Spectral and temporal speckle field measurements of a random medium

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The zero-mean circular complex Gaussian field statistics of a random medium are experimentally demonstrated in the optical domain, thus verifying this key assumption of statistical optics. Using a frequency-tunable laser source in a fixed-path-length interferometer, we obtain optical field fluctuations in the time and frequency domains that clearly show that the ensemble-averaged temporal intensity converges to the photon transit time distribution, which for the samples used is in excellent agreement with a diffusion model.

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Optical speckle arises from the interference of multiple, randomly phased contributions to the electric field associated with the different optical paths in scattering samples. As is well known, this results in random spatial variations in output intensity patterns observed for monochromatic illumination (spatial speckle).[1] However, for broadband illumination, random variations will occur in each of the space, time, and frequency domains. The statistics of spatial speckle as a function of optical bandwidth has been studied.[2] Correlations in the spectral intensity speckle (fluctuations in the optical power spectrum) observed at a single spatial location from a random medium have been connected to the ensemble-average behavior of the temporal intensity (impulse response)[3] and used to derive images of simple objects within scattering media.[4] We recently demonstrated that measurements of third-order frequency correlations in spectral intensity speckle can be used to extract the ensemble-average temporal intensity response of a random medium.[5] Angular averaging (by using a suitable imaging lens) of measured temporal intensity speckle also leads to the ensemble-average temporal intensity response.[6] All this work relies on measurements of intensity fluctuations, as opposed to measurements of the optical field directly.

Studies of field measurements from random media have been performed in the microwave domain on randomly filled waveguide structures.[7] There, near-Gaussian field statistics were obtained but not with zero mean, which was believed to be associated with the bounded waveguide geometry. This bounded geometry is a limitation when these results are extended to three-dimensional samples, which are typically required for important applications such as biomedical imaging and environmental sensing. Recently, measurement of the field statistics from a three-dimensional random medium in the terahertz domain was also demonstrated.[8] The measured field statistics were non-Gaussian because of the nonconstant power spectrum of the terahertz pulse over the measurement bandwidth.

In this Letter we report on measurements of the random optical field variations in both the frequency and the time domains. We demonstrate experimentally the widely held assumption that under sufficient scattering conditions, the optical field is a zero-mean circular complex Gaussian random process.[9] Also, we show directly that, while the field and intensity profiles of individual speckle spots fluctuate randomly in time, the ensemble average of the temporal intensity profiles converges to yield the photon travel time probability distribution function. This ties together the random field statistical point of view with the ensemble-average perspective of a photon transport model such as the diffusion approximation.

Our experimental setup is shown in Fig. 1. The tunable laser source used was an external-cavity laser diode (New Focus Vortex 6017) that has a narrow-linewidth (nominally 5-MHz) single-mode output with a center wavelength of 850 nm. We swept the laser center frequency linearly over a range of 60 GHz in less than 500 ms to acquire frequency-resolved data. The output was linearly polarized with an average

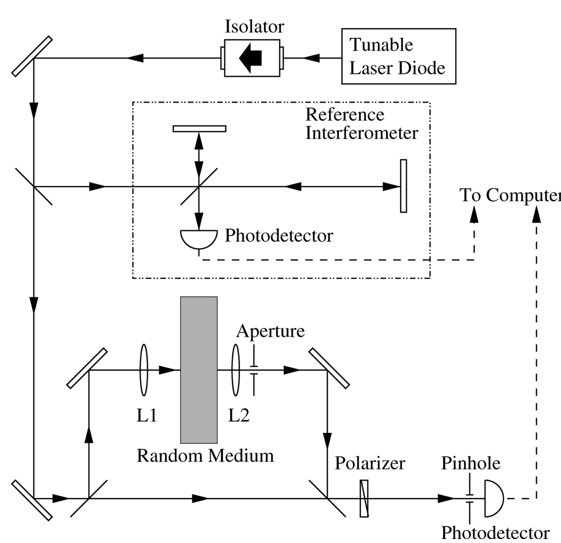

Fig. 1. Experimental setup for characterizing a random medium by use of frequency-resolved interferometer measurements.

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power of 10 mW. An optical isolator was used to prevent backreflections from destabilizing the laser diode output. A small portion of the output power was coupled into a fixed-path-length reference interferometer (path-length difference of 3.96 m), as shown in Fig. 1, and was used to count fringes as the laser center frequency was scanned. This was found to be essential for providing the correct frequency scale for the measured data.

