Microresonator Frequency Combs for Integrated Microwave Photonics

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***Abstract*— Microresonator-based Kerr frequency combs (microcombs) have many outstanding features, including chip-level integration, high repetition rate, and ultrabroad bandwidth. These merits make microcombs very promising for integrated microwave photonic applications. Here, we provide a brief review on recent progress of microcomb generation and its application in microwave photonics. Both coherent and incoherent examples are discussed, including low-phase-noise microwave and optical frequency synthesis, programmable photonic microwave signal processing, true time delay beamforming, and channelized receiver. An outlook on the future research direction and key problems is also given.**

***Index Terms*— Delay lines, finite impulse response filters, microwave photonics, optical resonators, optical solitons, phased arrays.**

I. Introduction

Microwave photonics, also named radiofrequency (RF) photonics, is an interdisciplinary area that involves microwaves and light – two types of electromagnetic waves with nearly 3- to 5-order difference in their frequencies [1]. The concept was first proposed in the 1960s when the research was focused on high-speed optoelectronic devices operating at the microwave frequencies. The first successful and perhaps the most important application is analog fiber links which are now widely used in radio-over-fiber access networks, radar, remote clock synchronization, and square-kilometer telescope arrays. The unique advantage of fiber wireless is largely due to the ultralow loss and broad bandwidth of optical fibers compared to copper cables. Besides transmission, more microwave functions may be achieved with the help of photonics, such as signal generation (including low-noise oscillators and arbitrary waveform generation), signal processing (filtering or frequency conversion), signal control and manipulation (time delay or phase shifting), measurement and sensing. The research in microwave photonics nowadays has become quite versatile with a continuously increasing number of publications. Recently, integrated microwave photonics has emerged as a promising direction [2]. Integrated microwave photonics takes advantage of photonic integration techniques to combine the basic building blocks on a single chip or in a compact package, thus to reduce the volume and increase the stability. Great successes have been demonstrated in integrating high-speed modulators, photodetectors, low-loss delay lines, filters, couplers, etc. Integrated laser sources with low noise and narrow linewidth are important for microwave photonic applications. One kind of particularly useful source is the optical frequency comb, the spectrum of which contains a series of equally spaced lines [3]. The most famous application of optical frequency combs has been in optical frequency synthesis and metrology where an optical frequency is linked to a microwave frequency with an ultrabroad comb [4]. Traditional comb sources include mode-locked lasers and electro-optic combs. Both have integrated on-chip versions demonstrated now [5]–[7]. In this letter, we provide a brief review on another type of very promising integrated comb source – microresonator based Kerr frequency combs (Kerr combs or microcombs for short) [8]. The unique advantages of Kerr combs include high repetition rate from GHz to THz and ultrabroad spectral bandwidth even larger than one octave.

The key component for Kerr comb generation is a nonlinear microresonator with high quality (Q) factor. There are basically two kinds of microresonators fabricated with different techniques. One is whisper gallery mode (WGM) resonators fabricated from crystalline (CaF_2, MgF_2) or fused silica by machining and polishing [9], [10]. The second is on-chip integrated microring resonators formed with planar waveguides or WGM. The microresonator materials for which on-chip comb generation has now been generated include silica [19], SiN [11], Hydex glass [12], AlN [13], silicon [14], diamond [15], InGaAs [16], GaP [17], and $LiNbO_3$ [18]. Although the highest Qs (10^8–10^{10}) have been reported with crystalline WGM resonators, amorphous silica WGM resonators have also achieved Qs in the order of 10^8. Integrated planar waveguide microresonators usually have Q levels between 10^5 and 10^7.

Figure 1(a) shows the basic scheme of Kerr comb generation. The insets show images of integrated SiN microring resonators fabricated at Purdue. Depending on the size of the resonator, different shapes such as simple rings, racetracks, and spirals can be used to reduce the device footprint. For comb generation, the pump laser is usually tuned into the resonance from the blue side (i.e., initial laser wavelength shorter than the resonant wavelength). The microresonator is red shifted due to thermal and Kerr nonlinearities in this process, forming a typical triangular transmission response

