# Optical Comb Sources and High-Resolution Optical Filtering for Measurement of Photodiode Harmonic Distortion

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***Abstract*—In this work we preset a new technique for measurement of harmonic distortion in microwave photodiodes. Our technique combines optical combs and high-resolution optical filtering to provide a pure optical two-tone excitation of the photodiode. Our technique removes the challenges presented by laser frequency drift and modulator bias control by utilizing a single laser and phase-modulation to synthesize the optical combs. We demonstrate our technique through measurement of the harmonic distortion generated in a commercial InGaAs photodiode and compare our results with those obtained via phase-locked laser measurements. We show that this technique provides essentially equivalent measurement fidelity to phase-locked laser techniques while offering decreased system complexity. We additionally provide a general discussion of the limits of nonlinearity measurements performed using optical heterodyne techniques.**

***Index Terms*—Harmonic distortion, microwave photonics, optical combs, photodiodes.**

## I. Introduction

Analog photonic links are finding increased usage in a variety of military and commercial radio-frequency (RF) systems. Diverse applications ranging from optical signal processing [1] and optical analog-to-digital conversion [2] to antenna remoting [3], [4] continue to drive enhanced link performance. In conventional externally intensity-modulated direct-detection (IMDD) analog optical links employing Mach–Zehnder intensity modulators (MZMs), link performance is predominantly determined by two quantities—the efficiency of the MZM and the average received photocurrent [5], [6]. As a result, there has been much interest in achieving very low halfwave voltages (a measure of modulator efficiency) and very high average photocurrent levels as these enable extremely low noise figure analog links [7], [8]. In addition to increased modulator efficiency, there is also significant interest in improving link linearity by mitigating the inherent nonlinearity of the MZM through the use of various linearized modulators [9], [10], the wavelength-[11] or polarization-dependence [12] of Mach–Zehnder modulators, or through linearized phase-modulated links [13]–[15]. While these techniques have been utilized to achieve spur(ious)-free dynamic ranges in excess of 130 dB (in a 1-Hz bandwidth), such demonstrations have only been performed in modest-photocurrent links ($I_{\mathrm{avg}} \leq 10$ mA). As the average current is increased to improve link performance, nonlinearities in the photodiode (PD) will eventually surpass those of the MZM [4]. Therefore, it is increasingly important to look beyond the well-understood nonlinearity of Mach–Zehnder modulators to quantify and, eventually, mitigate the nonlinearity of other components—of particular interest are the photodiodes utilized in the receiver.

In order to measure the nonlinearity arising from the photodiode (PD) one must ensure the optical signal exciting the photodiode is sufficiently pure such that the measured distortion may be correctly attributed to the photodiode. This is why a simple analog photonic link may not be used to characterize PD nonlinearity—for typical photodiodes and MZMs, the MZM distortion is significantly larger than that of the PD. Several techniques have been devised to provide pure optical excitation signals; the purest excitation is achieved through heterodyning a pair of offset phase-locked lasers [16]. This two optical-tone technique produces a single RF tone at the offset frequency and is ideally suited to characterize PD nonlinearity through measurements of harmonic distortion levels. The addition of a second pair of phase-locked lasers then enables two RF-tones and, hence, allows measurement of intermodulation distortion (IMD) [17]. While these techniques yield the cleanest optical excitation signal they require customized lasers (sufficiently close in wavelength that they may be locked and heterodyned), along with the necessary control and tuning electronics. Stringent laser frequency stabilization is required as drift in optical frequency directly translates to drift in the RF frequency. A technique based on two parallel IMDD links has also been demonstrated for measurement of intermodulation distortion [18]. Here, the parallel links operate at different carrier wavelengths such that there is no beating between sidebands arising from different links. While this technique may be applied at a variety of wavelengths and the RF frequency is easily tuned and stable, it requires exacting control of the MZM bias to ensure the measurement is not corrupted by modulator nonlinearities [19]. An improved technique would posses the purity of phase-locked laser techniques and the tunability and wavelength flexibility of parallel-link techniques, without the requisite complex control electronics of either.

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In this work we demonstrate the use of optical combs [20], [21] in concert with high-resolution optical filtering [22] for measurement of photodiode harmonic distortion [23]. The use of parallel optical phase modulators to generate two optical combs from the same erbium fiber laser removes both the effects of laser frequency drift and the need for modulator bias control. High-fidelity optical filtering provides the necessary excitation purity through deep suppression of unwanted optical comb lines. To illustrate our technique, we characterize the harmonic distortion of a commercially-available PIN photodiode. We demonstrate our technique exhibits sufficient sensitivity to measure small bias voltage-dependent changes in PD nonlinearity and show the results obtained with our technique compare exceedingly well with those obtained via phase-locked laser measurements. Additionally, we discuss the limits and requirements of our technique as well as several general guidelines for PD nonlinearity measurements.

We note modelocked laser-based optical combs (in particular in the 800–1300 nm regime from Ti:sapphire or Cr:forsterite lasers) are finding increased usage as ultra-stable microwave sources [24]. As a result there has been significant research in determining the primary sources of phase noise in these systems [25]. Time-domain saturation of the photodiode used to obtain the microwave signal has been shown to contribute a significant amount of phase noise in these sources [26], [27]. Recently, mode-filtering of the optical comb prior to photodetection has been demonstrated to provide substantial increase in microwave signal-to-noise ratio leading to potentially shot noise-limited phase noise performance [28]. To our knowledge, the work presented here represents the first use of optical combs to characterize the nonlinearity of high-speed photodiodes.

