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Effect of Group Velocity Mismatch on the Measurement of Ultrashort Optical Pulses via Second Harmonic Generation

ANDREW M. WEINER

Abstract—**The use of second harmonic generation as a technique for optical pulse width measurements is analyzed to determine the effects of both phase velocity and group velocity mismatch between fundamental and second harmonic fields. An expression for the time average second harmonic energy, which except for special cases differs from the intensity autocorrelation function, is derived. For Type I phase matching, the measurement yields an apparent correlation width which can be either shorter or longer than the actual intensity correlation width, depending on the specific pulse shape. When the group velocity mismatch is nonzero, the measurement becomes sensitive to the phase matching condition. Two special pulse shapes for which the measurement is independent of group and phase velocity mismatch are the Gaussian and the single-sided exponential.**

I. Introduction

RECENT advances [1], [2] in the generation of ultrashort optical pulses less than 0.1 ps long necessitate a reexamination of the methods used to measure the duration of these pulses. In one of the most popular methods, a pair of time delayed but otherwise identical pulses are mixed via phase matched second harmonic generation (SHG) in a nonlinear crystal [3]. It is well known that one can obtain the intensity autocorrelation function by measuring the time average second harmonic energy as a function of the relative time delay τ between the pulses, provided that phase matching can be maintained over the entire frequency spectrum of the pulse. It is not well known, however, what corrections arise when the pulses become so short that their bandwidth approaches or exceeds the phase matching bandwidth. The purpose of this paper is to make the lowest order corrections.

It has long been recognized that group velocity mismatch between fundamental and second harmonic fields, which results in a finite phase matching bandwidth, has an important effect on the up conversion of ultrashort optical pulses. Conversion efficiencies and second harmonic pulse shapes have been calculated [4]-[6] as a function of group velocity mismatch. However, the exact effect on SHG pulse width measurements has not been previously calculated. In this paper the SHG pulse width measurement technique is analyzed allowing for both group velocity and phase velocity mismatch. The next higher order effect, pulse spreading due to group velocity dispersion in the nonlinear crystal, is not considered in this publication.

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In order to justify this approach, let us consider the following numerical example. The nonlinear medium is a 0.3 mm long KDP crystal, angle tuned (57.7° phase matching angle) to achieve phase matching for SHG of a 620 nm fundamental [7]. The group velocity mismatch (defined as the difference in the group delay for the fundamental and the second harmonic) is 56 fs, assuming that the two pulses interact over the entire crystal length. If the interaction length is less than the 0.3 mm crystal length (e.g., due to beam crossing in a noncollinear geometry), a proportionately smaller value for the group velocity mismatch should be used. For pulses of less than 100 fs duration, we would expect the effect of the group velocity mismatch to become evident. The spreading of the fundamental pulses due to group velocity dispersion, however, will be extremely small; even the shortest pulses on record [2], with a duration of 30 fs full width half maximum, will spread only about 0.1 percent, assuming a Gaussian pulse shape [8]. Thus, our approach appears to be justified.

The calculation yields several interesting results. In the case of Type I phase matching, the measurement gives an apparent correlation width which can be either shorter or longer than the actual intensity correlation width, depending on the specific pulse shape. Furthermore, when the group velocity mismatch is nonzero, the measurement becomes sensitive to the phase matching condition and to frequency chirp, unlike the case where group velocities are matched. Two special pulse shapes for which the measurement is independent of the group and phase velocity mismatch are the Gaussian and the single-sided exponential.

The body of this paper is divided into four sections. First, the SHG measurement is analyzed assuming a noncollinear geometry with Type I phase matching (two ordinary waves at ω_0 interact to produce an extraordinary wave at $2\omega_0$). An expression for the time average second harmonic energy as a function of the relative delay τ is derived. Second, the expression is examined using several different pulse shapes as examples. Third, the collinear geometry, still with Type I phase matching, is considered. Finally, Type II phase matching (in which an ordinary wave ω_0 and an extraordinary wave at ω_0 interact to produce an extraordinary wave at $2\omega_0$) is discussed.

II. Noncollinear Geometry Type I Phase Matching

We consider first the noncollinear measurement geometry with Type I phase matching [9], [10] shown in Fig. 1. The phase matching is adjusted so that second harmonic is produced only when both fundamental beams are present. The two input pulses have the ordinary polarization; the second harmonic pulse has the extraordinary polarization perpendicular to that of the input pulses. Geometrical effects due to beam crossing, incomplete beam overlap in the crystal, double refraction, and beam divergence are not considered here; in fact, the fundamental and second harmonic fields are treated as plane waves. The second harmonic field propagates in the $+z$ direction. Rationalized MKS units are used throughout.

