

# Spectral line-by-line pulse shaping

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Received January 3, 2005

We experimentally demonstrate pulse-shaping experiments in which the individual spectral lines that are present in the output of a mode-locked laser (8.5 GHz mode spacing, centered at 1542 nm) are resolved. The shaped pulses overlap in time, and this leads to a new way to observe fluctuations of the comb-offset frequency in the time domain. © 2005 Optical Society of America

OCIS codes: 320.5540, 320.7160, 120.3940.

In the frequency domain, mode-locked laser pulses are characterized by a series of discrete spectral lines (an optical frequency comb) with a frequency interval equal to the longitudinal mode spacing of the laser or, equivalently, to the pulse repetition rate. Spectral lines and their stabilization in mode-locked lasers have recently played a critical role in the progress of optical frequency metrology and optical carrier-envelope phase control.<sup>1,2</sup> It has been shown that the optical spectral lines (the optical frequency comb) for a mode-locked laser can be expressed as<sup>1</sup>

$$f_n = nf_{\text{rep}} + \epsilon, \quad (1)$$

where  $n$  is a large integer,  $f_{\text{rep}}$  is the frequency interval between two spectral lines (also the repetition rate of a mode-locked laser), and  $\epsilon$  is the comb-offset frequency. Offset frequency  $\epsilon$  is related to the evolution of the carrier-envelope phase, which occurs as a result of a mismatch in the group and phase velocities inside the laser cavity. Adjacent pulses in the mode-locked pulse train have relative phase difference  $2\pi\epsilon/f_{\text{rep}}$ .<sup>3</sup> Therefore, stable (unstable) offset frequency  $\epsilon$  implies a fixed (fluctuating) phase relationship between adjacent pulses. If the adjacent pulses can be manipulated in a way such that they overlap, this phase relationship can be observed through their interference.

From the pulse-shaping perspective,<sup>4</sup> if it is possible to manipulate both the amplitude and the phase of individual spectral lines independently, essentially complete pulse-shape control can be achieved. Specifically, waveforms formed from adjacent pulses can overlap. However, in typical pulse shapers<sup>4</sup> the spectral lines are manipulated in groups rather than individually. This is primarily due to the practical difficulty of building a pulse shaper that is capable of resolving each spectral line for typical mode-locked lasers with repetition rates below 1 GHz.

Group-of-lines pulse shaping is illustrated in Fig. 1(a), where  $f_{\text{rep}}$  is the repetition rate defined above. Assuming that the pulse shaping occurs  $M$  lines at a time, the shaped pulses have maximum duration  $\sim 1/(Mf_{\text{rep}})$  and repeat with period  $T = 1/f_{\text{rep}}$ . Accordingly, the pulses are isolated in time. In contrast, for line-by-line pulse shaping ( $M=1$ ) as shown in Fig. 1(b), the shaped pulses can overlap, which leads to interference between contributions from different input pulses in the overlapped region. Previously a hy-

perfine wavelength-division multiplexing filter was used for spectral line-by-line phase manipulation with 5 GHz line spacing in an optical code division multiaccess system<sup>5</sup> and with 12.4 GHz spacing in photonic rf arbitrary waveform generation experiments<sup>6</sup> but without investigation of this pulse-overlap issue. More importantly, the hyperfine wavelength-division device has a periodic spectral response, which means that independent manipulation of the spectrum is possible only within one free spectral range, which was only 75–80 GHz in the experiments described in Refs. 5 and 6. In this Letter we report a high-resolution grating-based pulse shaper that is able to resolve individual spectral lines from an 8.5 GHz repetition rate actively mode-locked fiber laser. By performing amplitude and phase line-by-line pulse shaping experiments we are able to generate waveforms in which shaped pulses that arise from different input pulses clearly overlap. This leads to a new way to observe fluctuations of the comb-offset frequency in the time domain.