The scattering random medium was placed in the longer arm of a Mach–Zehnder interferometer configuration. In the absence of the sample the path-length difference between the two arms was $\Delta L = 1$ m, corresponding to a time delay of $\tau = 3.33$ ns. Lens L1 (focal length 100 mm) was used to focus the optical power on the front surface of the scattering sample. Lens L2 (focal length 50 mm) and an aperture imaged the output field onto an amplified p-i-n diode photodetector (New Focus 2001-FS), producing a speckle field whose spatial coherence area was ~16 times greater than the photodetector area. A polarizer was used to analyze the same linear polarization of the scattered output field as that of the input field.

Note that our approach is based on spectral interferometry, used in ultrafast optics for pulse measurement, and for optical fiber characterization.[10] The experimental implementation is slightly different (spectral interferometry uses a broad-spectrum source and a spectrometer to measure the output spectrum, while we use a frequency-tunable narrowband source and a single photodiode). In a related interferometric technique[11] the frequency of a laser diode was sinusoidally modulated, then through mathematical processing of the measured data a coherence gate was synthesized, giving a measure of the intensity temporal response of the random medium but not the field behavior or the field statistics.

We assume elastic scattering in the random medium and thus can treat it as a linear system. The optical field applied to the random medium, with a center frequency of ν_o, is $e_i(t) = a_i(t)\exp(j2\pi\nu_o t) +$ c.c., where $a_i(t)$ is the input electric field's complex envelope and c.c. represents the complex conjugate, and the output field is $e_o(t) = a_o(t)\exp(j2\pi\nu_o t) +$ c.c. The Fourier transforms of $e_i(t)$ and $e_o(t)$ are related by $E_o(\nu) = H(\nu)E_i(\nu)$, where $H(\nu)$ is the complex frequency response of the random medium. In terms of the Fourier transforms of the envelope functions, we have $A_o(\tilde{\nu}) = H(\tilde{\nu} + \nu_o)A_i(\tilde{\nu})$, where $\tilde{\nu} = \nu - \nu_o$. The power spectrum at the output of the interferometer in Fig. 1, with the random medium in the longer arm of time delay τ, is given by

$$|A_{\det}(\tilde{\nu})|^2 \sim [H(\nu)\exp(-j2\pi\nu\tau) + H^*(\nu) \times \exp(j2\pi\nu\tau) + |H(\nu)|^2 + 1]\,|A_i(\tilde{\nu})|^2. \quad (1)$$

To recover $H(\nu)$, we employ the strategy of spectral interferometry by inverse Fourier transforming expression (1), applying a time window to the data near time τ, time shifting the windowed data to the origin, and then Fourier transforming the windowed data. Dividing by the input power spectrum, $|A_i(\tilde{\nu})|^2$, which is constant over the bandwidth of 60 GHz for $\tilde{\nu}$ in our case, gives the desired complex frequency response, $H(\nu)$. In an experiment it is necessary to choose τ to be larger than the effective duration of the impulse response for the random medium to ensure that the features in the inverse Fourier transform of Eq. (1) do not overlap.

For our experiment we used a commercial white acrylic (Cyro Industries, Acrylite FF) with small TiO_2 particles (average diameter 50 nm) in a transparent acrylic background as the random medium. Two sample thicknesses of $d = 6$ mm and $d = 12$ mm (formed by using two 6-mm slabs) were measured. The real component of $A_o(\tilde{\nu})$ from four measurements is shown in Fig. 2(a). The imaginary component has similar features but was not plotted for clarity. To estimate the ensemble statistics of the electric field we performed 100 measurements, moving the sample laterally a small distance (500 μm) to give an independent configuration of scatterers between each measurement and then averaging the results. Histograms of the real and the imaginary components of $A_o(\tilde{\nu})$ show excellent agreement with a Gaussian distribution and are plotted in Fig. 2(b). The real and the imaginary components also had equal variances, $\langle A_r^2\rangle = \langle A_i^2\rangle = 1$ (in normalized units), and a small correlation coefficient of $\langle A_r A_i\rangle \simeq 0.01$, where A_r and A_i are the real and the imaginary field components, respectively, which for A_r and A_i Gaussian makes them virtually independent. Thus we have directly demonstrated the widely held assumption[1] of zero-mean circular complex Gaussian statistics for the scattered field from a random medium in the optical domain.