The approach here is similar to that of Glenn [5]. One

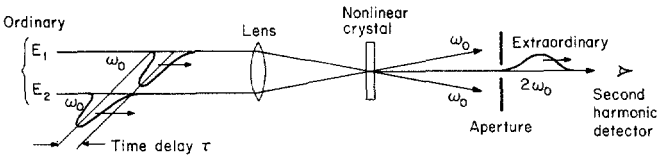

Fig. 1. Experimental setup for pulse width measurement, with noncollinear geometry and Type I phase matching.

assumes that the fundamental fields are undepleted, so that they can be written as a Fourier superposition of monochromatic waves whose amplitudes do not vary with z. The positive frequency part of the field $E_{f1}(z, t)$ is written as the product of a carrier and an envelope function, using the notation $\dot{k}_f = dk_f/d\omega$.

$$E_{f1}(z, t) = f(t - \dot{k}_f z) \exp [j(\omega_0 t - k_f z)]$$

$$= \exp [j(\omega_0 t - k_f z)] \cdot \frac{1}{2\pi} \int d\omega_1 F(\omega_1)$$

$$\cdot \exp [j(\omega_1 t - \dot{k}_f \omega_1 z)]. \quad (1)$$

The envelope function $f(t - \dot{k}_f z)$ is a complex quantity and includes the phase (which may be time varying). The positive frequency parts of the time delayed field $E_{f2}(z, t)$ and the second harmonic field $E_s(z, t)$ can be written similarly as

$$E_{f2}(z, t) = E_{f1}(z, t - \tau) = f(t - \tau - \dot{k}_f z)$$

$$\cdot \exp (-j\omega_0 \tau) \exp [j(\omega_0 t - k_f z)]$$

$$= \exp [j(\omega_0 t - k_f z)] \exp (-j\omega_0 \tau) \cdot \frac{1}{2\pi} \int d\omega_2 F(\omega_2)$$

$$\cdot \exp (-j\omega_2 \tau) \exp [j(\omega_2 t - \dot{k}_f \omega_2 z)] \quad (2)$$

and

$$E_s(z, t) = s(z, t - \dot{k}_s z) \exp [j(2\omega_0 t - k_s z)]$$

$$= \exp [j(2\omega_0 t - k_s z)] \cdot \frac{1}{2\pi} \int d\omega S(z, \omega)$$

$$\cdot \exp [j(\omega t - \dot{k}_s \omega z)]. \quad (3)$$

The subscripts f and s refer to the fundamental and second harmonic fields, respectively. $\dot{k}_f$ and $\dot{k}_s$ are the derivatives of k_f and k_s with respect to angular frequency. Components of the propagation vector $\bar{k}$ orthogonal to the z direction are omitted; these will cancel out anyway when the nonlinear polarization term is written. With Type I phase matching, both fundamental fields have the same k_f and the same $\dot{k}_f$. Although it is unimportant that the same k_f is used, the fact that both k_f's are identical is essential to the following analysis (compare Section V).

The nonlinear polarization at the second harmonic frequency is proportional to the product of the nonlinear susceptibility χ and the two fundamental fields. The nonlinear susceptibility χ, written here as a scalar, actually depends on both the actual

nonlinear susceptibility tensor and the orientation of the various fields. By making the substitution $\omega = \omega_1 + \omega_2$, the following expression for the polarization is obtained:

$$P_{NL}(z,t) = P_{NL}(t - \dot{k}_f z)\exp\,[j(2\omega_0 t - 2k_f z)]$$
$$= \exp\,[j(2\omega_0 t - 2k_f z)] \cdot \epsilon_0 \chi \exp\,(-j\omega_0 \tau)$$
$$\cdot \left(\frac{1}{2\pi}\right)^2 \int d\omega \exp\,[j(\omega t - \dot{k}_f \omega z)]$$
$$\times \int d\omega_2 F(\omega - \omega_2) F(\omega_2) \exp\,(-j\omega_2 \tau). \quad (4)$$

According to the standard [11] formalism of nonlinear optics, the nonlinear polarization serves as a source in the wave equation for the second harmonic field $E_s(z,t)$. In the slowly varying envelope approximation, the wave equation is written

$$\frac{\partial s(z,t)}{\partial z} = -j\,\frac{(2\omega_0)\,\mu_0 c}{2n} P_{NL} e^{j(k_s - k_p)z} \quad (5)$$

where $k_p = 2k_f$ is the propagation constant of the polarization and n is the refractive index experienced by the s field. We now substitute the Fourier expansions (3) and (4) of the second harmonic field and the nonlinear polarization into the wave equation (5) and equate terms which vary as $e^{j\omega t}$. The result is an equation which describes the growth of the individual Fourier components of the second harmonic field, as follows:

$$\frac{\partial S(z,\omega)}{\partial z} = -j\,\frac{A}{2\pi}\exp\,(-j\omega_0\tau)\exp\,[j(\Delta k + \alpha\omega)\,z]$$
$$\cdot \int d\omega_2 F(\omega - \omega_2) F(\omega_2)\, e^{-j\omega\tau} \quad (6)$$

where $A = \omega_0 \chi / nc$, $\Delta k = k_s - 2k_f$, and $\alpha = \dot{k}_s - \dot{k}_f$. Δk represents the phase mismatch and α is the group mismatch.