## II. Theory and Measurement Limits

It is useful to briefly consider the theory behind the use of optical heterodyne techniques to measure photodiode nonlinearities in order to set bounds on the level of photodiode distortion that may be measured accurately. Consider the two-tone optical field

$$e_{\mathrm{out}}(t) = \mathrm{Re}\{E_1 \exp[j2\pi(f_o + f_1)t] + E_2 \exp[j2\pi(f_o + f_2)t]\}, \quad (1)$$

where, $f_o$ is the optical carrier center frequency, $f_1$ and $f_2$ are offset frequencies relative to $f_o$, and $E_1$, $E_2$ are the field amplitudes. When this field is incident on a photodiode it may be readily shown that the photocurrent is given by

$$i(t) = \alpha_o\{(P_1 + P_2) + 2\mathrm{H}_{\mathrm{pd}}(\Delta f)\sqrt{P_1 P_2}\cos[2\pi\Delta f t]\}. \quad (2)$$

Here $P_1$, $P_2$ are the time-average optical powers intercepted by the photodiode, $\alpha_o$ is the photodiode DC responsivity (A/W) and $\mathrm{H}_{\mathrm{pd}}(\Delta f)$ is the diode frequency response at $\Delta f = f_2 - f_1$ (unitless, normalized $\alpha_o$). The time-average RF power delivered to a load resistance $R_o$ is then

$$P_{\mathrm{rf}} = 2\alpha_o^2 |\mathrm{H}_{\mathrm{pd}}(\Delta f)|^2 P_1 P_2 R_o. \quad (3)$$

Note, the RF power varies as the product of the powers of each optical tone. When the optical tones are derived by passing optical combs through an imperfect optical filter (one exhibiting finite extinction), the RF power in spurious tones arising from unwanted comb-lines will be determined by the product of the optical power transmission of the filter at the optical frequencies of interest. This will be further discussed in the following section with respect to the purity of the photodiode excitation in our apparatus.

As is common in microwave system analysis, we characterize the nonlinearity of the photodiode using the $n$-th order intercept points—the RF output power at which the fundamental and $n$-th order distortion powers would be equal. Mathematically, the relation between the fundamental output power $P_{\mathrm{rf}}$, the $n$-th order distortion power $(P_n)$ and the intercept point $(\mathrm{IP}_n)$ is [29]

$$\mathrm{IP}_n = \left[\frac{{P_{\mathrm{rf}}}^n}{P_n}\right]^{1/(n-1)}. \quad (4)$$

We see that we may readily determine $\mathrm{IP}_n$ by measuring the power in the nonlinear sidebands $(P_n)$ and fundamental signal. The maximum intercept point that may be determined is tied to the minimum measurable power; in turn, the minimum measurable power is set by the noise floor of the measurement apparatus (in the absence of significant nonlinearity in the measurement device itself). For a measurement system (electrical) bandwidth $B_e$, we may define the measurement dynamic range (unitless) to be

$$\mathrm{DR} = \frac{P_{\mathrm{rf}}|_{\max}}{N_o \times B_e}. \quad (5)$$

Here, $N_o$ is the output noise power spectral density and $P_{\mathrm{rf}}|_{\max}$ is the maximum fundamental RF power that may be generated in the system. Realizing the maximum intercept occurs when the fundamental power $(P_{\mathrm{rf}})$ is maximized and the distortion power is minimized $(P_n = N_o \times B_e)$ we find maximum $n$-th order intercept and measurement dynamic range are related by

$$\mathrm{IP}_{n,\max} = \mathrm{DR}^{n/(n-1)} \times N_o \times B_e. \quad (6)$$

Note, the maximum intercept that may be determined depends on the distortion order $n$; lower distortion orders grow less rapidly, therefore, higher intercept points may be determined for a fixed measurement noise floor.

We may now explore the measurement limits of optical heterodyne techniques in general by analyzing the measurement dynamic range. For two fixed-intensity optical tones, the maximum RF power is achieved for 100% modulation-depth, that is $P_1 = P_2 = P_o$. Since the average photocurrent is given by $I_{\mathrm{avg}} = 2\alpha_o P_o$ [see (2)], we see the maximum RF power that may be achieved for a fixed average photocurrent is

$$P_{\mathrm{rf}} = \frac{1}{2} I_{\mathrm{avg}}^2 R_o \times |\mathrm{H}_{\mathrm{pd}}(\Delta f)|^2. \quad (7)$$

When a low-noise laser is utilized to generate the optical combs and there are no optical amplifiers utilized in the system, the noise power spectral density at the photodiode output consists of thermal noise and shot noise ($k$ is Boltzmann's constant, $T = 290$ K, and $q$ is the magnitude of the electronic charge)

$$N_o = kT + 2qI_{\mathrm{avg}}R_o. \tag{8}$$

Substituting (7) and (8) into (4) and (5) we see the measurement dynamic range and maximum measurable $n$th-order intercept points are given by

$$\mathrm{DR} = \frac{1}{4}\frac{I_{\mathrm{avg}}}{q \times B_e} \times \frac{1}{1 + 1/2(I_{\mathrm{th}}/I_{\mathrm{avg}})} \tag{9}$$

and

$$\mathrm{IP}_{n,\max} = \left[\left(\frac{1}{2}\right)^{n+1} \frac{{I_{\mathrm{avg}}}^{2n-1}}{q \times B_e} \times \frac{1}{1 + 1/2(I_{\mathrm{th}}/I_{\mathrm{avg}})}\right]^{1/(n-1)} \tag{10}$$

where $I_{\mathrm{th}} = kT/(qR_o)$ is the thermal noise current delivered to the load; this current becomes negligible for photocurrents above $I_{\mathrm{avg}} \sim 1.0$ mA. The key point here is that the measurable intercept point and average photocurrent are linked—to accurately measure the intercept requires that the average photocurrent is sufficiently high to bring the nonlinear sidebands above the noise floor. The calculated system dynamic range (gray curve), and measurable intercept points (black curves for $n = 2$: dashed, and $n = 3$: solid) are shown in Fig. 1 for a measurement bandwidth of 1-Hz. Here, we have taken the photodiode frequency response as $|\mathrm{H}_{\mathrm{pd}}(\Delta f)|^2 = 1$ for simplicity. As is evident from Fig. 1, very high intercepts may be measured at very modest photocurrent levels, provided the measurement bandwidth is sufficiently narrow. Practically, larger bandwidths are typically required due to stability or time constraints. For a shot noise-limited system with reasonable average photocurrents ($I_{\mathrm{avg}} \geq 5$ mA) and measurement bandwidth ($B_e = 100$ Hz), the system dynamic range is on the order of $\mathrm{DR} \sim 139$ dB and the maximum intercept points are approximately $\mathrm{IP}_2 = 137$ dBm and $\mathrm{IP}_3 = 67$ dBm, respectively.

