1 & c_2 \\
c_3 & c_4
\end{bmatrix},
\tau =
\begin{bmatrix}
A_1 x_5 - F_{R_1} - R_1 \\
A_2 x_6 - F_{R_2} - R_2
\end{bmatrix},
\tag{6}
\]

and

\[
\begin{align*}
\dot{x}_5 &= h_1(x) + g_1(x)u_1 \\
\dot{x}_6 &= h_2(x) + g_2(x)u_2
\end{align*}
\]

\[
M =
\begin{bmatrix}
c_1 & c_2 \\
c_3 & c_4
\end{bmatrix},
\tau =
\begin{bmatrix}
A_1 x_5 - F_{R_1} - R_1 \\
A_2 x_6 - F_{R_2} - R_2
\end{bmatrix},
\tag{6}
\]

and

\[
\begin{align*}
\dot{x}_5 &= h_1(x) + g_1(x)u_1 \\
\dot{x}_6 &= h_2(x) + g_2(x)u_2
\end{align*}
\]

\[
M =
\begin{bmatrix}
c_1 & c_2 \\
c_3 & c_4
\end{bmatrix},
\tau =
\begin{bmatrix}
A_1 x_5 - F_{R_1} - R_1 \\
A_2 x_6 - F_{R_2} - R_2
\end{bmatrix},
\tag{6}
\]
is a linear MIMO system with load uncertainties from the force inputs \(\tau = [\tau_x, \tau_z]^T\) to position output \([x_i, x_j]^T\). In addition, as can be seen from Eq. (4), the nonlinear pressure dynamics do not explicitly include any load uncertainties. These two characteristics are the motivation to separate the synchronization problem into two separate portions: one is the linear MIMO motion synchronization control problem and the other is the nonlinear force control problem. For the MIMO linear motion synchronization, linear robust MIMO design techniques can be used to handle load uncertainties. The force control problem for each individual cylinder is a SISO nonlinear control problem with unmodeled nonlinearity and parameter uncertainties. Backstepping design technique is applied to combine the two parts together such that the desired force tracking trajectory for each cylinder is obtained from the linear motion synchronization control, i.e. the pressure/force control is an inner loop to the motion synchronization control.

In Eq. (6), the force input vector \([\tau_x, \tau_z]^T\) is the resultant force acting on the cylinders that include the forces produced by the pressures in the cylinders, the friction forces and equivalent load for each cylinder. The facts that the load is unknown and the frictional forces are difficult to model lead to the uncertainty of the term (\(F_p + R\)). Since the initial pressures in the cylinders, \(x_{50}\) and \(x_{60}\), can be used to estimate \((F_p + R)\), let

\[
d_f = \begin{bmatrix} d_{f_1} \\ d_{f_2} \end{bmatrix} = \begin{bmatrix} A_1 x_{50} - F_{f_1} - R_1 \\ A_2 x_{60} - F_{f_2} - R_2 \end{bmatrix},
\]

then \(\tau\) can be written as

\[
\tau = \begin{bmatrix} A_1 (x_5 - x_{50}) \\ A_2 (x_6 - x_{60}) \end{bmatrix} + \begin{bmatrix} A_1, x_{50} - F_{f_1} - R_1 \\ A_2, x_{60} - F_{f_2} - R_2 \end{bmatrix} = \mathbf{f}_p + \mathbf{d}_f.
\]

where \(\mathbf{f}_p = \begin{bmatrix} f_{p_1} \\ f_{p_2} \end{bmatrix}\) are the desired cylinder force output.

Without loss of generality, \(d_f\) is assumed to be bounded. Substitute Eq. (7) into Eq. (5) the rigid body motion dynamic can be written as:

\[
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}
\]

\[
\begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \mathbf{M} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + \mathbf{B}_p \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \mathbf{f}_p + \mathbf{d}_f.
\]

(8)

To emphasize the synchronization performance, the motion dynamics described in Eq. (8) will be transformed from the physical coordinates to a new task orientated coordinates that will be described next.