Equation (6) is easily solved by integration; the result for a nonlinear medium of length L is given by

$$S(L,\omega) = \frac{AL}{2\pi} e^{-j\omega_0\tau}$$
$$\cdot \left[\exp\,[j(\Delta k + \alpha\omega)\,L/2]\;\frac{\sin\,(\Delta k + \alpha\omega)\,L/2}{(\Delta k + \alpha\omega)\,L/2}\right]$$
$$\times \int d\omega_2 F(\omega - \omega_2) F(\omega_2)\, e^{-j\omega_2\tau}. \quad (7)$$

We have implicitly assumed that the two fundamental fields E_{f1} and E_{f2} interact over the entire length L of the nonlinear medium. In the noncollinear geometry, this assumption may often prove invalid; for example, the interaction length of two beams focused to a 35 μm spot diameter and crossing at an angle 10° is limited to approximately 0.17 mm. The length L in equation (7) should be interpreted as the interaction length when this is less than the crystal length.

It is clear from (7) that the phase matching condition depends on the second harmonic frequency ω; the claim that group velocity mismatch leads to a finite phase matching bandwidth is therefore justified. Effectively, this results in a filter which acts only on the second harmonic field and not on the fundamental fields.

Equation (7) is now inverse Fourier transformed to obtain the time dependence of the second harmonic field at $z = L$. The result is

$$s(L,t) = AL\exp\,(-j\omega_0\tau)\;[f(t)\,f(t-\tau)] * \left[\frac{1}{\alpha L}\,sq\left(\frac{t}{\alpha L} + \frac{1}{2}\right)\right.$$
$$\left.\exp\,(-j\Delta k t/\alpha)\right]$$
$$= AL\exp\,(-j\omega_0\tau)\,g(t) * h(t) \quad (8)$$

where "$*$" denotes convolution,

$$g(t) = f(t)\,f(t-\tau)\exp\,(-j\omega_0\tau),$$
$$h(t) = \frac{1}{\alpha L}\,sq\left(\frac{t}{\alpha L} + \frac{1}{2}\right)\exp\,(-j\Delta k t/\alpha)$$

and

$$sq(x) = \begin{cases} 1, & |x| \leqslant 1/2 \\ 0, & \text{otherwise}. \end{cases}$$

According to (8), the resultant second harmonic field can be viewed as the response of a linear system. The function $g(t) = f(t)\,f(t-\tau)\,e^{-j\omega_0\tau}$ is just the driving term for the SHG process and serves as the input to the linear system. The function $h(t)$ can be thought of as the impulse response; even if the input $g(t)$ is a delta function, the second harmonic field which results is a square pulse of duration $|\alpha L|$ and height proportional to $1/|\alpha L|$ which either trails or leads the input delta function depending on the sign of αL. The phase of this square pulse is constant if $\Delta k = 0$ (perfect phase matching); otherwise, the phase varies by an amount $\Delta k L$ across the square pulse. The total second harmonic field is just the convolution integral of the input function $g(t)$ and the impulse response $h(t)$. Thus, the second harmonic field at time t depends not only on the fundamental fields at time t but also on the fundamental fields over the entire time interval between t and $t - \alpha L$.

Note that the actual second harmonic field at $z = L$ as defined by (3) is not the function $s(L,t)$ discussed above but a time delayed version of this, $s(L, t - \dot{k}_s z)$. This time delay ensures that the "linear system" above is, in fact, causal.

The quantity one actually measures is the time average intensity at frequency $2\omega_0$, which we denote $\bar{I}_s(\tau)$. An expression for this quantity is obtained by integrating the second harmonic intensity over the pulse duration; the intensity in turn is proportional to the product of the second harmonic field given by (3) and (8) and its complex conjugate.

$$\bar{I}_s(\tau) \sim |AL|^2 \int dt[(g * h)\,e^{j2\omega_0 t})]\;[(g * h)\,e^{2j\omega_0 t})]^*$$
$$= |AL|^2 \int dt \int dt'\, g(t')\,h(t - t') \int dt''$$
$$\cdot g^*(t'')\,h^*(t - t''). \quad (9)$$

The "*" symbol denotes complex conjugation when written as a superscript and convolution otherwise. By making the substitution $\eta = t' - t''$ and interchanging the orders of various integrations, one obtains

$$\overline{I_s}(\tau) \sim |AL|^2 \int d\eta \int dt'' \, g(\eta + t'') \, g^*(t'') \int dt \cdot h(t - \eta) \, h^*(t). \tag{10}$$