Finally, it is important to address potential nonlinearities in the electrical measurement apparatus. While some of those familiar with photodiode measurements are aware of the necessary precautions to ensure a valid nonlinearity measurement, to our knowledge the topic has not been discussed in the literature. To use third-order distortion as an example, it may be readily shown [29] that the third-order intercept point of a two-element cascaded system satisfies (in linear units)

$$\frac{1}{\mathrm{IP}_3} = \frac{1}{G_2\mathrm{IP}_{3,1}} + \frac{1}{\mathrm{IP}_{3,2}}, \tag{11}$$

where $G_2$ is the RF power gain of the second stage and $\mathrm{IP}_{3,1}$ and $\mathrm{IP}_{3,2}$ are the intercept points of the first and second stages, respectively. Clearly, the largest term in (11) dominates. Therefore, if the nonlinearity of the measurement device approaches or exceeds that of the photodiode ($\mathrm{IP}_{3,\mathrm{meas.}} \leq \mathrm{IP}_{3,\mathrm{pd}}$) one must reduce the gain of the measurement device sufficiently to ensure the photodiode nonlinearity dominates the measurement. We note, the presence of spurious tones in the optical excitation may also impact the measurement dynamic range—the impact of imperfect two-tone excitation is discussed in the following section.

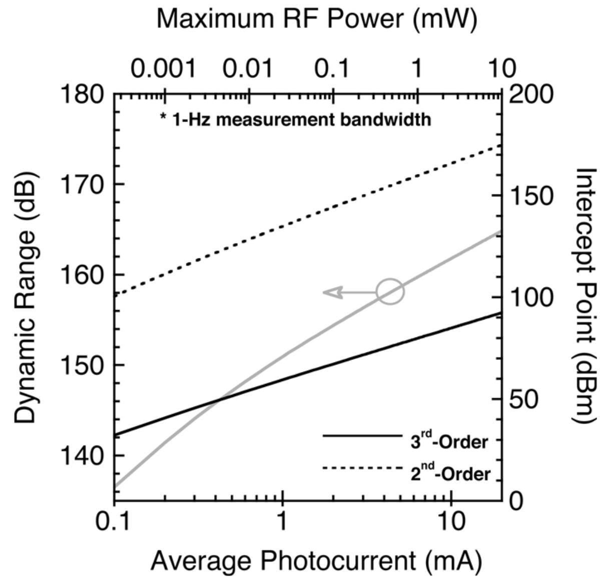


Fig. 1. Measurement dynamic range and maximum second- and third-order intercepts as a function of average photocurrent. Here, we assume a low-noise laser is used and that the system does not employ optical amplifiers [noise floor is determined by (8)].

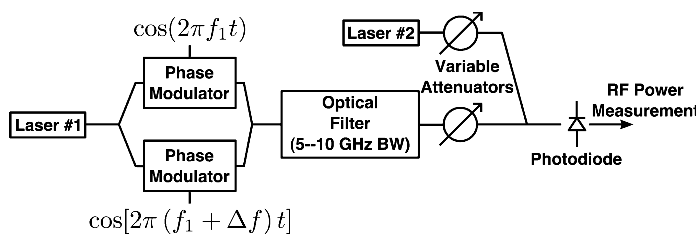


Fig. 2. Measurement apparatus. Here, the laser wavelengths are sufficiently far apart to ensure there is no beat note between the combs and the second laser source.

## III. Experiment

The basic concept of our technique is to emulate a pure two-tone excitation through high-resolution filtering of a pair of optical combs. Our measurement apparatus is shown in Fig. 2. Here, the output of a ~150 mW erbium fiber laser (NP Photonics, Inc., $\lambda \approx 1554$ nm) is split via a 50/50 fiber coupler and directed to two parallel low-$\mathrm{V}_\pi$ phase modulators ($\mathrm{V}_\pi \sim 3$ V, EOSpace, Inc.). The applied RF signals are at center frequencies of $f_1 = 20$ GHz and $f_1 + \Delta f = 21.24$ GHz (or 21 GHz). The phase-modulator outputs are recombined with a second 50/50 coupler and directed into a high-resolution optical filter designed to select one line from each comb. The optical filter is a fiber-coupled double-pass Fourier transform pulse shaper designed for ~12 GHz frequency resolution [30]. The output of the optical filter is combined with the output of a second laser (80 mW distributed feedback laser, EM4 Inc., $\lambda \approx 1550$ nm) using another 50/50 coupler and the composite signal is directed onto the photodiode under test. As in other techniques [16], [19], the second laser is sufficiently offset in wavelength from the first that there is no beating between the two at the photodiode. Therefore, the second laser acts only as a source of average photocurrent allowing the output RF power and DC photocurrent to be varied independently.