A. Task-space coordinates

Since the motion synchronization objective can be written as \(x_1 - x_2 = 0\), a task coordinate frame expand by the basis vectors \([q_i, q_j]^T = [x_i + x_2, x_i - x_2]^T\). The transformation between the original physical coordinate frame expand by \([x_i, x_2]\) and the task coordinate frame is

\[
T = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}.
\]

The motion dynamics expressed in the task coordinated frame can be written as:

\[
\begin{bmatrix} \dot{q}_i \\ \dot{q}_j \end{bmatrix} + \mathbf{B}_p \begin{bmatrix} \dot{q}_i \\ \dot{q}_j \end{bmatrix} = \begin{bmatrix} f_{p_1} \\ f_{p_2} \end{bmatrix} + \begin{bmatrix} d_{f_1} \\ d_{f_2} \end{bmatrix},
\]

(9)

where \(\mathbf{M} = T^T \mathbf{M} T\) and \(\mathbf{B}_p = T^T \mathbf{B}_p T\) and

\[
\begin{bmatrix} f_{p_1} \\ f_{p_2} \end{bmatrix} = T^{-1} \mathbf{f}_p = \begin{bmatrix} f_{p_1} + f_{p_2} \\ f_{p_1} - f_{p_2} \end{bmatrix}, \quad \begin{bmatrix} d_{f_1} \\ d_{f_2} \end{bmatrix} = T^{-1} \mathbf{d}_f = \begin{bmatrix} d_{f_1} + d_{f_2} \\ d_{f_1} - d_{f_2} \end{bmatrix}.
\]

The control design for the motion dynamics will be completed in the task coordinates where the control law \([f_{p_1}, f_{p_2}]\) will be determined. The actual desired force input in the physical coordinates will be obtained through the inverse transform \(T\).

B. Controller design

Although any MIMO controller design methodology can be applied, a sequential loop Qualitative-Feedback-Theory (QFT) design is used as an example to obtain the desired pressures that are needed to maintain synchronization of two cylinders under various loading conditions. A nonlinear observer based control technique will then be applied on the hydraulic pressure dynamics to achieve the given desired pressure using the back-stepping procedure.

1) Linear MIMO design for motion synchronization

Given the parameter uncertainties in the rigid body motion dynamics, adaptive control or robust control techniques can be used to design a desired outer loop synchronization controller. However, since the adaptive control tends to introduce more complicated controllers, robust control approach is used to design an outer loop motion synchronization controller. Among the various robust control design method, the QFT design is employed. It should be noted that although the QFT approach is used in the subsequent discussion, the proposed approach is able to accommodate any robust control design methodology.

For motion synchronization, regulating \(q_t\) to zero is more important compared with \(q_i\) tracking a desired profile. The synchronization problem can be formulated as a disturbance rejection problem, see Figure 3, where the 2x2 controller matrix \(G\) is to be designed to reject the disturbance \(\mathbf{d} = [d_1, d_2]^T\) given the plant transfer function matrix \(P(s) = (\mathbf{M}^T + \mathbf{B}_p)^{-1}\) with bounded load uncertainties.

Let \(S_{zd}\) be the transfer function matrix from the disturbance \(\mathbf{d}\) to the output \(\mathbf{q}\), then

\[
\begin{bmatrix} q_x(s) \\ q_y(s) \end{bmatrix} = S_{zd} \begin{bmatrix} d_1(s) \\ d_2(s) \end{bmatrix}, \quad S_{zd} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}.
\]

For a step disturbance, the synchronization performance can be emphasized by choosing \(0 < \mathbf{e}_{zd} < \mathbf{e}_{zd}\) such that

\[
|q_x(s)| = |s_{11} d_1 + s_{12} d_2| \leq \mathbf{e}_{zd}.
\]

(10)

Following Franke\[\text{h}\], Eq. (10) also implies time domain constraints, \(|q_x(t)| \leq \mathbf{e}_{zd}\) and \(|q_y(t)| \leq \mathbf{e}_{zd}\). Hence, \(\mathbf{e}_{zd}\) and \(\mathbf{e}_{zd}\) will also be used as appropriate time domain specifications.
To further simplify the design, the worst-case disturbance system could encounter in the direction is considered. Hence $d$ can be expressed by $d = \lambda \Gamma$, where $\lambda$ is a positive constant that represents the largest magnitude of the disturbance that system could encounter in the $\Gamma$ direction. Substituting $d$ into Eq. (10), one set of the performance specification can be written as:

$$
\begin{align*}
|q_i(s)| &= |s_1 \gamma_i + s_2 \gamma_i| |\lambda| \leq \epsilon_{ud}, \\
|q_i(s)| &= |s_1 \gamma_i + s_2 \gamma_i| |\lambda| \leq \epsilon_{ud}, \\
|q_i(s)| &= |t_1 d| \leq \epsilon_{ud},
\end{align*}
$$

(11)