Finally, if one plugs in for the functions g and h using (8) and substitutes $\eta' = \eta/\alpha L$, the result is

$$\begin{aligned}\overline{I_s}(\tau) &\sim |AL|^2 \, G(\tau) \\ &= |AL|^2 \int d\eta' \, tri(\eta') \exp(j\Delta kL\eta') \int dt \\ &\quad \cdot f^*(t) f^*(t - \tau) f(t + \alpha L\eta') f(t + \alpha L\eta' - \tau)\end{aligned} \tag{11}$$

where

$$G(\tau) = \int d\eta' \, tri(\eta') \exp(j\Delta kL\eta') \, R(\tau, \alpha L\eta'),$$

$$R(\tau, \alpha L\eta') = \int dt \, f^*(t) f^*(t - \tau) f(t + \alpha L\eta') \cdot f(t + \alpha L\eta' - \tau)$$

and

$$tri(x) = \begin{cases} 1 - |x|, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}.$$

Equation (11) is the desired result. If the group velocity mismatch αL is exactly zero, then one obtains the usual result, i.e., the second harmonic intensity is proportional to the intensity correlation function, independent of the phase matching

$$\overline{I_s}(\tau) \sim \int dt \, |f(t)|^2 \, |f(t - \tau)|^2 . \tag{12}$$

When αL is nonzero, one no longer obtains the intensity correlation function, except for special pulse shapes; furthermore, the result is no longer independent of the phase matching. Note that (11) depends on the complex field amplitude $f(t)$, whereas (12) depends only on the magnitude $|f(t)|$. Therefore, when the group velocity mismatch becomes nonzero, the pulse shape measurement becomes sensitive to chirp and to phase variations; two pulses with identical intensity profiles but with different phase profiles will in general lead to different measurement results. The second harmonic energy does remain an even function of the delay τ.

III. Examples and Discussion

We now apply the results of the previous section to several different pulse shapes.

A. Gaussian Pulse

The form of the electric field envelope is $f(t) = \exp[-t^2/2tp^2]$, with tp^2 real. Substituting this pulse shape into (11), we find that

$$R(\tau, \alpha L\eta') = \sqrt{\pi/2}\, tp \exp(-\tau^2/2tp^2) \exp[-(\alpha L\eta')^2/2tp^2]. \tag{13}$$

The dependence of the second harmonic energy on the delay τ is given by $G(\tau)$

$$\begin{aligned}G(\tau) &= \int d\eta' \, tri(\eta') \exp(j\Delta kL\eta') \, R(\tau, \alpha L\eta') \\ &= tp\sqrt{\pi/2}\, e^{-\tau^2/2tp^2} \int d\eta' \, tri(\eta') \exp(j\Delta kL\eta') \\ &\quad \cdot \exp[-(\alpha L\eta')^2/2tp^2 \\ &\sim \exp(-\tau^2/2tp^2).\end{aligned} \tag{14}$$

The function $G(\tau)$ is proportional to the intensity correlation function, independent of the group velocity mismatch αL and the phase velocity mismatch ΔkL. If the parameter $1/tp^2$ is allowed to be complex, then $f(t)$ represents a Gaussian pulse with a linear chirp, and the measurement still yields the intensity correlation function. However, if the Gaussian pulse is multiplied by a nonlinear chirp function, the result of the measurement will in general change.

B. Single-Sided Exponential Pulse

The electric field envelope is $f(t) = \exp(-t/2tp)\, u(t)$ where $u(t)$ is a unit step function and tp is real and positive. Plugging into (11) yields

$$R(\tau, \alpha L\eta') = \frac{tp}{2} \exp(-|\tau|/tp) \exp(-|\alpha L\eta'|/tp)$$

and

$$\begin{aligned}G(\tau) &= \frac{tp}{2} \exp(-|\tau|/tp) \int d\eta' \, tri(\eta') \exp(j\Delta kL\eta') \\ &\quad \cdot \exp(-|\alpha L\eta'|/tp) \\ &\sim \exp(-|\tau|/tp).\end{aligned} \tag{15}$$

Once again the intensity correlation function is obtained. If the parameter $1/tp$ is allowed to be complex (with positive real part), then $f(t)$ represents a single-sided exponential pulse with a special nonlinear chirp, and the same result still applies. However, if the single-sided exponential is chirped in any other way, then the intensity correlation function is no longer obtained.