The RF measurement apparatus (not shown) consists of a 6-dB waveguide power divider followed by a pair of electrical spectrum analyzers (ESA, Agilent Technologies 8563EC). One ESA is used to measure the fundamental output power. The input to the second ESA is filtered to remove the fundamental, thereby providing a measurement of the distortion present in the photodiode output which is unaffected by potential ESA nonlinearity. The filtered signal is then amplified and measured with the ESA [the measured distortion powers are adjusted to account for the measurement gain of 34 dB (2.48 GHz) and 38 dB (3 GHz)]. The frequency offset between the combs is varied from 1.24 GHz to 1 GHz because of the available filter center frequencies (2.48 GHz and 3 GHz). The 240 MHz change in fundamental frequency has negligible impact on our experiment. This apparatus allows one to probe photodiode nonlinearity as a function of both modulation depth for a fixed average photocurrent (as presented here) or as a function of average photocurrent for a fixed modulation depth. In the latter measurement, the second laser is removed (or simply switched off), the optical combs are adjusted to have equal amplitudes (~100% modulation depth), and the post-filter variable optical attenuator is adjusted to vary the average photocurrent. The measurements presented here (nonlinearity as a function of modulation depth) are indicative of how the photodiode operates in an actual analog link—that is, the average photocurrent is determined by the available laser and/or optical amplifier while the modulation depth is determined by the input RF signal amplitude. In this case, the photodiode nonlinearity is largely due to static redistribution of the photodiode's internal electric field arising from the average photocurrent level (the nonlinear sideband amplitudes obey a largely power-law dependence on average photocurrent) [31]–[33]. As the average photocurrent increases for a fixed modulation depth (the second measurement mode) the peak photocurrent will cause a dynamic redistribution of the junction electric field. This results in a sharp increase in nonlinearity where the nonlinear sideband amplitudes tend to follow a greater-than-power-law dependence on average photocurrent. Thus, these measurement modes are complementary and provide data for two distinctly different operating regimes of the photodiode.

There are several benefits to this technique. First since both combs are generated from the same laser, laser frequency drift is a substantially smaller (or negligible) problem than in phase-locked laser techniques. Here, laser frequency drift is mapped to an amplitude fluctuation of the fundamental RF tone (if the frequency drifts far enough to push the selected comb lines to the edge of the optical filter passband), in contrast to the frequency drift it causes in phase-locked laser systems. So long as the fundamental and distortion powers are measured simultaneously—or if sufficient averaging is utilized—amplitude fluctuations will have negligible effect on the measurement results. Secondly, any distortion arising from impurity of the optical excitation is fixed at a level determined by the optical filter extinction. This is achieved by generating the optical combs via phase- instead of amplitude-modulation—modulator bias control is not required. Adjustment of the RF drive amplitudes may be used to compensate for differences in halfwave voltage between the modulators (to achieve the desired amplitude relation between the optical comb lines) at the beginning of the measurement. The modulators are co-located and operate with fixed optical and RF input powers; therefore, thermal drift is not a problem in this technique. Finally, our technique may be easily extended to intermodulation distortion measurements by introducing another laser and pair of phase modulators prior to the optical filter.

### *A. Comb/RF Tone Generation and Measurement Limits*

In this work, the phase modulators are excited with approximately 1-W of RF power apiece—this corresponds to a drive voltage approximately three-times greater than the modulator halfwave voltage. The optical combs consist of approximately 9-lines apiece as shown in Fig. 3(a). Note that, since the line spacing is different for each comb (solid black curve: 20.00 GHz, dashed black curve: 21.24 GHz), the spacing between comb lines of the same order increases monotonically with the magnitude of the order. For example, the first-order lines ($\mathrm{N} = \pm 1$ yield an RF beat note at $\Delta f = 1.24$ GHz, the second-order lines ($\mathrm{N} = \pm 2$) give an RF frequency of $2\Delta f = 2.48$ GHz, and so on. This illustrates one method by which the RF excitation frequency may be tuned. Alternatively, the offset frequency $\Delta f$ may be continuously varied by tuning the RF drive frequency of one phase modulator. We note the small-amplitude lines in the 21.24 GHz comb ($\pm 10, 30, 50$ GHz) are due to the spectral impurity of the driving synthesizer. The 21.24 GHz tone is derived from a fundamental signal at ~5.31 GHz and the measured spectrum contains low-level tones (~30–60 dBc) at the fundamental through sixth-harmonic. In this work, the optical filter is adjusted to select the $\mathrm{N} = +1$ sideband; the filter passband as measured with an amplified spontaneous emission source and an optical spectrum analyzer is shown by the solid black curve in Fig. 3(b) and the passband calculated from the optical filter parameters is shown by the dashed gray curve. Here, the passband shape and bandwidth are primarily determined by the spatial dispersion ($df/dx \approx 119.54$ GHz/mm) of the spectrometer (filter) and the focused spot-size ($w_o \approx 67.9\ \mu$m) of the Gaussian beam within the filter [34]. The measured $-10$ dB bandwidth is approximately 12.5 GHz which agrees extremely well with the predicted value of $\mathrm{BW} = 12.3$ GHz. The peak insertion loss of IL ~16 dB is typical for filters employing 2-in gratings and optics. The filtered combs are shown in Fig. 3(c). Here, the $\mathrm{N} = +1$ lines from each comb are selected, with greater than 34 dB suppression of unwanted lines. To examine the purity of the photodiode excitation, we can determine the upper bounds on the in-band distortion arising from imperfectly suppressed optical comb lines. From Fig. 3(c), we see the $\mathrm{N} = -1$ lines are suppressed to a level of $\sim -34$ dB relative to the $\mathrm{N} = +1$ lines. If we assume both $\mathrm{N} = -1$ lines are suppressed equally, the result is an additional photocurrent contribution roughly three-orders-of-magnitude below that arising from the $\mathrm{N} = +1$ lines [see (2)]. This additional current has negligible impact on our results. If we consider the second- and third-order optical sidebands, we see these have been sup-