Let $T_{yd}$ be the transfer matrix from the reference $r$ to the output $q$:

$$
\begin{bmatrix}
q_1(s) \\
q_2(s)
\end{bmatrix} = T_{yd} \begin{bmatrix}
q_{id}(s) \\
0
\end{bmatrix},
\end{equation}

where $T_{yd} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}.$

Note that the synchronization error is also affected by the tracking error $q_{id}$ through the cross-coupling terms. To minimize the effect of the tracking error on the synchronization error, the following specification is applied.

$$
|q_1(s)| = |s_1 \gamma_1 + s_2 \gamma_1| |\lambda| \leq \epsilon_{ud},
$$

(12)

where $\epsilon_{ud}$ is a given positive constant. Combining Eq. (11) and (12), the specifications for the robust QFT design are:

$$
\begin{align*}
|q_1(s)| &= |s_1 \gamma_1 + s_2 \gamma_1| |\lambda| \leq \epsilon_{ud}, \\
|q_2(s)| &= |s_1 \gamma_2 + s_2 \gamma_2| |\lambda| \leq \epsilon_{ud}, \\
|q_1(s)| &= |t_1 d| \leq \epsilon_{ud},
\end{align*}
$$

(13)

Following the standard MIMO QFT design procedure [4], a diagonal controller

$$
G(s) = \begin{bmatrix}
g_1(s) & 0 \\ 0 & g_2(s)
\end{bmatrix}
$$

can be designed to satisfy the desired performance inequalities under bounded uncertainties. The input to the individual internal pressure loop can be obtained by:

$$
\begin{bmatrix}
f_{p1} \\
f_{p2}
\end{bmatrix} = T_\Gamma \begin{bmatrix} g_1(s) & 0 \\ 0 & g_2(s) \end{bmatrix} T_\Gamma^{-1} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = G(s) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.
$$

(14)

Note that if we choose $g_1(s) = K_{p1}$ and $g_2(s) = K_{p2} + K_{i} / s$, then the resulting controller is a cross-coupled PI controller.

### 2) Nonlinear force control design

Once the desired force (pressure) profile is given by the outer motion synchronization loop, a force feedback controller is needed for each of the lift cylinder. The nonlinear perturbation observer-based force control technique will be applied to the pressure dynamics. The pressure dynamics in Eq. (4) can be written as:

$$
\begin{align*}
\dot{x}_{q1} &= h_1(x, \beta) + \phi_1(x) \beta K_{gb} u_1 + d_{i1}, \\
\dot{x}_{q2} &= h_2(x, \beta) + \phi_2(x) \beta K_{gb} u_2 + d_{i2},
\end{align*}
$$

(15)

where $\beta = \beta_{gb} + \beta_{qu}$, and

$$
\begin{align*}
\dot{h}_i &= h_i(x, \beta), \\
\dot{h}_i &= h_i(x, \beta) + \phi_i(x) \beta K_{gb} u_i + d_{i1},
\end{align*}
$$

(16)

with $h_i(x, \beta) = h_i(x, \beta) - h_i(x, \beta)$, $i = 1, 2$, represent the parametric uncertainties in the flow gain of the servo valves and bulk modulus.

The desired cylinder forces $f_{p1}$ and $f_{p2}$ have been obtained from the outer motion synchronization loop. However, since we can only control the cylinder output force through the hydraulic pressure dynamics, the inner pressure loop design is to minimize the difference between the actual cylinder output force and the desired cylinder force. Let

$$
P_d = \begin{bmatrix} P_{d1} \\ P_{d2} \end{bmatrix} = \begin{bmatrix} f_{p1} / A_i \\ f_{p2} / A_i \end{bmatrix}
$$

be the desired pressure profile, then the pressure tracking error $z_i$ can be defined by

$$
\begin{align*}
z_{i1} &= x_1 - x_{i0} - P_{d1}, \\
z_{i2} &= x_2 - x_{i0} - P_{d2},
\end{align*}
$$

(17)

The pressure tracking error dynamics can be written as:

$$
\begin{align*}
z_i &= -\frac{\beta}{V_i(x_i)} A_{x_i} + \frac{\beta}{V_i(x_i)} K_{gb} \sqrt{\frac{P_i}{2}} + \text{sgn}(u_i) \left( \frac{P_i}{2} - x_i \right) - P_{d1} + d_1, \\
z_i &= -\frac{\beta}{V_i(x_i)} A_{x_i} + \frac{\beta}{V_i(x_i)} K_{gb} \sqrt{\frac{P_i}{2}} + \text{sgn}(u_i) \left( \frac{P_i}{2} - x_i \right) - P_{d2} + d_2,
\end{align*}
$$

(18)

where $d_1$ and $d_2$ are the perturbations that include parameter uncertainties and unmodeled nonlinearities. Let the state space representation of the controller $G_d(s)$ be:

$$
\eta = A_d \eta + B_d \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},
$$

(19)

then the time derivative of the $P_d$ can be calculated from the above equation:

$$
P_d = \begin{bmatrix} A_{d1} & 0 \\ 0 & A_{d2} \end{bmatrix} C_{\eta} \begin{bmatrix} A_d \eta + B_d \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + D_d \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{bmatrix}.
$$

(20)

Note that the control law described by Eq. (17) does not require $\dot{x}_i$ and $\dot{z}_i$ explicitly. This avoids the complication of propagating the loading uncertainties into the design of the pressure dynamics.