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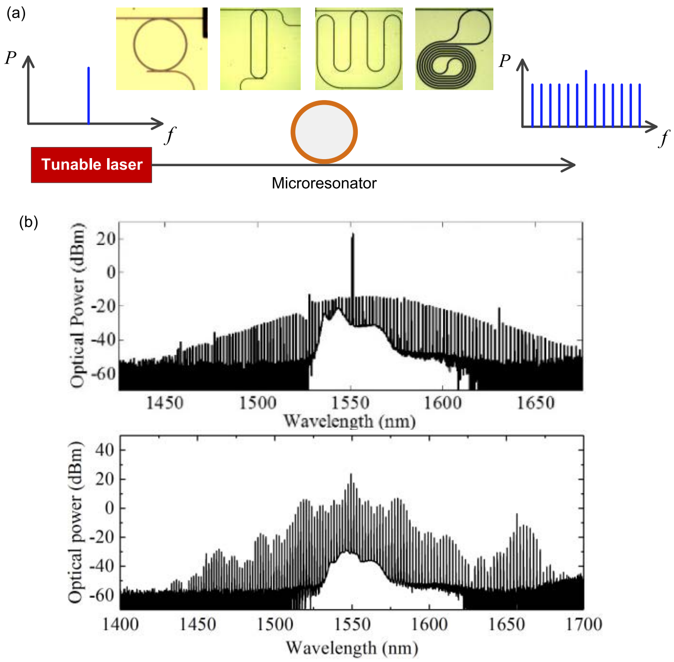

Fig. 1. (a) Microresonator-based Kerr frequency comb generation. The insets show images of integrated SiN microring resonators with different shapes (fabricated by X. Xuan and M. Qi at Purdue University). The free spectral ranges from left to right: 231 GHz, 75 GHz, 50 GHz, 25 GHz. (b) Mode-locked comb states. Upper: bright soliton in the anomalous dispersion region [27]; lower: dark soliton in the normal dispersion region [26].

if probed at a through port. The comb generation process is explained by cascaded four-wave mixing in the frequency domain, and can be described by a coherently driven nonlinear Schrodinger equation (also called the Lugiato-Lefever equation) in the time domain [20]. The group velocity dispersion of the microresonator plays a critical role in affecting the nonlinear dynamics. For comb initiation a natural modulational instability can be exploited in the anomalous dispersion region, but without perturbations this modulational instability is absent when the dispersion is normal. Special methods are thus required to trigger the comb generation in the normal dispersion region, including spatial mode coupling [21], [22], second-harmonic assisted four-wave mixing [23], or dual-coupled microresonators [24]. Comb generation usually starts with generation of a relatively small number of lines (often termed a Turing roll), then usually proceeds through a chaotic state with broad bandwidth but high noise, and finally can be brought into a mode-locked regime with low noise and high optical coherence. For microwave photonics and most other applications, it is important to reach the low noise, mode-locked state. Depending on the different signs of dispersion, the mode-locked solitons may be bright pulses in the anomalous dispersion region [25] or dark pulses in the normal dispersion region [26]. Figure 1(b) shows two examples of bright- and dark-soliton combs generated with SiN microring resonators.

II. Microwave Photonic Applications of Microcombs

The applications of microcombs in microwave photonics can be divided into coherent ones and incoherent ones. For coherent applications, a stable phase relation between the comb lines is essential. One typical coherent application is microwave generation by detecting the beat note of a mode-locked microcomb (see Fig. 2). In the simplest configuration, the microcomb is free running without locking to any reference. The spectral purity of the microwave signal critically depends on the microresonator Q and the pump laser noise. Low-phase-noise microwave generation has been demonstrated by using crystalline WGM resonators and silica wedge resonators with Q factors between 10^8 and 10^9 [28]–[30]. Both Turing roll and soliton states are applicable. Comb extraction from a drop port is also helpful for filtering out the pump laser noise. In [28], a 10 GHz microwave was generated with phase noise better than -60 dBc/Hz @ 10 Hz, -90 dBc/Hz @ 100 Hz, and -170 dBc/Hz @ 10 MHz. This performance makes the microcomb oscillator among the best photonic microwave oscillators with similar volume and power consumption – compare for example to [31] and [32].