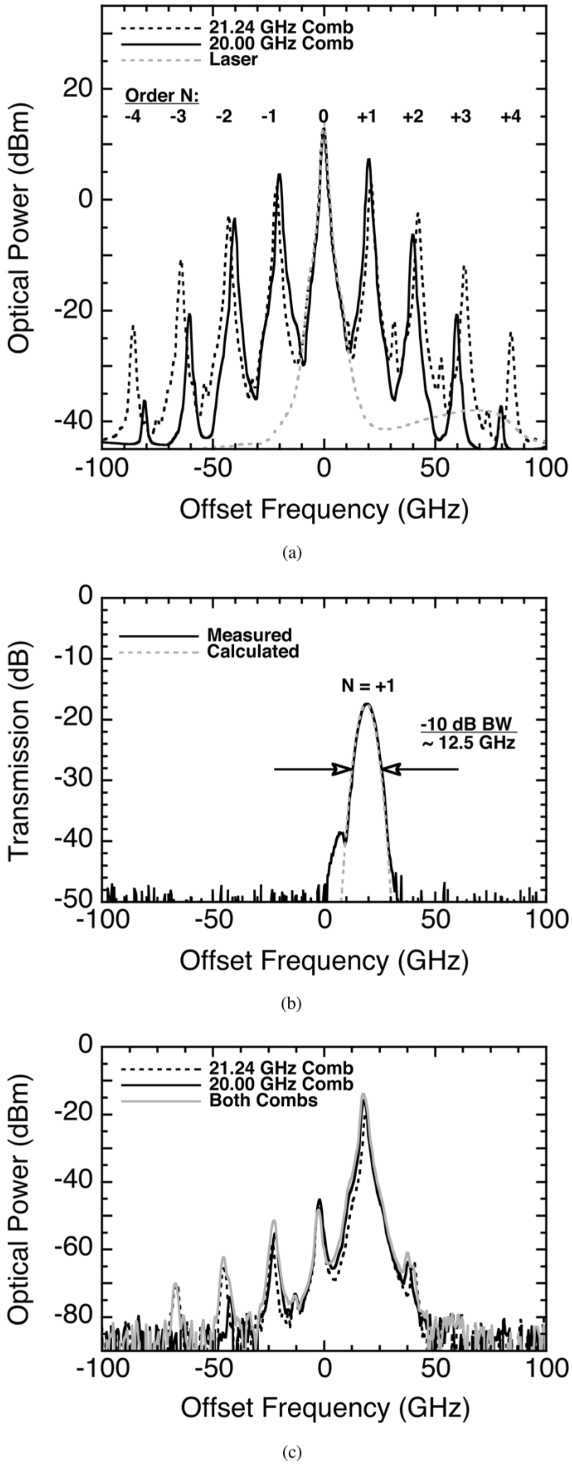


Fig. 3. (a) Optical combs at the input to the optical filter. (b) Measured (solid black) and calculated (dashed gray) optical filter passband. (c) Filtered optical combs demonstrating better than 34 dB suppression of unwanted comb lines.

pressed to levels of roughly $-48$ dB $(\mathrm{N} = \pm 2)$ and $-56$ dB $(\mathrm{N} = -3)$ below the $+1$-order lines in the optical domain. In the RF domain the distortion signals at $2\Delta f$ and $3\Delta f$ are then at least 96 dB and 112 dB below the fundamental signal, respectively. To cast these into intercept values, the maximum fundamental RF power arising from the $+1$-lines is approximately $P_{\mathrm{rf}} = +4$ dBm; since the $n$-th order intercept point may be expressed as (in dBm)

$$\mathrm{IP}_n = P_{\mathrm{rf}} + \frac{1}{n-1}\mathrm{CD} \tag{12}$$

where CD is the carrier-to-distortion ratio (in dB), we see the intercept points corresponding to the apparent distortion levels are $\mathrm{IP}_2 = 100$ dBm and $\mathrm{IP}_3 = 60$ dBm—these values are substantially higher that the intercepts thus-far demonstrated in state-of-the-art photodiodes [35].

It is worth noting that the above values assume equal filtering for both combs and, thus, represent the worst-case excitation purity. If we consider the individual 20 GHz and 21.24 GHz combs in Fig. 3(c) we see that the combs are not equally-filtered; there is an approximately 10 dB power difference between the $\mathrm{N} = -2$ lines, while the $\mathrm{N} = -3$ line from the 20 GHz comb is at least 10 dB below the corresponding line of the 21.24 GHz comb (limited by the measurement noise floor). Thus, the excitation-limited intercept points could increase by $\Delta\mathrm{IP}_2 = 10$ dBm and $\Delta\mathrm{IP}_3 = 5$ dBm, or more. While there is also beating between different orders of the optical combs, the resulting distortion products are mapped out-of-band. Therefore, they do not impact our measurements.

Though not shown in Fig. 2, our apparatus utilizes erbium-doped fiber amplifiers (EDFAs) just prior to, and following, the optical filter. It is well known that the amplified spontaneous emission (ASE) from the EDFA leads to an increased noise floor in the RF (electrical) domain [36] [an additive noise term in (8)]. The EDFAs in our system result in a $\sim$24 dB noise penalty—this also lowers the measurement dynamic range from the shot noise-limited value. We compare the measurement limits imposed by ASE noise and spurious excitation tones in our apparatus to the shot noise-limited performance in Table I. In the current apparatus, the dynamic range and measurable intercept points are clearly limited by the purity of the photodiode excitation. We note, however, that the optical filter extinction may be further improved (increased by $\sim$10 dB) through proper choice of components and RF drive frequencies. Such a system would then be limited by the additional noise arising from the EDFAs. In either case, the current apparatus provides sufficient purity to characterize the linearity of current and near-future photodiodes. We note, similar filtering and purity performance was achieved in a separate apparatus [23]; in this system, however, the measurement sensitivity was slightly lower due to an additional EDFA used to amplify the second CW laser source (used to control DC photocurrent).

