

Reply to Comment on “Generalized grating equation for virtually-imaged phased-array spectral dispersions”

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I respond to a comment on [Appl. Opt. **42**, 4152 (2003)] and discuss some unusual features that arise in a plane-wave-inspired analysis of the virtually imaged phased-array spectral disperser. © 2012 Optical Society of America
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Reference [1] comments on the spectral dispersion law for a virtually imaged phased-array (VIPA) spectral disperser [2]. In previous papers published by my group, we first presented a law based on a simple plane-wave analysis [3] and later published an improved law based on paraxial wave theory [4]. These laws differ significantly in the small-angle regime where VIPAs are generally used. The improved law based on paraxial wave theory was shown to be in excellent agreement with experiment [4].

Reference [1] makes several useful contributions: it adds insight to the deficiency of the treatment in [3], it points out important earlier references of which I was previously unaware, for instance [5], and it shows how the improved spectral dispersion law of [4] can be obtained directly via expansion of the standard Fabry–Perot transmission formula. These observations are useful and contribute to our understanding of VIPA spectral dispersers.

In the remainder of this brief note, I would like to highlight some unusual features that arise in a plane-wave-inspired analysis of the VIPA. I hope this discussion will further refine the understanding of this interesting device.

The VIPA may be understood as a side-entrance, plane-parallel Fabry–Perot, or more accurately as a Fabry–Perot in which cylindrically focused light is injected at a small angle through an antireflection-coated window on the input mirror. For sketches

please refer to [1], [3], or [4]. Ideally the remainder of the input mirror is perfectly reflecting, while the second mirror offers partial transmission. Ideally, then, all of the input light should exit through the second surface. Due to multiple beam interference effects, however, different frequencies exit at different angles; unlike the conventional Fabry–Perot, the VIPA acts as a spectral disperser.

Referring to [5], the comment [1] invokes the standard Fabry–Perot etalon result:

$$2kL[\cos \theta_i] = 2m\pi, \quad (1)$$

where θ_i is the angle of incidence, L is the separation between reflecting surfaces, and the index of refraction is taken as unity. As is well known, this result may be obtained through an infinite summation of partial plane waves:

$$\frac{E_{\text{out}}}{E_{\text{in}}} = t_1 t_2 e^{-j\delta/2} \sum_{m=0}^{\infty} (r_1 r_2 e^{-j\delta})^m, \quad (2)$$

where $\delta = 2kL \cos \theta_i$ is a round-trip phase shift. The intensity transmission is then obtained as

$$\frac{I_{\text{out}}}{I_{\text{in}}} = \frac{(1 - R_1)(1 - R_2)}{1 + R_1 R_2 - 2r_1 r_2 \cos \delta}. \quad (3)$$

Here r_i and t_i are respectively the field reflection and transmission coefficients for surface i ($i = 1, 2$), $R_i = |r_i|^2$, and we neglect loss. We take r_1 and r_2

as real and positive. Equation (3) yields a transmission peak at angle θ_i satisfying Eq. (1).

The interesting question is the manner in which this plane-wave analysis applies to the VIPA. The most obvious approach is to repeat the summation of Eq. (2), but dropping the first transmission coefficient t_1 , because now the beam enters through a non-reflecting window. This procedure yields

$$\frac{E_{\text{out}}}{E_{\text{in}}} = t_2 e^{-j\delta/2} \sum_{m=0}^{\infty} (r_1 r_2 e^{-j\delta})^m = \frac{t_2 e^{-j\delta/2}}{1 - r_1 r_2 e^{-j\delta}}. \quad (4)$$

This is in agreement with Eq. (1) of [5]. The intensity transmission is given by

$$\frac{I_{\text{out}}}{I_{\text{in}}} = \frac{(1 - R_2)}{1 + R_1 R_2 - 2r_1 r_2 \cos \delta}. \quad (5)$$

When Eq. (1) is satisfied, we have peak transmission given by

$$\frac{I_{\text{out}}}{I_{\text{in}}} = \frac{1 - R_2}{(1 - r_1 r_2)^2}. \quad (6)$$

For the ideal case of a perfectly reflecting front surface ($r_1 = 1$), this simplifies to

$$\frac{I_{\text{out}}}{I_{\text{in}}} = \frac{1 + r_2}{1 - r_2}. \quad (7)$$

This quantity exceeds unity by a factor that becomes large when the second mirror reflectivity is high. Such behavior, which does not appear to conserve energy, is nonphysical within the context of a plane-wave treatment. Hence, our attempt to use purely plane-wave concepts to analyze the VIPA leads to a paradox.