Spectral line-by-line manipulation is implemented by the well-developed ultrashort pulse-shaping techniques<sup>4</sup> with a fiber coupled Fourier-transform pulse shaper that incorporates a  $2 \times 128$  pixel liquid-crystal modulator (LCM) array to independently control both amplitude and phase of each spectral line. To achieve line-by-line pulse shaping we take great care in the pulse-shaper design to improve resolution. A fiber coupled pulse shaper with reflective geometry is built, which includes a collimator–telescope combination to produce a collimated beam with  $\sim 18$  mm diameter, an 1100 groove/mm grating, a lens with 750 mm focal length, a LCM with a 12.8 mm aperture and  $2 \times 128$  independent pixels, a retroreflecting mirror, and a circulator. The fiber

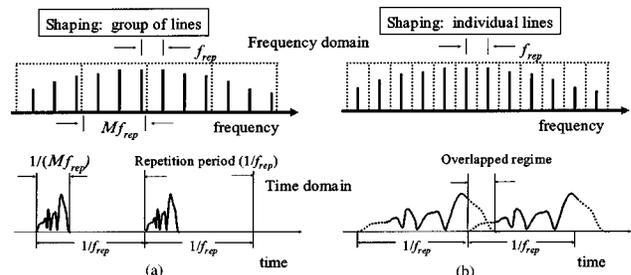


Fig. 1. Illustration of pulse shaping with (a) groups of lines and (b) individual lines manipulated.

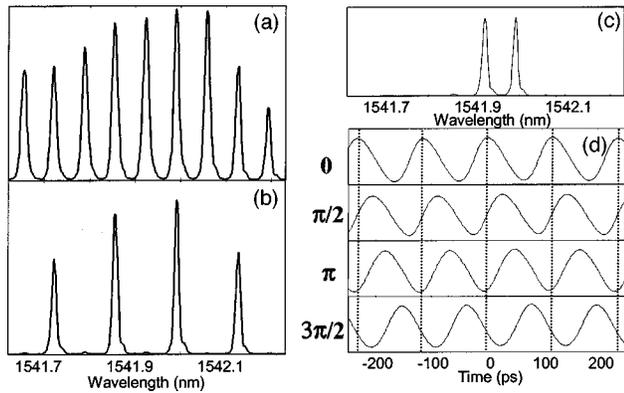


Fig. 2. (a) Spectral lines without amplitude modulation. (b) Amplitude modulation with every other line blocked. (c) Two spectral lines. (d) Sampling scope traces with phase modulation ( $0$ ,  $\pi/2$ ,  $\pi$ , and  $3\pi/2$ ) on one spectral line. The traces are the average of 100 measurements.

coupled input–output loss of the pulse shaper is 15 dB. The measured resolution (bandwidth controlled by an individual LCM pixel) agrees well with the calculated value of 8.5 GHz. To the best of our knowledge, this is the highest resolution ever reported for a grating-based pulse shaper.

Our experiments are performed with a home-built harmonically mode-locked fiber laser producing  $\sim 3$  ps pulses at 1542 nm center wavelength with a repetition rate that can be tuned from 8 to 13 GHz. The output pulses are used as the input to our pulse shaper. Figure 2(a) shows a portion of the power spectrum when the laser is tuned for an 8.5 GHz repetition rate without amplitude or phase modulation from the pulse shaper. The optical linewidths are limited by the 0.01 nm resolution of the optical spectrum analyzer used for this measurement. Importantly, even without active stabilization, at an 8.5 GHz repetition rate the absolute frequency positions of the individual spectral lines are stable to within the measurement resolution for periods of tens of minutes. This makes possible stable line-by-line pulse-shaping experiments. For example, Fig. 2(b) demonstrates line-by-line amplitude control by programming the LCM to block every other spectral line. In this example the blocked spectral lines are almost completely suppressed (less than  $-18$ -dB cross talk), while the transmitted lines remain essentially untouched, clearly illustrating that line-by-line pulse shaping with an excellent resolution of 8.5 GHz has been achieved.

Figures 2(c) and 2(d) demonstrate the capability of line-by-line phase control by phase shifting one of two spectral lines. Figure 2(c) shows two spectral lines; we blocked all other lines by programming the pulse shaper. As there are only two spectral lines, ideally the waveform intensity profile in the time domain corresponds to a cosine function (with a dc offset) in which the temporal phase of the cosine function is determined by the relative spectral phase between the two spectral lines. Figure 2(d) shows the intensity profiles of waveforms in the time domain detected by a 60 GHz photodiode and measured by a

sampling oscilloscope (averaged 100 times). The predicted temporal shifts are observed as expected.