The following control law is applied to achieve pressure tracking in each of the cylinders:

$$
u_i = V_i(x_i) \frac{\beta}{\sqrt{V_i(x_i)}} A_{x_i} + \frac{\beta}{V_i(x_i)} P_{d1} - \dot{z}_i - K_{gb} z_i,
$$

(21)

where $K_{gb} > 0$ are the design parameters that represent the proportional feedback gain. Note that in the above control law, $P_{d1}$ can be calculated from Eq. (20) and no differentiation of $P_{d1}$ is required. $\dot{z}_i$ is the estimated perturbation which can be obtained from the following equations:

$$
\begin{align*}
q_1(s) &= |s_1 \gamma_1 + s_2 \gamma_1| |\lambda| \leq \epsilon_{ud}, \\
q_2(s) &= |s_1 \gamma_2 + s_2 \gamma_2| |\lambda| \leq \epsilon_{ud}, \\
q_1(s) &= |t_1 d| \leq \epsilon_{ud},
\end{align*}
$$

(16)
\[
\begin{align*}
\ddot{d}_1 &= k_p (\hat{x}_0 - x_0) + k_l \int (\hat{x}_0 - x_0) \, dt , \\
\ddot{d}_2 &= k_p (\hat{x}_0 - x_0) + k_l \int (\hat{x}_0 - x_0) \, dt \\
\end{align*}
\]

where \( k_p \) and \( k_l \) are the perturbation observer design parameters. In the above equation, \( \hat{x}_0 \) and \( \dot{x}_0 \) are the solutions to the following nominal pressure dynamics:

\[
\begin{align*}
\dot{x}_0 &= -\frac{\beta_0}{V_1(x_1)} A x_0 + \frac{\beta_0}{V_1(x_1)} K_{20} \sqrt{\frac{P}{2} + \text{sgn}(u_l) \left( \frac{P}{2} - x_0 \right)} x_1 + \dot{d}_1 , \\
\dot{x}_0 &= -\frac{\beta_0}{V_2(x_2)} A x_0 + \frac{\beta_0}{V_2(x_2)} K_{20} \sqrt{\frac{P}{2} + \text{sgn}(u_l) \left( \frac{P}{2} - x_0 \right)} x_1 + \dot{d}_2 .
\end{align*}
\]

The stability of the combined system, pressure tracking error dynamics together with the perturbation observer, can be shown through the application of the passivity theorem. Note that uncertainties of both bulk modulus and flow coefficient are considered in design. The larger the lumped perturbation \( d \) is, the larger perturbation estimated errors are. Therefore for a given control gain, a small perturbation will have better pressure tracking performance than the large one. The stability proof of the combined inner and outer loop design can be shown through the back-stepping procedure that is detailed in Appendix A.

IV. EXPERIMENTAL RESULTS

Experiments were performed on a two-cylinder hydraulic lifting apparatus to verify the proposed control algorithm. Several experiments were conducted under different loading conditions. The sampling rate is set at 500Hz. Parameters used in the control algorithm are: \( K_{20} = K_{21} = 40 \) and the perturbation observer bandwidth is set at 15 Hz (\( k_p = 42\pi \) and \( k_l = (30\pi)^2 \)).

Figure 4 shows the synchronization error and tracking error under a 3500lb centered load. Even though there is large tracking error, the synchronization error of the EH controlled system is significantly smaller because of the emphasizing of the synchronization error dynamics in design process. Figure 5 illustrates the synchronization error with and without load. Even though the synchronization error with a 3500lb centered load is larger than with no load, the synchronization error is still less than 0.012 in. This is 25 times better than the existing mechanical solution. Figure 6 shows the effect of uncertain load locations on synchronization error. Overall, the synchronization error is still an order of magnitude better than the existing mechanical (cable) solution. The large difference between the synchronization error of the positive and negative off-center loading conditions suggested the significant difference in the dynamics of the two cylinders. One potential contribution factor is the length of the hydraulic hoses, there is an 800% difference between the two cylinders (9 m versus 1 m). However, the performance is still significantly better than using mechanical linkages.