The observation that the group velocity mismatch has no effect of the shape of $G(\tau)$ in examples A and B is not indicative of a general property but rather results because the Gaussian and the single-sided exponential are special pulse shapes. For these two envelope functions, the correlation function $R(\tau, \alpha L\eta')$ separates into the product of the τ dependence and the η' dependence, i.e.,

$$R(\tau, \alpha L\eta') = r_1(\tau)\, r_2(\alpha L\eta'). \tag{16}$$

Thus, the τ dependence $r_1(\tau)$ can be taken outside the η' integral in equation (11), yielding a $G(\tau)$ whose shape does not change as αL and ΔkL are varied. Equation (16) applies because the product $f(t)\,f(t-\tau)$, which serves as the input for the SHG process, can be written

$$f(t)\,f(t-\tau) = a(\tau)\, I(t - b(\tau)). \tag{17}$$

Although the product has an amplitude $a(\tau)$ and a time shift $b(\tau)$ which do depend on the delay τ, its shape does not depend on τ. Therefore, the total second harmonic energy can depend on the properties of the nonlinear medium only through the multiplicative factor $|a(\tau)|^2$. For these special pulse shapes, corrections arise only when one includes the effect of group velocity dispersion on the pulse themselves.

Below we consider some pulse shapes which do not satisfy the special conditions (16) and (17).

C. Square Pulse

We assume that the electric field envelope is a square pulse of width T with $f(t) = 1$ for $|t| \leqslant T/2$ and $f(t) = 0$ otherwise. The phase is assumed constant. The resultant $G(\tau)$ is plotted in Fig. 2 with normalized group velocity mismatch $|\alpha L/T|$ as a parameter. Perfect phase matching $\Delta kL = 0$ is assumed. The apparent correlation width decreases with increasing $|\alpha L/T|$; the curve for $|\alpha L/T| = \infty$ (not shown) is only 59 percent as wide as the curve for $|\alpha L/T| = 0$.

D. Secant Hyperbolic Pulse

The envelope of the electric field is $f(t) = \operatorname{sech}(t/tp)$, with tp real. In order to evaluate $R(\tau, \alpha L\eta')$, we employ the method of reference [12], which uses the fact that the Fourier transform of the autocorrelation of a function $g(t)$ is equal to the power spectrum $|G(\omega)|^2$. The result is obtained after two contour integrations.

$$\begin{aligned} R(\tau,\eta) = {} & \frac{4}{\sinh^2(\tau/tp)} \\ & \cdot \left\{ \frac{-2|\eta|}{\exp(2|\eta|/t_p) - 1} + \frac{|\tau+\eta|}{\exp(2|\tau+\eta|/t_p) - 1} \right. \\ & + \frac{|\tau-\eta|}{\exp(2|\tau-\eta|/t_p) - 1} + 1/2\,[-2|\eta| \\ & \left. + |\tau+\eta| + |\tau-\eta|] \right\} \end{aligned} \tag{18}$$

where the shorthand $\eta = \alpha L\eta'$ is employed. In the limit that $\alpha L = 0$, this reduces to

$$R(\tau, 0) = \frac{4t_p}{\sinh^3(\tau/t_p)} \left[\left(\frac{\tau}{t_p}\right) \cosh\left(\frac{\tau}{t_p}\right) - \sinh\left(\frac{\tau}{t_p}\right) \right]. \tag{19}$$

This is exactly the intensity correlation function [12], as required.

The results for $G(\tau)$ are given below for two limiting cases. First, if the group velocity mismatch αL is small compared to the pulse width, then $R(\tau, \eta)$ can be expanded in η up to quadratic terms. The resulting expression for $G(\tau)$ is as follows.

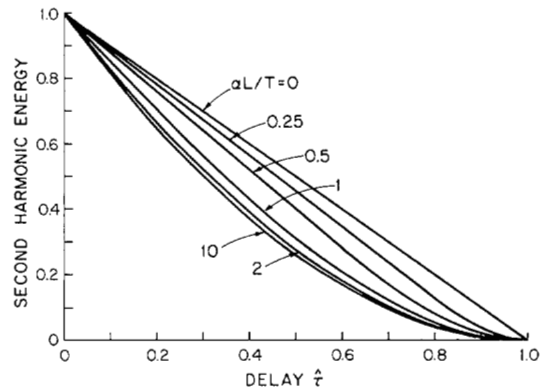

Fig. 2. Second harmonic energy $G(\hat{\tau})$ versus normalized delay $\hat{\tau} = \tau/T$ for a square pulse of duration T. $\alpha L/T = 0$, 0.25, 0.5, 1.0, 2.0, and 10.0. $\Delta kL = 0$. The second harmonic energy is normalized differently for each curve by the value at $\hat{\tau} = 0$. The numerical value of α in KDP for a 620 nm fundamental is 187 f/mm.