Intuitively, line-by-line pulse shaping requires a stable mode-locked laser source, especially stable spectral line positions. In our system, when the laser is operating at an 8.5 GHz repetition rate we observe relatively stable performance, which makes line-by-line pulse shaping possible, as shown in Fig. 2. To investigate this issue further, we repeated the line-by-line phase control experiment and recorded both the optical spectra and sampling scope traces consecutively to show their fluctuations. Figures 3(a) and 3(b) show an overlap of 100 scans for the two spectral lines and sampling scope traces for an 8.5 GHz pulse repetition rate, which show relatively stable features. If there is no pulse shaping (corresponding to a  $0$  phase shift), the sampling scope traces are clear. If there is pulse shaping with a  $\pi$  phase shift on one spectral line, the sampling scope traces become slightly noisy because of the small fluctuations of spectral lines as shown in Fig. 3(a). Nevertheless, the positions of the spectral lines are stable enough for line-by-line control, as demonstrated above.

When we tune the laser source to an 11.0 GHz repetition rate, we observe empirically that the absolute frequency positions of the spectral lines become considerably less stable, as shown in Fig. 3(c), with frequency fluctuations observable on a time scale of seconds. We attribute the spectral line fluctuations in our actively mode-locked laser to comb-offset frequency fluctuations; we attribute the difference in optical frequency stability at different laser repetition rates to the frequency-dependent response of the microwave components used for feedback control of the cavity length. Thus we can investigate the role of optical comb frequency fluctuations on line-by-line shaping. If there is no pulse shaping (corresponding to a  $0$  phase shift), under the conditions of our experiments the sampling scope traces are clear even if the spectral lines are relatively unstable, as shown in Fig. 3(d). However, if there is pulse shaping with a  $\pi$  phase shift on one spectral line, the sampling scope traces become extremely noisy because of the large fluctuations of the spectral line positions. This result can be understood from the overlap effect: For a  $\pi$

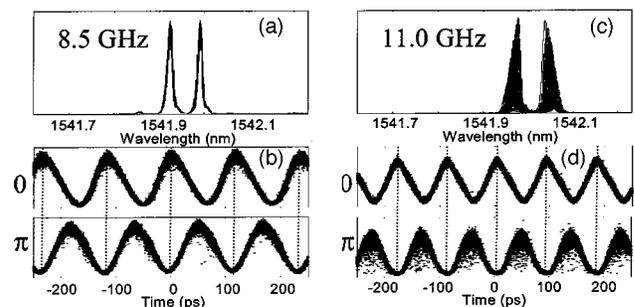


Fig. 3. (a) Two relatively stable spectral lines at 8.5 GHz. (b) Sampling scope traces with phase modulation ( $0$  and  $\pi$ ) on one spectral line. The traces are scanned 100 times. (c) Two relatively unstable spectral lines at 11.0 GHz. (d) Sampling scope traces with phase modulation ( $0$  and  $\pi$ ) on one spectral line. The traces are scanned 100 times.

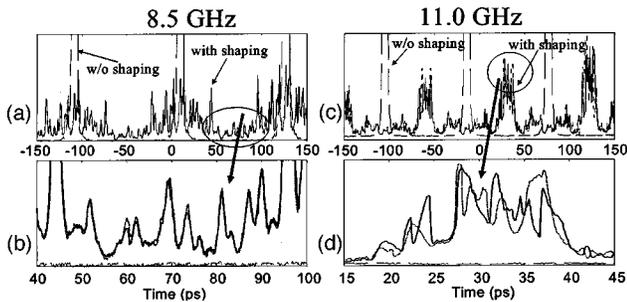


Fig. 4. (a) Intensity cross-correlation traces at 8.5 GHz for multiple spectral lines. (b) Details in an overlapped region. (c) Intensity cross-correlation traces at 11.0 GHz for multiple spectral lines. (d) Details in an overlapped region. Dashed curves, pulses without shaping. Solid and dotted curves, pulses with shaping (two sequential measurement scans).

phase shift, the original laser pulses (corresponding to a 0 phase shift) are reshaped to form waveforms with intensities in the temporal region where contributions from adjacent input pulses overlap. Because the adjacent original pulses have an unstable phase relationship (intimately related to unstable comb-offset frequency), their interference in the overlapped region as a result of pulse shaping leads to large fluctuations. Much weaker fluctuations, if any, are observed at the time locations of the original input pulses, as there is little temporal overlap at those times. Clearly, this overlap effect leads to observation of time-dependent noise that is directly linked to variations in the comb-offset frequency.

