Breathers are localized waves in nonlinear systems that undergo a periodic variation in time or space. The concept of breathers is useful for describing many nonlinear physical systems including granular lattices, Bose-Einstein condensates, hydrodynamics, plasmas, and optics. In optics, breathers can exist in either the anomalous or the normal dispersion regimes, but they have only been characterized in the former, to our knowledge. Here, externally pumped optical microresonators are used to characterize the breathing dynamics of localized waves in the normal dispersion regime. High-$Q$ optical microresonators featuring normal dispersion can yield mode-locked Kerr combs whose time-domain waveform corresponds to circulating dark pulses in the cavity. We show that with relatively high pump power these Kerr combs can enter a breathing regime, in which the time-domain waveform remains a dark pulse but experiences a periodic modulation on a time scale much slower than the microresonator round trip time. The breathing is observed in the optical frequency domain as a significant difference in the phase and amplitude of the modulation experienced by different spectral lines. In the highly pumped regime, a transition to a chaotic breathing state where the waveform remains dark-pulse-like is also observed, for the first time to our knowledge; such a transition is reversible by reducing the pump power.

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consistent with numerical simulations based on the LLE. Moreover, simulations reveal the importance of modulation of switching waves (also termed as domain walls or fronts) [24,34,35] in the breathing dynamics.

We start our investigation with simulations based on the LLE [12,30,33], which can be written as

\[
\left( t_\nu \frac{\partial}{\partial t} + \frac{\alpha}{2} + i \delta_0 + \frac{i \beta_2 L \partial^2}{2} \right) E - i \gamma L |E|^2 E = \sqrt{\kappa} E_{\text{in}},
\]

where \( E \) is the envelope of the intracavity field, \( t \) and \( \tau \) are slow time and fast time variables, respectively, \( t_R \) and \( L \) are the round trip time and cavity length, \( \beta_2 \) and \( \gamma \) are the group velocity dispersion and nonlinearity coefficient, \( \alpha \) and \( \kappa \) are the total loss and coupling loss, \( E_{\text{in}} \) is the amplitude of the pump field, and \( \delta_0 \) is the pump phase detuning (angular frequency detuning to the closest resonance multiplied by \( t_R \)). Simulation parameters used are \( t_R = 4.3 \text{ ps} \), \( \beta_2 = 187 \text{ ps}^2/\text{km} \), \( L = 628 \mu \text{m} \), \( \gamma = 0.9 \text{ (Wm)}^{-1} \), \( \alpha = 0.0068 \), \( \kappa = 0.0054 \), which are close to experimental parameters and enable direct comparison with experiments. The simulation starts with a noise field in the microresonator, and the detuning \( \delta_0 \) is tuned to generate the comb [see the Supplemental Material [36] for a detailed view of the path to comb generation]. A breathing dark pulse is obtained with a pump power of 700 mW (64% of the experimental on-chip power) and a final detuning of 0.0629 radians (equivalent to a laser frequency 2.3 GHz to the red of the resonance). We show the simulated breathing dynamics together with representative spectra and waveforms in Fig. 1. Both the power spectrum and the temporal intensity profile of the comb vary periodically, repeating with a period of 1.3 ns (782 MHz), ~300 times longer than \( t_R \). The formation of dark pulses can be understood as arising from interlocked switching waves, which are traveling front solutions connecting the bistable homogeneous state of the LLE [24,34,35]. The background field (i.e., the high power part of the dark-localized waveform) stays nearly unchanged during the breathing cycle, because it follows the homogeneous stable state [22,23,34]. Because the background field carries most of the comb power (average power over one round trip time), the breathing depth of the comb power, which is defined as \( (P_{\text{max}} - P_{\text{min}})/(P_{\text{max}} + P_{\text{min}}) \) with \( P_{\text{max(min)}} \) being the maximum (minimum) power, is relatively low. The simulated breathing depth of the intracavity comb power including (excluding) the pump is 3% (6%). During the breathing, the switching waves are modulated as illustrated both by Fig. 1(b) and Animation 1. The corresponding modulation in the tails of the switching waves results in the stronger breathing in the waveform hole (the low power part of the dark-localized waveform). Because the dark pulse has a complex waveform, the breathing within the waveform hole is also complex.

We have performed experiments in three different (see Table I) optical microresonators which provide data supporting the simulated dynamics. We focus our discussion on results from a silicon nitride microresonator (device 1 in Table I) with a radius of 100 \( \mu \text{m} \), a loaded \( Q \) factor of \( 0.8 \times 10^6 \), geometry of \( 2000 \text{ nm} \times 600 \text{ nm} \) (waveguide width vs height), a free spectral range (FSR) of 231 GHz, and strong normal group velocity dispersion \( \beta_2 = 190 \text{ ps}^2/\text{km} \). The experimental setup is sketched in Fig. 2(a). Similar to bright soliton breathers [12,14–16], the breathing dark pulse state exists in a regime with relatively strong pump power and small detuning [22,23]. Hence, we generate our comb with a relatively large on-chip pump power of 1.1 W (obtained by measuring the off-chip power and adjusted with ~2 dB fiber-to-chip coupling loss). By tuning the continuous wave (cw) pump laser into resonance from the blue to the red, we are able to generate a broadband, single FSR comb [Fig. 2(b)]. In this state the comb power measured by a fast photodetector exhibits narrow radio-frequency (rf) tones as shown in Fig. 2(c). The fundamental frequency of the rf tones is 740 MHz, in close agreement with the simulated breathing frequency and much larger than the resonance linewidth which has half-width-half-maximum (HWHM) of 125 MHz. The other normal dispersion devices listed in Table I also show narrow rf tones that can be attributed to dark pulse breathing, supporting the generality of our observation. The observed breathing frequencies decrease with increasing loaded \( Q \) factor (decreasing resonance linewidth).