TABLE I
COMPARISON OF MEASUREMENT LIMITS ($I_{\mathrm{avg}} = 10.0$ mA, $B_e = 1$ Hz)

| | Shot Noise | ASE Noise | Excitation Purity |
|---|---|---|---|
| *DR* (dB) | 164 | 140 | 96 (2nd), 112 (3rd) |
| $IP_{2,\max}$ (dBm) | 170 | 146 | 100 |
| $IP_{3,\max}$ (dBm) | 88 | 76 | 60 |

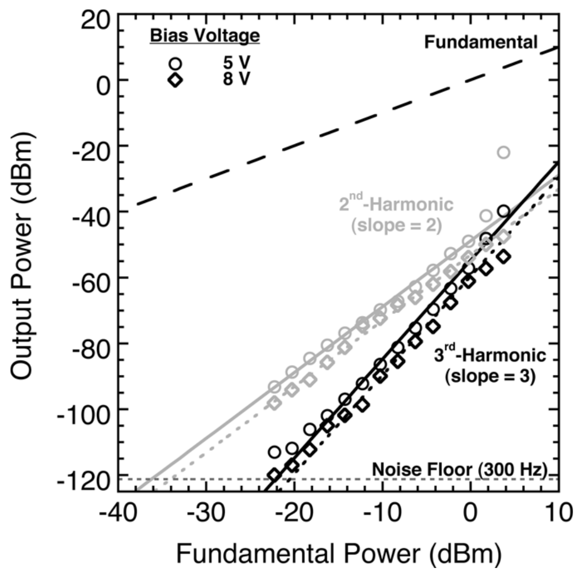


Fig. 4. Measured harmonic distortion levels as a function of fundamental output power for bias voltages of $V_{\rm bias} = 5$ V (circles) and $V_{\rm bias} = 8$ V (diamonds).

## B. Example Nonlinearity Measurement

To demonstrate the efficacy of our technique, we probe the nonlinearity of a commercial 40 $\mu$m (18 GHz) InGaAs PIN photodiode. For these measurements, the fundamental frequency is near $\Delta f = 1.0$ GHz (as described above, it is tuned over ~240 MHz to place the nonlinear sidebands within the passbands of the available RF filters) and the measurement bandwidth is $B_e = 300$ Hz. The total average photocurrent is maintained at 10 mA by appropriately adjusting the power of the second laser as the filtered comb power is varied. The fundamental power is varied over the range of approximately $-20$–$+4$ dBm; the lower limit corresponds to the point where the smallest nonlinear sideband $(3\Delta f)$ drops below the noise floor and the upper limit is set by the available laser power and RF drive parameters. The measurement is performed with photodiode bias voltages of $V_{\rm bias} = 5$ V and $V_{\rm bias} = 8$ V—the ability to resolve the small improvement in linearity with increasing bias [33] provides a simple way to verify the sensitivity of our technique.

Fig. 4 shows the measured harmonic distortion levels versus the measured fundamental output power. Here the symbols (circles: $V_{\rm bias} = 5$ V, diamonds: $V_{\rm bias} = 8$ V) represent the measured data and the lines are linear fits to the log-scale data. The linear fits for the $V_{\rm bias} = 5$ V exclude points that obviously deviate from the appropriate slope (for example those very near the noise floor, or those occurring at very high fundamental powers where there are sharp increases in nonlinearity at low/moderate bias voltages). From the data we extract intercepts of $\mathrm{IP}_2 = 49$ dBm and $\mathrm{IP}_3 = 27$ dBm for $V_{\rm bias} = 5$ V. At the increased bias voltage of $V_{\rm bias} = 8$ V, both intercepts are found to increase reaching $\mathrm{IP}_2 = 53$ dBm and $\mathrm{IP}_3 = 30$ dBm. We see a clear increase in linearity with the intercept points increasing by $\Delta\mathrm{IP}_2 = 4$ dBm and $\Delta\mathrm{IP}_3 = 3$ dBm, respectively. Such improvement in linearity is well documented [33] and verifies the sensitivity of this technique.

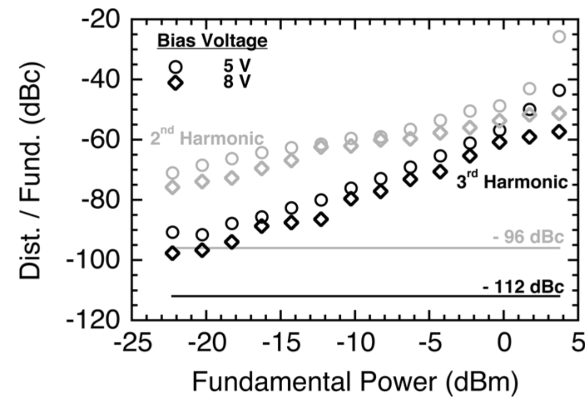


Fig. 5. Ratio of distortion to fundamental powers as compared to the maximum distortion which could arise directly from the optical excitation.

TABLE II
COMPARISON OF INTERCEPT POINTS OBTAINED VIA OPTICAL COMB AND PHASE-LOCKED LASER TECHNIQUES

| | $V_{\rm bias} = 5$ V | | $V_{\rm bias} = 8$ V | |
|---|---|---|---|---|
| | Opt. Comb | Laser Het. | Opt. Comb | Laser Het. |
| $IP_2$ (dBm) | 49 | 46 | 53 | 52 |
| $IP_3$ (dBm) | 27 | 26 | 30 | 29 |

In Fig. 5 we compare the distortion-to-fundamental ratio for the measured distortion levels to the worst-case distortion present in the excitation signal. Here, the second-harmonic data are shown in gray and the third-harmonic in black; the circles show data for $V_{\rm bias} = 5$ V and the diamonds correspond to $V_{\rm bias} = 8$ V. At the lowest fundamental and distortion powers (where excitation fidelity matters most) we see that the measured distortion levels exceed the potential excitation distortion by almost two orders-of-magnitude for both the second- and third-harmonic. This clearly illustrates our technique exhibits the fidelity necessary to characterize the linearity of state-of-the-art photodiodes.