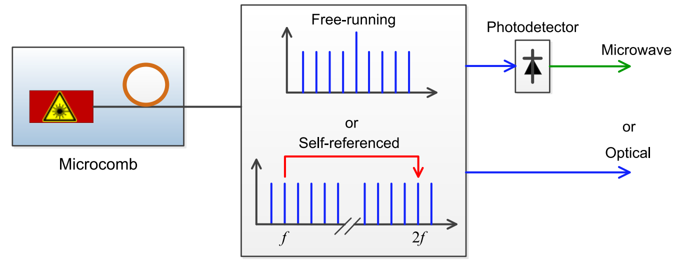

Fig. 2. Microwave and optical frequency synthesis with a microcomb. A microwave signal can be generated by detecting the beat note of a free-running comb or a self-referenced comb locked to a stable optical reference.

The microwave phase noise may be further reduced by locking the microcomb to a stable optical frequency standard [33]. This approach has the potential to generate microwaves with state-of-the-art spectral purity. Compared to a free-running microcomb, the complexity of optical frequency division is much higher because it needs not only octave-spanning microcomb generation for self-referencing but also a miniature optical frequency standard. There is thus a longer way to go to get to integrated photonic microwave generators based on optical frequency division. In either way, photonics based microwave generation can potentially achieve phase noise levels much lower than the state-of-the-art electronics and provides a promising way to break the electronic bottleneck.

Converse to optical frequency division which translates an optical frequency to a microwave frequency, optical frequency synthesis translates a microwave frequency to optical frequencies. Optical frequency synthesis may also benefit microwave photonic applications as it can provide ultra-stable and narrow-linewidth optical carriers for some instrumental and sensing applications. A miniature optical frequency synthesizer based on integrated microcombs has been demonstrated recently [34].

In incoherent applications, a microcomb provides carriers at multiple wavelengths to make copies of the microwave signals that are to be processed. As shown in Fig. 3, the microcomb is first modulated by the microwave via an electro-optic modulator. The microwave-carrying microcomb then passes through optical signal processing units and is detected by photodetectors. In principle, the role of microcombs in incoherent applications can be replaced by a multi-wavelength laser array. High coherence between the comb lines is not required. However, for microcombs, mode-locking and intensity noise are usually correlated [35]. Mode-locked combs usually show low intensity noise, while incoherent combs usually show high intensity noise. Therefore, mode-locking of microcombs is still required even for incoherent applications.

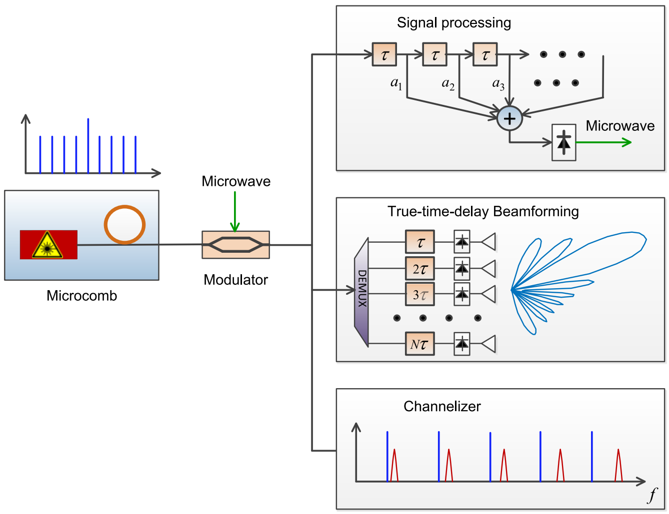

Fig. 3. Examples of incoherent applications with a microcomb, including signal processing, true-time-delay beamforming, and channelizer. When a second comb is used for coherent detection in a channelizer, a stable phase relation between the combs is required (i.e., a coherent application).

In microcomb based microwave signal processing, the microcomb after modulation passes through a dispersive delay element to introduce differential time delays between the comb lines and the corresponding microwave signals. By shaping the comb spectrum, arbitrary transfer functions can be synthesized by summing the microwave signals with a photodetector. One unique advantage of microcombs compared to traditional combs such as electro-optic combs is that their comb line spacing can be much larger. The large comb line spacing corresponds to a large Nyquist zone which equals half of the comb line spacing and denotes the maximum microwave frequency that can be handled without interference from adjacent comb lines. In [36], a complex-tap photonic microwave filter is demonstrated with a microcomb generated from a SiN microring resonator. The comb line spacing is 231 GHz corresponding to a Nyquist zone of 115.5 GHz. The large comb line spacing also makes it possible to process the modulation sidebands with commercial spectral shapers. Although the frequency responses of discrete taped delay line filters are periodic in theory, single passband can be achieved in microcomb based filters by combining optical and electrical filtering. Since the transfer function can be arbitrarily programmed, functions other than bandpass filtering may be also achieved, such as Hilbert transformer and differentiator [37], [38].