Reference [5] does not explicitly acknowledge this paradox but does consider the transmission averaged over phase shift, $\delta = 2kL \cos \theta_i$. This is appropriate because the angular content of a beam small enough to be injected efficiently into a VIPA is sufficient to span a free spectral range (i.e., δ varies by at least 2π). The transmission averaged over phase shift can be shown to satisfy

$$\left\langle \frac{I_{\text{out}}}{I_{\text{in}}} \right\rangle_{\delta} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{I_{\text{out}}}{I_{\text{in}}} d\delta = \frac{1 - R_2}{1 - R_1 R_2}. \quad (8)$$

For the case of a perfect front reflector, $R_1 = 1$, this reduces to

$$\left\langle \frac{I_{\text{out}}}{I_{\text{in}}} \right\rangle_{\delta} = 1. \quad (9)$$

Reference [5] concludes from this result that “diffraction ensures that energy is conserved.” Here we should understand “diffraction” to be associated with the finite size and finite angular spread of the

incident beam. Equation (9) also gives the desired result that all the input light emerges at the output of the second Fabry–Perot surface.

Although as shown by [1], a standard plane-wave Fabry–Perot result, Eq. (1), can be used to get the VIPA spectral dispersion law, we see from the discussion above that the justification for this approach is rather nuanced. For any specific optical frequency, the VIPA evidently acts as an angular focusing element, converting power from angles that do not satisfy the resonance condition of Eq. (1) into a narrow band of angles that do. Such angular conversion could not occur in a spatially invariant medium. Unlike a conventional Fabry–Perot, however, a VIPA is a component that lacks invariance in the transverse spatial direction; an abrupt step in reflectivity along the surface of the input mirror is inherent. This reflectivity step evidently works together with the multiple beam interference geometry to yield the focusing of the angular spectrum that physically occurs. Multiple reflections at an angle within the VIPA lead to an elongated spatial distribution; the elongated spatial distribution corresponds to proportional narrowing of the angular distribution.

A final point: the analogy between spatial and temporal propagation is well known [6]. This motivates me to propose a time-domain analog of the VIPA, sketched in Fig. 1. A Fabry–Perot cavity, uniform and effectively infinite in the transverse direction, is constructed of a time-varying front mirror $R_1(t)$ and a time-invariant partial back reflector R_2 . $R_1(t)$ is taken as a step function, switching abruptly from zero to one, in analogy with the spatial step in the reflectivity of the VIPA front mirror. An appropriately timed input pulse of duration just shorter than the cavity round trip, $t_p \lesssim 2L/c$, can be trapped in the cavity. Here we consider the pulse as a spatial plane wave arriving at normal incidence. Because $R_1 = 1$ after switch-on, all the light (barring loss) must emerge to the right of mirror R_2 . The bandwidth of our input signal is of order $c/2L$, the same as the free spectral range of the cavity. The VIPA analogy informs us that only frequencies close to the resonance condition will be emitted. Therefore, the power spectrum of the input signal is spectrally focused to a narrower band of output frequencies, determined by the linewidth of the steady-state cavity. This corresponds to an output pulse elongated in time according to the number of bounces before it leaks out of the cavity. Furthermore, if the incidence angle of the plane-wave input pulse is changed, the spectral focusing action will still occur, but the

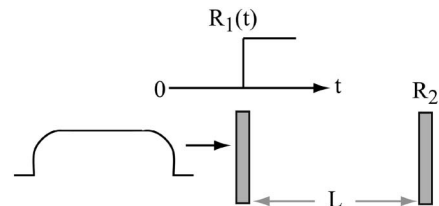


Fig. 1. Proposed time-domain analogue of the VIPA.

output frequency will be tuned in accord with the resonance condition, Eq. (1).

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