For comparison we also measure the same photodiode using a pair of offset phase-locked Nd:YAG lasers at approximately 1320 nm [16]. The experimental parameters remain fixed—the fundamental excitation is at $\Delta f = 1.0$ GHz, the average photocurrent is held at 10 mA and the measurements are made for both $V_{\rm bias} = 5$ V and $V_{\rm bias} = 8$ V. Table II summarizes the results of our measurements. We see that the intercept points derived from these two techniques agree quite well; the 1–3 dB differences are readily attributed to measurement accuracy (the ESA amplitude accuracy is $\sim$1 dB for both systems). There are likely small wavelength-dependent variations in the measured nonlinearity as well [37]; however, this area warrants further research. This comparison illustrates that both techniques provide equivalent measurement fidelity for current photodiode structures.

## IV. SUMMARY

In this work we present a new technique for measurement of photodiode harmonic distortion based on an optical heterodyne between lines of two distinct optical frequency combs.

Both combs are generated from the same erbium fiber laser, thereby removing any adverse effects arising from laser frequency drift. By utilizing optical phase modulation as opposed to intensity modulation we eliminate modulator bias drift thereby increasing the measurement fidelity and simplifying the measurement system. The current apparatus may be used to characterize photodiodes exhibiting harmonic-distortion intercept points as high as $\mathrm{IP}_2 = 100$ dBm and $\mathrm{IP}_2 = 60$ dBm (limited by the optical filter extinction). We note our technique may be readily extended to measurement of intermodulation distortion. The addition of a second laser and pair of phase-modulators before the optical filter would enable generation of two independently-tunable fundamental signals—this addition would further extend the facility of the technique as harmonic and intermodulation distortion are not necessarily easily linked in photodiodes.

## REFERENCES


[1] R. A. Minasian, "Photonic signal processing of microwave signals," *IEEE Trans. Microw. Theory Tech.*, vol. 54, no. 2, pp. 832–846, Feb. 2006.

[2] P. W. Juodawlkis, J. C. Twichell, G. E. Betts, J. J. Hargreaves, R. D. Younger, J. L. Wasserman, F. J. O'Donnell, K. G. Ray, and R. C. Williamson, "Optically sampled analog-to-digital converters," *IEEE Trans. Microw. Theory Tech.*, vol. 49, no. 10, pp. 1840–1853, Oct. 2001.

[3] A. J. Seeds and K. J. Williams, "Microwave photonics," *J. Lightw. Technol.*, vol. 24, no. 12, pp. 4628–4641, Dec. 2006.

[4] A. Hastings, V. J. Urick, C. Sunderman, J. Diehl, J. D. McKinney, D. Tulchinsky, P. S. Devgan, and K. J. Williams, "Suppression of even-order photodiode nonlinearities in multi-octave photonic links," *J. Lightw. Technol.*, vol. 26, no. 15, pp. 2557–2562, Aug. 2008.

[5] C. H. Cox, III, *Analog Optical Links: Theory and Practice.*. New York: Cambridge Univ. Press, 2004.

[6] V. J. Urick, M. S. Rogge, F. Bucholtz, and K. J. Williams, "The performance of analog photonic links employing highly compressed erbium-doped fiber amplifiers," *IEEE Trans. Microw. Theory Tech.*, vol. 54, no. 7, pp. 3141–3145, Jul. 2006.

[7] J. D. McKinney, M. Godinez, V. J. Urick, S. Thaniyavarn, W. Charczenko, and K. J. Williams, "Sub-10 db noise figure in a multiple-ghz analog optical link," *IEEE Photon. Technol. Lett.*, vol. 19, pp. 465–467, Apr. 2007.

[8] H. V. Roussell, M. D. Regan, J. L. Prince, C. H. Cox, J. X. Chen, W. K. Burns, G. E. Betts, E. I. Ackerman, and J. C. Campbell, "Gain, noise figure and bandwidth-limited dynamic range of a low-biased external modulation link," in *Proc. IEEE Int. Topical Meeting Microw. Photon.*, Oct. 3–5, 2007, pp. 84–87.

[9] J. L. Brooks, G. S. Maurer, and R. A. Becker, "Implementation and evaluation of a dual parallel linearization system for am-scm video transmission," *J. Lightw. Technol.*, vol. 11, no. 1, pp. 34–41, Jan. 1993.

[10] G. E. Betts, "Linearized modulator for suboctave-bandpass optical analog links," *IEEE Trans. Microw. Theory Tech.*, vol. 42, no. 12, pp. 2642–2649, Dec. 1994.

[11] E. I. Ackerman, "Broad-band linearization of a mach-zehnder electrooptic modulator," *IEEE Trans. Microw. Theory Tech.*, vol. 47, no. 12, pp. 2271–2279, Dec. 1999.

[12] L. M. Johnson and H. V. Roussell, "Reduction of intermodulation distortion in interferometric optical modulators," *Opt. Lett.*, vol. 13, pp. 928–930, 1988.

[13] B. M. Haas and T. E. Murphy, "A simple, linearized, phase-modulated analog optical transmission system," *IEEE Photon. Technol. Lett.*, vol. 19, no. 10, pp. 729–731, May 2007.

[14] B. M. Haas, V. J. Urick, J. D. McKinney, and T. M. Murphy, "Dual-wavelength linearization of optically phase-modulated analog microwave signals," *J. Lightw. Technol.*, vol. 26, no. 15, pp. 2748–2753, Aug. 2008.

[15] J. D. McKinney, K. Colladay, and K. J. Williams, "Linearization of phase-modulated analog optical links employing interferometric demodulation," *J. Lightw. Technol.*, vol. 27, no. 9, pp. 1212–1220, May 2009.

[16] K. J. Williams, L. Goldberg, R. D. Esman, M. Dagenais, and J. F. Weller, "6–34 GHz offset phase-locking of Nd:YAG 1319 nm nonplanar ring lasers," *Electron. Lett.*, vol. 25, no. 18, pp. 1242–1243, Aug. 1989.