![FIG. 1. (a) Simulated breathing dynamics of optical power spectrum, showing complex breathing behavior. (b) Simulated breathing dynamics in the time domain. The switching waves (SWs) are modulated during breathing and the waveform hole breathes strongly. Above each panel we show snapshots of the spectrum and time-domain waveform at moments indicated by the dashed lines.](image_url)
We can transition to a stable dark pulse state by reducing the pump power to 1.0 W at the same wavelength and can return to the breathing state by increasing the pump power back to 1.1 W. At the lower pump power, the comb shows low-noise operation [Figs. 2(d) and 2(e)]. Again, the simulated stable dark pulse gives an optical spectrum in agreement with experiment. The time averaged optical spectra of the breathing and stable dark pulse combs show no significant difference. This is a consequence of the weak overall breathing and is a key difference with respect to bright soliton breathers [14,16]. There is a spike in the spectrum around 1657 nm due to interaction with another spatial mode, which is reproduced by inclusion of mode interaction in the simulations [see the Supplemental Material [36]]. Note that the existence of mode interaction was observed to cause intermode breathing in one recent report on anomalous dispersion microresonators [17]. We have verified that mode interaction is not responsible for the observed breathing in our experiments by changing the chip temperature to a value where the mode interaction effects vanish [see the Supplemental Material [36]].

To establish the observation of a breathing dark pulse, we first need to verify that the comb in Figs. 2(b) and 2(c) has a dark-localized waveform. We use line-by-line pulse shaping to probe the corresponding waveform [24,39,40]. By adjusting the phases of the comb lines to compress them into a transform-limited pulse train, we can retrieve the comb phase profile and the corresponding waveform [see the Supplemental Material [36] for methods]. In experiments, we first adjust the pump power to reach the stable dark pulse state and apply pulse shaping to compress the output into transform-limited pulses, whose intensity autocorrelations (ACs) are shown in Fig. 3(a). We then adjust the pump power to transition into the breathing state, while keeping the same phase profile on the pulse shaper. The output is still compressed into a transform-limited pulse. This comparison suggests that the breathing dark pulse has a spectral phase profile very close to that of the stable dark pulse. Because the breathing dark pulse only exhibits a gentle modulation, we deduce that the breathing comb retains a dark-localized waveform [inset of Fig. 3(a)]. The simulated AC of the breathing dark pulse after numerical pulse compression and averaging over slow time [i.e., \( t \) in Eq. (1)] is close to the AC of the stable comb (see the Supplemental Material [36] for methods to obtain the numerical AC) [Fig. 3(b)]. In experiments on another microresonator (device 2), we have confirmed that a dark-localized waveform is retained during breathing by using an alternative approach based on an optical sampling oscilloscope [see the Supplemental Material [36]].

Next, we confirm that the form of the comb varies periodically and examine the breathing of individual lines. To do that, we program the pulse shaper to select individual comb lines and then measure their time-dependent power using a photodetector. A sample of the full comb is also detected; its power modulation under breathing is used as a
FIG. 3. (a),(b) By line-by-line pulse shaping, stable dark pulses can be compressed into transform-limited pulses (the shaped comb lines contain 90% of the comb power). Applying the same phase on the pulse shaper, the breathing dark pulses can also be compressed into transform-limited pulses. The intensity autocorrelations (ACs) of the compressed pulses are plotted in the figure. Using the phase retrieved in the line-by-line pulse shaping, the actual intracavity field is deduced to have a dark-localized waveform for the breathing Kerr comb (inset). The averaged simulated AC trace of the pulse-shaped breathing dark pulse comb [see the Supplemental Material [36] for numerical methods] is close to the AC of the pulse-shaped stable dark pulse comb. The inset is an example of a snapshot of the breathing dark pulse (SW: switching wave). The AC in simulations is shorter, because it calculates the full spectrum while only a portion of the spectrum is shaped in experiments. (c),(d) The breathing depth (blue) and phase (red) of individual comb lines. The breathing depths tend to be larger at the wings of the spectrum; different lines also breathe with different phases. Both the breathing depth and phase can change abruptly between adjacent lines.

In summary, optical breathing dark pulses are clearly observed and comprehensively modeled for the first time, using normal dispersion microresonators. The breathing dark pulse features a high frequency modulation and a weak modulation depth. Different comb lines breathe in distinct manners, a behavior which is related to a modulation of the switching waves and their oscillating tails. This behavior is in sharp contrast to the dynamics observed in breathing bright solitons in anomalous dispersion microresonators. At higher pump power, the breathing dark pulse undergoes a
and Curtis Menyuk.

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reversible chaotic transition, where a dark-localized wave-
form is retained despite the broadband intensity noise. The
breathing instability can impair applications which need
stable combs; knowledge of breathing dark pulses can help
us avoid this instability. Our Letter also suggests high Q
microresonator can be a useful platform to study nonlinear
dynamics, especially in the highly driven regime.

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\*cbao@caltech.edu

Present address: T. J. Watson Laboratory of Applied
Physics, California Institute of Technology, Pasadena
91125, USA.

\*amw@purdue.edu

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