V. CONCLUSION

In this paper, an electro-hydraulic motion synchronization control algorithm is proposed. The controller is synthesized through a two-step process using a linear MIMO motion synchronization controller as the outer loop controller and a perturbation observer based pressure controller as the inner loop controller. Stability of the combined system is proved using a backstepping procedure. Experimental result validated the effectiveness of the proposed approach and demonstrated an order of magnitude improvement than the existing mechanical synchronization solution. Method to extend to 3 and above actuators will be investigated.

REFERENCES

Consider the pressure dynamics described in Eq. (15). \( h_i(x) \) and \( d_{si}(x) \) \((i = 1, 2)\) are assumed to be bounded. \( \phi(x) \) \((i = 1, 2)\) do not equal to zero under normal operation range. Substitute the control law described by Eq. (19) into Eq. (16) and notice that
\[
\dot{d}(s) = Q(s)d(s) = \frac{k_s s + k_i}{s^2 + k_s s + k_i} d(s),
\]
we can obtain the following:
\[
\dot{z}_i = \frac{1}{1 + \alpha Q(s)}(-K_{i0}(1 + \alpha)z_i + \bar{\eta}_i), \quad i = 1, 2,
\]
where \( \alpha = K_{i0}/K_{w0} \) \((i = 1, 2)\). Since in Eq. (A.3), \( \bar{\eta}_i = (1 - Q(s)) \left( K_{w0}d_{i0} + h_i(x, \beta_0) + d_s(x) \right) \) \((i = 1, 2)\) are bounded. Eq. (A.3) can then be written as:
\[
\dot{z}_i(s) = \frac{1}{1 + \alpha Q(s)s + K_{w0}(1 + \alpha)} \bar{\eta}_i, \quad i = 1, 2
\]
The dynamics represented by Eq. (A.4) is \( L_2 \) stable, which can be shown from the passivity theorem. Let the state space representation of Eq. (A.4) be
\[
\dot{X}_i^2 = A_i X_i^2 + B_i \bar{\eta}_i, \quad i = 1, 2.
\]
Let \( X_i = [X_{2i} \ 0]^T \), \( \bar{z} = [z_i \ \hat{z}_i]^T \) and \( \bar{\eta} = [\bar{\eta}_i \ \hat{\eta}_i]^T \), then the Eq. (A.5) can be written as:
\[
\dot{X}_i = A_i X_i + B_i \bar{\eta},
\]
where \( A_i = \text{diag}(\{A_{1i} \ A_{2i}\}) \), \( B_i = \text{diag}(\{B_{1i} \ B_{2i}\}) \), \( C_i = \text{diag}(\{C_{1i} \ C_{2i}\}) \), and \( D_i = \text{diag}(\{D_{1i} \ D_{2i}\}) \). Since Eq (A.4) is \( L_2 \) stable, this implies that \( A_i \) is negative definite. Combining the synchronized motion dynamics described by Eq. (A.2) and the pressure error dynamics described by Eq. (A.6), the augmented closed-loop system dynamics can be represented by:
\[
\begin{bmatrix}
\dot{X}_1 \\
\eta \\
\dot{X}_2
\end{bmatrix} = \begin{bmatrix}
A & B & 0 \\
B & C & 0 \\
0 & 0 & D
\end{bmatrix}
\begin{bmatrix}
X_1 \\
\eta \\
X_2
\end{bmatrix} + \begin{bmatrix}
B_1 \\
0 \\
B_2
\end{bmatrix} \begin{bmatrix}
d_1 \\
d_2
\end{bmatrix},
\]
(\text{A.7})
Since
\[
\text{eig} \begin{bmatrix}
A & B & 0 \\
B & C & 0 \\
0 & 0 & D
\end{bmatrix} = \text{eig} \begin{bmatrix}
A_i & B_i & 0 \\
B_i & C_i & 0 \\
0 & 0 & D_i
\end{bmatrix} \cdot \text{eig} \begin{bmatrix}A_z \end{bmatrix}
\]
and that both matrices
\[
\begin{bmatrix}
A & B & 0 \\
B & C & 0 \\
0 & 0 & D
\end{bmatrix}
\]
and \( A_z \) are Hurwitz, then the closed-loop system matrix
\[
\begin{bmatrix}
A & B & 0 \\
B & C & 0 \\
0 & 0 & A
\end{bmatrix}
\]
is also Hurwitz.