[17] D. C. Scott, T. A. Vang, J. Elliott, D. Forbes, J. Lacey, K. Everett, F. Alvarez, R. Johnson, A. Krispin, J. Brock, L. Lembo, H. Jiang, D. S. Shin, J. T. Zhu, and P. K. L. Yu, "Measurement of $\mathrm{IP}_3$ in P-I-N photodetectors and proposed performance requirements for rf fiber-optic links," *IEEE Photon. Technol. Lett.*, vol. 12, no. 4, pp. 422–424, Apr. 2000.

[18] H. Jiang, D. S. Shin, G. L. Li, A. Vang, D. C. Scott, and P. K. L. Yu, "The frequency behavior of the third-order intercept point in a waveguide photodiode," *IEEE Photon. Technol. Lett.*, vol. 12, no. 5, pp. 540–542, May 2000.

[19] M. N. Draa, J. Ren, D. C. Scott, W. S. C. Chang, and P. K. L. Yu, "Three laser two-tone setup for measurement of photodiode intercept points," *Opt. Exp.*, vol. 16, no. 16, pp. 12108–12113, Aug. 2008.

[20] H. Murata, A. Morimoto, T. Kobayashi, and S. Yamamoto, "Optical pulse generation by electrooptic-modulation method and its application to integrated ultrashort pulse generators," *IEEE J. Select. Topics Quantum Electron.*, vol. 6, no. 6, pp. 1325–1331, Nov./Dec. 2000.

[21] C.-B. Huang, S.-G. Park, D. E. Leaird, and A. M. Weiner, "Nonlinearly broadened phase-modulated continuous-wave laser frequency combs characterized using DPSK decoding," *Opt. Exp.*, vol. 16, no. 4, pp. 2520–2527, Feb. 2008.

[22] Z. Jiang, D. Seo, D. E. Leaird, and A. M. Weiner, "Spectral line-by-line pulse shaping," *Opt. Lett.*, vol. 30, pp. 1557–1559, 2005.

[23] J. D. McKinney, D. E. Leaird, A. M. Weiner, and K. J. Williams, "Measurement of photodiode nonlinearities using an optical comb source," in *Proc. CLEO*, Baltimore, MD, May 31–June 5 2009, Paper CWI5.

[24] A. Bartels, S. A. Diddams, C. W. Oates, G. Wilpers, J. C. Bergquist, W. H. Oskay, and L. Hollberg, "Femtosecond-laser-based synthesis of ultrastable microwave signals from optical frequency references," *Opt. Lett.*, vol. 30, no. 6, pp. 667–669, Mar. 2005.

[25] E. N. Ivanov, S. A. Diddams, and L. Hollberg, "Analysis of noise mechanisms limiting the frequency stability of microwave signgenerated with a femtosecond laser," *IEEE. J. Sel. Topics. Quantum Electron.*, vol. 9, no. 4, pp. 1059–1065, Jul./Aug. 2003.

[26] E. N. Ivanov, S. A. Diddams, and L. Hollberg, "Study of the excess noise associated with demodulation of ultrashort infrared pulses," *IEEE Trans. Ultrason., Ferroelect., Freq. Contr.*, vol. 52, no. 7, pp. 1068–1074, Jul. 2005.

[27] M. Currie and I. Vurgaftman, "Microwave phase retardation in saturated ingaas photodetectors," *IEEE Photon. Technol. Lett.*, vol. 18, no. 13, pp. 1433–1435, Jul. 2006.

[28] S. A. Diddams, M. Kirchner, T. Fortier, D. Braje, A. M. Weiner, and L. Hollberg, "Improved signal-to-noise ratio of 10 GHz microwave signals generated with a mode-filtered femtosecond laser frequency comb," *Opt. Exp.*, vol. 17, no. 5, pp. 3331–3340, Mar. 2009.

[29] D. M. Pozar, *Microwave Engineering*, 3rd ed. Hoboken, NJ: Wiley, 2005.

[30] A. M. Weiner, "Femtosecond pulse shaping using spatial light modulators," *Rev. Sci. Instr.*, vol. 71, no. 5, pp. 1929–1960, May 2000.

[31] K. J. Williams, L. T. Nichols, and R. D. Esman, "Photodetector nonlinearity limitations on a high-dynamic range 3 ghz fiber optic link," *J. Lightw. Technol.*, vol. 16, no. 2, pp. 192–199, Feb. 1998.

[32] K. J. Williams, R. D. Esman, and M. Dagenais, "Effects of high space-charge fields on the response of microwave photodetectors," *IEEE Photon. Technol. Lett.*, vol. 6, no. 5, pp. 639–641, May 1994.

[33] K. J. Williams, R. D. Esman, and M. Dagenais, "Nonlinearities in PIN microwave photodetectors," *J. Lightw. Technol.*, vol. 14, no. 1, pp. 84–96, Jan. 1996.

[34] R. N. Thurston, J. P. Heritage, A. M. Weiner, and W. J. Tomlinson, "Analysis of picosecond pulse shape synthesis by spectral masking in a grating pulse compressor," *IEEE J. Quantum Electron.*, vol. 22, pp. 682–696, 1986.

[35] A. Joshi, S. Datta, and D. Becker, "GRIN lens coupled top-illuminated highly linear InGaAs photodiodes," *IEEE Photon. Technol. Lett.*, vol. 20, no. 17, pp. 1500–1502, Sep. 2008.

[36] E. Desurvire, *Erbium-Doped Fiber Amplifiers: Principles and Applications.*. New York: John Wiley and Sons, Inc., 1994.

[37] A. S. Hastings, D. A. Tulchinsky, K. J. Williams, H. Pan, A. Beling, and J. C. Campbell, "Minimizing photodiode nonlinearities by compensating voltage-dependent responsivity effects," in *Proc. MWP'09*, Valencia, Spain, Oct. 4–6, 2009, vol. 1, pp. 1–4.

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