Dark optical solitons with finite-width background pulses

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Received August 19, 1988; accepted November 28, 1988

Using numerical solutions of the nonlinear Schrödinger equation, we show that for positive group-velocity dispersion, optical dark pulses superimposed upon background pulses only ~10× wider can exhibit stable soliton propagation in single-mode fibers. During propagation the background pulse spreads (which reduces its intensity) and develops a frequency chirp. Nevertheless, as the background pulse evolves the dark pulse adiabatically maintains its soliton characteristics. These numerical results are in excellent agreement with recent experimental investigations of dark-pulse propagation in fibers.

INTRODUCTION

Optical solitons in fibers are pulses that propagate without any change in pulse shape or intensity because, for appropriate combinations of pulse shape and intensity, the effects of the intensity-dependent refractive index of the fiber exactly compensate for the pulse-spreading effects of group-velocity dispersion (GVD). A basic analytical theory of optical solitons in fibers, including conditions for their existence, is well established.[1,2] For negative (or anomalous) GVD, which occurs in typical single-mode silica-based fibers for wavelengths ≥ 1.3 μm, the fundamental soliton is a simple pulse with a sech^2 intensity profile,[1] and the propagation of these bright solitons has been studied extensively and verified experimentally.[3] For positive (or normal) GVD, the theory[2] and numerical simulations[4] predict that the solitons are dark pulses (i.e., a short decrease or dip in the intensity of a cw background), but these have proven to be more difficult to study experimentally, primarily because of the difficulty of producing the required input pulses.

Recent experimental studies of dark-pulse propagation in single-mode optical fibers[5,6] have provided data on dark-soliton propagation, and the development of techniques for synthesizing short optical pulses with almost arbitrary intensity and phase profiles[7] has now made it possible to observe solitonlike propagation of individual dark pulses in fibers.[8] However, the interpretation of the results of Ref. 8 as soliton propagation could be questioned because the background pulses, upon which the dark pulses were superimposed, were only ~10× wider than the dark pulses, and thus the background pulses spread significantly and develop a substantial frequency chirp. These perturbations have not been considered in previous theoretical studies of dark solitons.[2,4] In this paper we present the results of numerical simulations that show that even with such a rapidly evolving background dark pulses can exhibit stable soliton propagation. The results of these studies are in excellent agreement with the experimental observations in Ref. 8.

THEORY

The propagation of pulses in single-mode optical fibers is described well by the nonlinear Schrödinger equation, which we write in the normalized form

$$\frac{\partial u}{\partial (z/z_0)} = i\,\frac{\pi}{4}\left[\pm\frac{\partial^2 u}{\partial (t/t_0)^2} - 2|u|^2 u\right], \tag{1}$$

in which u is the (complex) amplitude envelope of the pulse, z is the distance along the fiber, and the plus (minus) corresponds to positive (negative) GVD. The time variable t is a retarded time measured in a frame of reference moving along the fiber at the group velocity. The normalizing length z_0 is defined by

$$z_0 = \frac{\pi^2 c^2 {t_0}^2}{|D(\lambda_0)|\lambda_0}, \tag{2}$$

and the amplitude u is normalized such that $|u|^2 = 1$ corresponds to an effective intensity of

$$I_0 = 10^{-7} n c \lambda_0/(16\pi n_2 z_0)\ \mathrm{W/cm^2}. \tag{3}$$

In these expressions t_0 is a width parameter for the input pulse, $D(\lambda) \equiv \lambda^2 d^2 n/d\lambda^2$ is the GVD in dimensionless units, n is the refractive index of the core material, n_2 is its nonlinear coefficient in electrostatic units (1.1×10^{-13} esu for silica), c is the velocity of light in centimeters per second, and λ_0 is the vacuum wavelength. These normalizations have been used widely in the study of soliton propagation in fibers and in studies of the use of nonlinear effects in fibers for pulse compression.[9]

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It is well established[1] that for a negative GVD a bright pulse of the form

$$u(t) = A_0 \operatorname{sech}(A_0 t/t_0) \tag{4}$$

is a soliton solution of Eq. (1). (Input pulses with amplitudes NA_0, with N an integer, will exhibit higher-order soliton propagation, which is periodic with period z_0; however, that is not of concern in the present paper.[10])