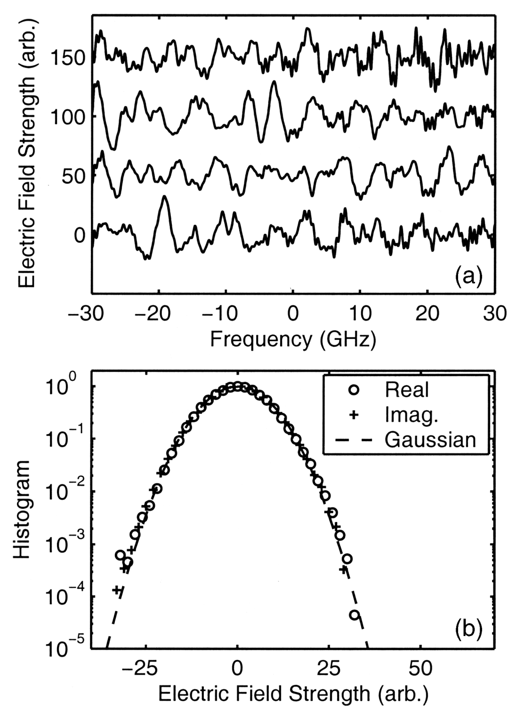

Fig. 2. (a) Real component of the complex electric field amplitude $A_o(\tilde{\nu})$ from four independent measurements of the $d = 6$ mm sample (offset for clarity). (b) Histogram of the real and the imaginary components of the field averaged over 100 measurements, plotted on a semi-log scale.

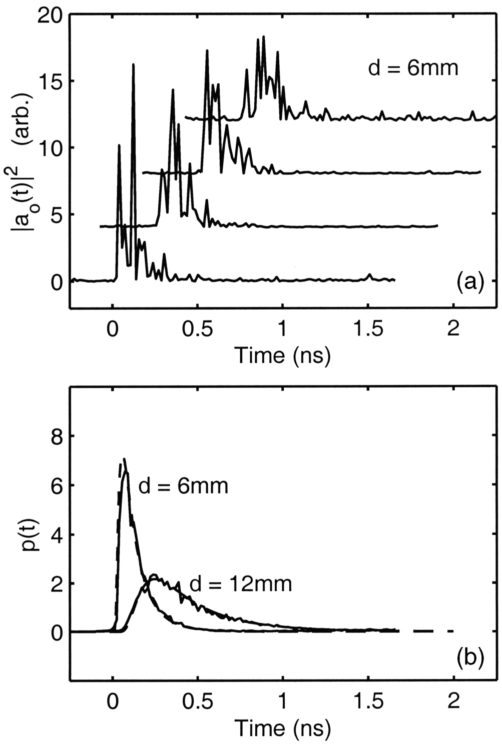

Fig. 3. (a) Typical intensity temporal responses obtained from inverse Fourier transforms of the data in Fig. 2(a) (offset for clarity). (b) Estimate of the ensemble-average intensity temporal response obtained by averaging 100 measurements (solid curves). Excellent agreement with a diffusion model for each sample thickness is shown (dashed curves).

We also report on measurements of the scattered temporal field, $a_o(t)$, showing speckle in the time domain. Figure 3(a) shows typical measured intensity responses, $|a_o(t)|^2$, for the $d = 6$ mm sample. The minimum temporal feature width in these plots is related to the inverse power spectrum bandwidth of the input field (a 60-GHz bandwidth, giving a temporal resolution of 17 ps).

An important parameter in describing the temporal behavior of a random medium is $p(t)$, the probability density function for the time of flight of the scattered partial waves traversing the medium, and it is equal to the normalized ensemble-average intensity temporal response. In terms of our measured field, $a_o(t)$,

$$p(t) = \langle |a_o(t)|^2 \rangle \bigg/ \int_0^\infty \mathrm{d}t \langle |a_o(t)|^2 \rangle , \qquad (2)$$

where the angle brackets $\langle \cdots \rangle$ represent the ensemble average, and for validity we require that $a_i(t)$ be a sufficiently narrow input pulse (to approximate a delta function), or conversely, the bandwidth of $A_i(\tilde{\nu})$ needs to be very broad compared with the spectral features in $H(\tilde{\nu} + \nu_o)$. In Fig. 3(b) we plot the ensemble-average intensity temporal responses for the two sample thicknesses of $d = 6$ mm and $d = 12$ mm. The analytic results for $p(t)$ were calculated from the diffusion equation with $\mu_s{}' = 14\ \mathrm{cm}^{-1}$ and $\mu_a = 0\ \mathrm{cm}^{-1}$ for a uniform slab using image theory[12] and show excellent agreement with the measured data. We averaged 100 independent measurements to estimate the ensemble-average intensity temporal response. The results clearly demonstrate that averaging over a large number of individually measured temporal speckle responses accurately reproduces the ensemble intensity temporal response obtained with other techniques.[5,13]

In conclusion, we have experimentally demonstrated zero-mean circular complex Gaussian field statistics of a random medium in the optical domain, which is a foundation of statistical optics. We have also measured the field statistics in the time and frequency domains, from which we clearly show that the ensemble-average temporal intensity profiles yield the photon transit time distribution, which for the samples measured is accurately modeled by the diffusion approximation.

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