In true time delay beamforming, the comb lines are demultiplexed and detected after a dispersive delay. The generated microwave signals with different time delays are amplified and sent to an antenna array. The far-field microwave beam pattern is the Fourier transform of the tap coefficients whose amplitude is generally determined by the comb spectral intensity. The beam direction can be steered by changing the differential time delay. In [39], a true time delay network with 21 channels was demonstrated. The dispersive element was a binary dispersion matrix constructed of MEMS optical switches and single mode fibers. By changing the dispersion, the true time delay network can support a beam scanning range of $\pm 60.2°$. The large comb line spacing also makes it possible to achieve bipolar taps by shaping the modulation sidebands, which opens up new possibilities for antenna beam shapes (e.g., steered notches). The number of channels can be further increased by employing broadband microcombs with proper comb line spacing [40]. For example, with a spacing of 50 GHz, the comb lines in the lightwave S, C and L bands will be able to support more than 400 antennas.

Another promising application of microcombs is for wideband microwave channelized receivers. The duplicated microwave spectra by the comb lines are separately sliced by photonic filters and then detected. Both the amplitude and phase information of the microwave signals may be obtained by mixing the comb with a second comb with slightly different line spacing [41]. In such a dual-comb coherent channelizer, a stable phase relation between the comb lines and the two combs is required (which makes it a coherent application). A preliminary demonstration of microcomb based channelizer was reported in [42].

III. Summary and Outlook

In summary, microcombs are very promising for integrated microwave photonics. For coherent applications, since the comb phase noise is closely related to the microresonator Q factor, fabrication of microresonators with ultrahigh Q (especially on integrated platforms) is still one priority for the microcomb field. For incoherent applications, another important figure of merit is the energy conversion efficiency which is defined as the fraction of the pump power converted to the comb lines. The power level of each comb line critically affects the insertion loss and noise figure of the microwave photonic system. In traditional comb generation schemes with a single microresonator, the conversion efficiency is closely related to the duty cycle of the cavity solitons. The conversion efficiency of single bright soliton combs is typically only a few percent (in many cases even $<1\%$). Dark soliton combs may achieve higher efficiency ($>30\%$) due to their larger duty cycle compared to bright solitons [43]. A scheme to greatly improve the conversion efficiency of bright soliton combs using mutually coupled microresonators was proposed recently [44].

Most microcomb generation experiments performed to date require a high-power optical amplifier to boost the pump power, especially for CMOS-integrated microring resonators. To achieve fully integrated compacted microcomb sources, integration or packaging of the pump laser and the microresonator is essential. Some encouraging advances have been reported recently with SiN microrings [45]–[47]. The microresonator Q factor is improved to a high level of $\sim 10^7$ so that the comb generation threshold is greatly reduced and no pump amplifier is required. Soliton microcomb powered by batteries has been successfully demonstrated in [46].

Flexible control of the comb spectrum is required in many comb based microwave photonic applications, and can be performed by a programmable optical spectral shaper. Current commercial spectral shapers are based on spatial light modulators whose volume is still too bulky for integrated microwave photonics. On-chip integrated spectral shapers can potentially greatly reduce the volume. In [48], a 32-channel spectral shaper integrated on an InP platform was demonstrated. The optical spectrum is dispersed by an arrayed waveguide grating (AWG) and then shaped by an array of semiconductor optical amplifiers (SOAs). A loopback structure was employed to use the same AWG to combine the spectral components after shaping. The channel spacing is 25 GHz. The channel cross-talk is around −15 dB. The use of SOAs as shaping elements not only provides gain to compensate the losses of the AWG, but also provides a high switching speed with a response time between 400-500 ns. Use of the integrated spectral shaper for fast reconfigurables photonic microwave filtering has been demonstrated in [48]. It can be expected that in the future microcombs will be integrated together with more function modules (monolithic or hybrid) to build compact complex microwave photonic systems.

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