For $\hat{\alpha}L \ll 1$

$$\begin{aligned} G(\hat{\tau}) = {} & \frac{2(1-\cos\Delta kL)}{(\Delta kL)^2} A(\hat{\tau}) \\ & + 2\left[\frac{4\sin\Delta kL}{(\Delta kL)^3} - \frac{\cos\Delta kL}{(\Delta kL)^2} \right. \\ & \left. - 6\,\frac{(1-\cos\Delta kL)}{(\Delta kL)^4} \right] (\hat{\alpha}L)^2\, B(\hat{\tau}) \end{aligned} \tag{20}$$

where

$$A(\hat{\tau}) = \frac{4t_p}{\sinh^3(\hat{\tau})} [\hat{\tau}\cosh\hat{\tau} - \sinh\hat{\tau}],$$

$$B(\hat{\tau}) = \frac{4t_p}{\sinh^5(\hat{\tau})} \left[\hat{\tau}\cosh\hat{\tau} - \sinh\hat{\tau} - \frac{1}{3}\sinh^3\hat{\tau} \right]$$

and

$$\hat{\tau} = \tau/t_p, \qquad \hat{\alpha}L = \alpha L/t_p.$$

$A(\hat{\tau})$ is the intensity correlation function [(21)], and $B(\hat{\tau})$ is the lowest order correction term. Note that when $\Delta kL = 2m\pi$ [corresponding to a node of the phase matching factor $(\sin^2(\Delta kL/2)/(\Delta kL/2)^2)$], the coefficient of $A(\hat{\tau})$ vanishes; only the $B(\hat{\tau})$ contribution remains.

If the group velocity mismatch is assumed large ($\alpha L \gg 1$), then $R(\tau, \alpha L\eta')$ will be appreciable only near $\eta' = 0$, so that $tri(\eta')$ can be replaced by unity. (Actually, this approximation will be valid only if $\hat{\tau} \ll \hat{\alpha}L$ in addition, but when this condition is violated, $G(\hat{\tau})$ will be very small.) The following expression for $G(\hat{\tau})$ is obtained.

For $\alpha L \gg 1$, $\quad \Delta kL \neq 0$

$$G(\hat{\tau})/G(0) = \frac{2}{(\Delta kL)^2} [1 - \cos(\Delta kL\hat{\tau})] \frac{1}{\sinh^2(\hat{\tau})}. \tag{21a}$$

For $\hat{\alpha}L \gg 1$, $\quad \Delta kL = 0$

$$G(\hat{\tau})/G(0) = C(\hat{\tau}) = \hat{\tau}^2/\sinh^2(\hat{\tau}). \tag{21b}$$

Fig. 3 shows the graphs of $A(\hat{\tau})$ and $B(\hat{\tau})$ (the $\hat{\alpha}L \ll 1$ limit) and $C(\hat{\tau})$ (the $\hat{\alpha}L \gg 1$ limit). For $\Delta kL = 0$ and $\hat{\alpha}L =$

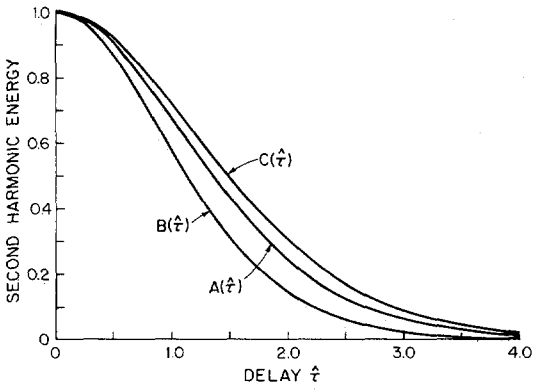

Fig. 3. Second harmonic energy $G(\hat{\tau})$ versus normalized delay $\hat{\tau} = \tau/t_p$ for a secant hyperbolic pulse $f(t) = \text{sech}\,(t/t_p)$. Curve A is the normal intensity correlation function which applies when $\alpha L \ll t_p$ and $\Delta kL = 0$. Curve B applies when $\alpha L \ll t_p$ and $\Delta kL = 2m\pi$. Actually, this is the lowest order correction to curve A; but when $\Delta kL = 2m\pi$, we are at a node of the phase matching factor and the contribution from curve A vanishes. Curve C is the result for large group mismatch and zero phase mismatch, i.e., $\alpha L \gg t_p$ and $\Delta kL = 0$.

anything, the plot of $G(\hat{\tau})$ lies in the narrow region between $A(\hat{\tau})$ and $C(\hat{\tau})$. $B(\hat{\tau})$, which applies for $\Delta kL = 2m\pi$ and $\hat{\alpha}L \ll 1$, is also fairly close in width to the other plots.

E. Chirped Pulses

We observe from examples A-D that although the shape of $G(\tau)$ can be significantly affected by αL, the width of $G(\tau)$ is in all these cases on the same order as the width of the intensity correlation function. For bandwidth limited pulses, one expects that this will always be the case.