We also implement line-by-line pulse shaping for multiple spectral lines (corresponding to short pulses instead of cosine waveforms). To achieve a large number of spectral lines,  $\sim 3$  ps pulses are compressed to  $\sim 0.4$  ps by a dispersion-decreasing fiber soliton compressor. Figure 4(a) shows two sequential intensity cross-correlation scans performed for an 8.5 GHz repetition rate. Without shaping, short pulses are well isolated (dashed curve). When spectral phase manipulation is applied to multiple spectral lines, the broadened pulses clearly overlap (solid and dotted curves). In the overlapped region shown in Fig. 4(b) the measured waveforms are essentially reproducible, consistent with the relatively clear sampling scope trace in Fig. 3(b). In contrast, at an 11.0 GHz repetition rate with relatively unstable spectral lines, the profiles of the shaped pulses in the overlapped region fluctuate, as shown for two sequential cross-correlation scans in Figs. 4(c) and 4(d). Again, this demonstrates that fluctuations in the comb-offset frequency give rise to fluctuating interference in overlapped time regions caused by pulse shaping.

To develop a simple theory to describe these results we can write the mode-locked laser spectrum with discrete spectral lines as

$$E(f) = G(f) \sum_n \delta(f - nf_{\text{rep}} - \epsilon), \quad (2)$$

where  $\delta()$  represents the impulse function and  $G(f)$  represents the shape of the complex spectrum

(shaped or unshaped). It can be shown that the temporal trace is<sup>1</sup>

$$e(t) = T \sum_k g(t - kT) \exp(i2\pi k\epsilon T), \quad (3)$$

where  $g(t)$  is the inverse Fourier transform of  $G(f)$ , representing a single pulse (shaped or unshaped). There is no interference between pulses if the duration of pulse  $g(t)$  is less than one period,  $T$ . Pulses broadened by line-by-line shaping can overlap and interfere with each other. Because overlap occurs mostly between adjacent pulses, for simplicity the overlap region between two adjacent pulses ( $k=0$  and  $k=1$ ) is investigated, which can be expressed as

$$\begin{aligned} |e(t)|^2/T^2 = & |g(t)|^2 + |g(t-T)|^2 + 2|g(t)g(t-T)|\cos[\theta(t) \\ & - \theta(t-T) - 2\pi\epsilon T], \end{aligned} \quad (4)$$

where  $g(t) = |g(t)|\exp[i\theta(t)]$ . The third term represents the overlap and interference effect and depends on comb-offset frequency  $\epsilon$ . As a result, fluctuations in the comb-offset frequency result in fluctuations of the pulse intensity profile in the overlapped region where  $|g(t)g(t-T)|$  is not zero. For a fixed degree of fluctuations in  $\epsilon$ , the size of the intensity fluctuations varies in time according to  $|g(t)g(t-T)|$ , in accord with our experimental observations.

In summary, we have experimentally demonstrated spectral line-by-line pulse shaping with an actively mode-locked fiber laser by using a high-resolution pulse shaper. The shaped pulses overlap, which leads to a new method used in the time domain for observation of variations in comb-offset frequency.

This Letter is based on research supported by the Defense Advanced Research Projects Agency under grant MDA972-03-1-0014. D. S. Seo is supported in part by the Korea Science and Engineering Foundation (R01-2003-000-10444-0) and the Inha University Engineering Research Center, Korea. Z. Jiang's e-mail address is zjiang@purdue.edu.

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## References

1. S. T. Cundiff, *J. Phys. D* **35**, R43 (2002).
2. T. M. Fortier, D. J. Jones, J. Ye, S. T. Cundiff, and R. S. Windeler, *Opt. Lett.* **27**, 1436 (2002).
3. L. Xu, C. Spielmann, A. Poppe, T. Brabec, F. Krausz, and T. W. Hänsch, *Opt. Lett.* **21**, 2008 (1996).
4. A. M. Weiner, *Rev. Sci. Instrum.* **71**, 1929 (2000).
5. S. Etemad, T. Banwell, S. Galli, J. Jackel, R. Menendez, P. Toliver, J. Young, P. Delfyett, C. Price, and T. Turpin, in *Optical Fiber Communication Conference (OFC)*, Vol. 95 of OSA Trends in Optics and Photonics Series (Optical Society of America, 2004), paper FG5.
6. T. Yilmaz, C. M. DePriest, T. Turpin, J. H. Abeles, and P. J. Delfyett, *IEEE Photonics Technol. Lett.* **14**, 1608 (2002).