However, this is not necessarily true for chirped pulses. It has been shown experimentally [13] that the apparent correlation width of a chirped pulse can be changed by filtering the second harmonic with a monochromator. The idea is that the low frequency end of the second harmonic spectrum arises only when low frequency parts of the fundamental spectrum mix with each other, and similarly for the high frequency end of the spectrum. Therefore, by tuning the monochromator to the low (high) frequency tail of the second harmonic, one obtains a correlation function which reflects the duration of only the low (high) frequency portion of the fundamental. For a chirped pulse, this duration can differ from the actual pulse width.

In the experiment we are analyzing, there is no monochromator; however, the nonlinear medium itself can play a similar role. Because of group velocity mismatch, phase matching can be achieved only for a certain range of second harmonic frequencies. In effect, this constitutes a filter whose bandwidth is inversely proportional to αL and whose center frequency is controlled by varying ΔkL. Thus, the measured correlation width of a strongly chirped pulse may in some cases vary appreciably from the true intensity correlation width if the group velocity mismatch αL is sufficiently large and if ΔkL is tuned so that phase matching occurs away from the spectral peak.

IV. Collinear Geometry Type I Phase Matching

Let us now consider what happens when a collinear geometry is used, still with Type I phase matching. The geometry is the same as Fig. 1, except that all the beams are collinear. The new feature here is that the measurement is no longer background free; second harmonic arises from each fundamental field individually as well as from cross terms. The standard result [3] for zero group velocity mismatch is that the second harmonic energy gives the intensity correlation function superimposed on a constant background, i.e.,

$$\bar{I}_s(\tau) \sim 1 + 2\,\frac{\int dt\,|f(t)|^2\,|f(t-\tau)|^2}{\int dt\,|f(t)|^4} + r(\tau) \tag{22}$$

where $r(\tau)$ represents interference terms which vary rapidly with τ and average to zero.

The formula for $\alpha L \neq 0$ is easily derived if we consider individually each of the several terms which arise when $E = E_1 + E_2$ is first squared to yield the second harmonic polarization and then after some intervening steps multiplied by the complex conjugate to yield the second harmonic intensity. The algebra involved in evaluating several of the terms is essentially identical to that in Section II and can be done by inspection; the remaining terms are rapidly varying and for our purposes need not be computed. The result is

$$I_s(\tau) \sim 1 + 2\,\frac{\int d\eta'\; tri(\eta')\exp(j\Delta kL\eta')\int dt\, f^*(t)\,f^*(t-\tau)\,f(t+\alpha L\eta')\,f(t+\alpha L\eta'-\tau)}{\int d\eta'\; tri(\eta')\exp(j\Delta kL\eta')\int dt\,[f^*(t)]^2\,[f(t+\alpha L\eta')]^2} + r'(\tau). \tag{23}$$

$r'(\tau)$ is different from $r(\tau)$ above but still varies rapidly and averages to zero. Equation (23) is quite similar to (22). The second harmonic energy still consists of a correlation function plus background; the correlation function is just the noncollinear result, (11). The ratio of correlation function to background, and hence the contrast ratio, is unaffected by the group velocity mismatch.

V. Type II Phase Matching

The pulse width measurement is discussed below for the case of Type II phase matching [9], [14], in which two nonidentical fundamental waves (i.e., one ordinary and one extraordinary wave) interact to produce an extraordinary wave at the second harmonic frequency. The experimental arrangement is shown in Fig. 4.

In comparison to the preceding analysis, the new feature here is that the group velocities of the two fundamental waves are no longer identical. This means that the delay between the

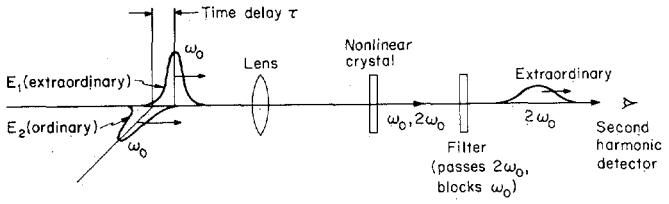

Fig. 4. Experimental setup for pulse width measurement, with collinear geometry and Type II phase matching.

two pulses wil vary according to the position in the nonlinear crystal. Because the faster pulse can catch up to and even pass through the slower one, we anticipate that the second harmonic peak near zero delay will broaden. Furthermore, because the sign of τ determines whether the two pulses get closer or separate in the medium, the second harmonic energy does not remain an even function of delay.

The analysis parallels closely that of Section II. If as before the fundamental fields are assumed undepleted, the nonlinear polarization at the second harmonic frequency can be written

$$P_{NL}(z,t) = \exp\left[j[2\omega_0 t - (k_1 + k_2)z]\right] \epsilon_0 \chi \exp(-j\omega_0\tau) \cdot \left(\frac{1}{2\pi}\right)^2 \int d\omega \exp\left[j\omega(t - \dot{k}_1 z)\right] \times \int d\omega_2 F(\omega - \omega_2) F(\omega_2) \exp(-j\omega_2\tau) \cdot \exp\left[-j(\dot{k}_2 - \dot{k}_1)\omega_2 z\right] \quad (24)$$

where k_1, $\dot{k}_1$, k_2, and $\dot{k}_2$ refer respectively to the propagation constant and its ω-derivative for the two fundamental fields. Using this polarization as a source in the wave equation for the frequency doubled field, one obtains the frequency domain solution

$$S(L,\omega) = \frac{AL}{2\pi} \exp(-j\omega_0\tau) \int d\omega_2 F(\omega - \omega_2) F(\omega_2) \cdot \exp(-j\omega_2\tau) \times \exp\left[j(\beta\omega_2 + \alpha\omega + \Delta k)L/2\right] \cdot \frac{\sin(\beta\omega_2 + \alpha\omega + \Delta k)L/2}{(\beta\omega_2 + \alpha\omega + \Delta k)L/2} \quad (25)$$

where $\Delta k = k_s - k_1 - k_2$, $\alpha = \dot{k}_s - \dot{k}_1$ and $\beta = \dot{k}_1 - \dot{k}_2$. This equation is more complicated than (7), the corresponding result for Type I phase matching because both frequency variables ω and ω_2 are involved in the argument of the $\sin(x)/x$ function. In contrast to the earlier result, the phase matching filter effect depends not only on the second harmonic frequency but also on the particular frequency components which mix together to produce it. For this reason one is unable to obtain a time domain formula analogous to (8) for $s(L,t)$ because it is not possible to inverse Fourier transform equation (25).

In order to make the mathematics tractable, let us assume that $\alpha L = 0$, i.e., the group velocity at $2\omega_0$ is the same as the group velocity of one of the fundamental fields. This is a special case, in general untrue, but it does preserve the principal difficulty with this measurement scheme, the fact that the two fields at ω_0 have different group velocities.

With the proviso $\alpha L = 0$, we can now inverse Fourier transform equation (25), with the result

$$s(L,t) = AL \exp(-j\omega_0\tau) f(t) g(t)$$

where

$$g(t) = f(t-\tau) * \left[\frac{1}{\beta L} sq\left(\frac{t}{\beta L} + \frac{1}{2}\right) \exp\left[-j(\Delta kL)\, t/\beta L\right]\right]. \quad (26)$$

The "*" sign stands for convolution, and the function $sq(x)$ was defined previously in (8). The time average intensity is obtained as usual by multiplying (26) by its complex conjugate and integrating over time.

According to (26), the time average second harmonic intensity is in effect the cross correlation of an input pulse with a broadened version of itself. If $\Delta kL = 0$, the broadening function is just a square pulse of width βL; if $\Delta kL \neq 0$, the broadening function is more complicated. Thus, the SHG measurement gives a result which is broader than the actual intensity correlation function by an amount which increases with inceasing group velocity mismatch βL between fundamental fields. Furthermore, because the second harmonic energy gives a cross correlation measurement rather than an autocorrelation measurement, the result need not be a symmetric function of the delay.

VI. Conclusion

We have examined the use of SHG for optical pulse width measurements, allowing for phase velocity and group velocity mismatch. The effect of group velocity dispersion on the pulses themselves has not been included.

In the case of Type I phase matching, the shape of the measured correlation function depends on the group velocity mismatch αL between fundamental and second harmonic waves, but for bandwidth limited pulses, its width does not vary greatly from the $\alpha L = 0$ result; i.e., the group velocity mismatch can distort the correlation function but cannot broaden it. For strongly chirped pulses, however, the width can in principal vary appreciably with both αL and the phase velocity mismatch ΔkL. Unlike the zero group velocity mismatch case, the shape of the SHG versus delay plot can be affected by the amount of phase velocity mismatch. The plot does remain an even function of the delay τ.

In the case of Type II phase matching, the group velocity mismatch βL between the two fundamental waves has the dominant effect on the pulse width measurement. As a result, the measured correlation function does broaden and may become an asymmetric function of τ. As above, the measurement does depend on the setting of the phase mismatch ΔkL.

In terms of the frequency domain, the group velocity mismatch means that phase matching can occur over a certain range of frequencies. The result is a filter function whose bandwidth is inversely proportional to the group velocity mismatch and whose center frequency is determined by the phase

velocity mismatch. For Type I phase matching, the filter function depends only on the second harmonic frequency; for Type II phase matching, the filter function depends on both the second harmonic frequency and on the particular fundamental frequencies which mix together to produce it.

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