For positive or normal GVD it has been shown that dark pulses of the form

$$u(t) = A_0[B^{-2} - \operatorname{sech}^2(A_0 t/t_0)]^{1/2} \exp[i\varphi(A_0 t/t_0)], \tag{5a}$$

$$\varphi(\xi) \equiv \sin^{-1}\left\{\frac{B \tanh(\xi)}{[1 - B^2 \operatorname{sech}^2(\xi)]^{1/2}}\right\} \tag{5b}$$

are soliton solutions of Eq. (1) for $|B| \le 1$.[2,11] Representative plots of the dark-soliton intensity ($|u|^2$) and phase are given in Fig. 1, along with corresponding plots for bright solitons. As illustrated in Fig. 1, in the limiting case of $B = \pm 1$, for which Eqs. (5) reduce to $u(t) = \pm A_0 \tanh(A_0 t/t_0)$, the intensity goes to zero at the center of the pulse, and this has been referred to as the fundamental dark soliton.[2] We suggest that a shorter and more descriptive name for this case is a black soliton. In addition to the fundamental dark soliton, Eqs. (5) show that there is a continuous range of lower-contrast dark solitons, described by the parameter B (note the right-hand side of Fig. 1). We suggest that pulses with $|B| < 1$ be referred to as gray solitons, with $|B|$ as the blackness parameter. We use this terminology in the balance of this paper.

A major difference between bright and dark solitons is their symmetry. Fundamental bright solitons are even pulses, with a constant phase across the entire pulse, and black solitons are odd pulses, with a π phase jump at the center (where the intensity is zero). Gray solitons have a similar but smaller and more gradual phase shift at their center (note Fig. 1). As can be seen from Eq. (5b), the total phase shift across a gray soliton is $2 \sin^{-1} B$. [For gray solitons the time-dependent phase shift described by Eq. (5b) represents an effective frequency shift, and thus input pulses of the form given by Eqs. (5) have propagation velocities slightly different from the group velocity at λ_0 used to define our moving coordinate system.[2]] The phase function of dark solitons is a major reason why they are difficult to study experimentally, and it is only recently that it has been possible to create pulses with the necessary phase variation to excite a single dark soliton.[6,8]

The soliton solutions [Eqs. (4) and (5)] have the common features that the normalized peak intensity (or depth) is $I_p = A_0^2$, and the full width at half-peak (or depth) intensity is $\tau = 1.76/A_0$ (measured in units of t/t_0), so that the product $I_p\tau^2 = 1.76^2$ is a constant for all solitons (note Fig. 1). In physical units this constant has the value

$$I_0(1.76t_0)^2 = \left(\frac{3.1 \times 10^{-7}}{16\pi^2 c}\right)\lambda_0^2\left(\frac{n|D(\lambda_0)|}{n_2}\right) \text{W cm}^{-2} \sec^2. \tag{6}$$

In the present study we use the constancy of this product of peak intensity and width squared as our basic indicator of soliton propagation. Other more rigorous tests, such as collisions, may be applied in subsequent studies.

BASIC ASSUMPTIONS AND TECHNIQUES

In the present paper we have restricted ourselves to the consideration of two forms of input pulses, representing odd and even dark pulses. To study the propagation of odd dark pulses we have used input pulses of the form

$$u(t, z = 0) = A \tanh(t/t_0)\exp[-(t/15t_0)^2]. \tag{7}$$

The Gaussian background pulse has a full width at half-peak intensity that is $\approx 10\times$ that of the superimposed dark pulse. For $A \approx 1$, the dark pulse should generate the fundamental black soliton, except for possible perturbations by the finite width of the background.

In a previous experimental study of dark-pulse propagation in fibers,[5] the background pulses were long enough that their evolution could be neglected, but the input dark pulses were even functions of time. It was found experimentally[5] and shown in numerical simulations[4,5] that even dark pulses split into complementary pairs of gray solitons. To study the evolution of even input dark pulses with finite-width backgrounds, we have used pulses of the form

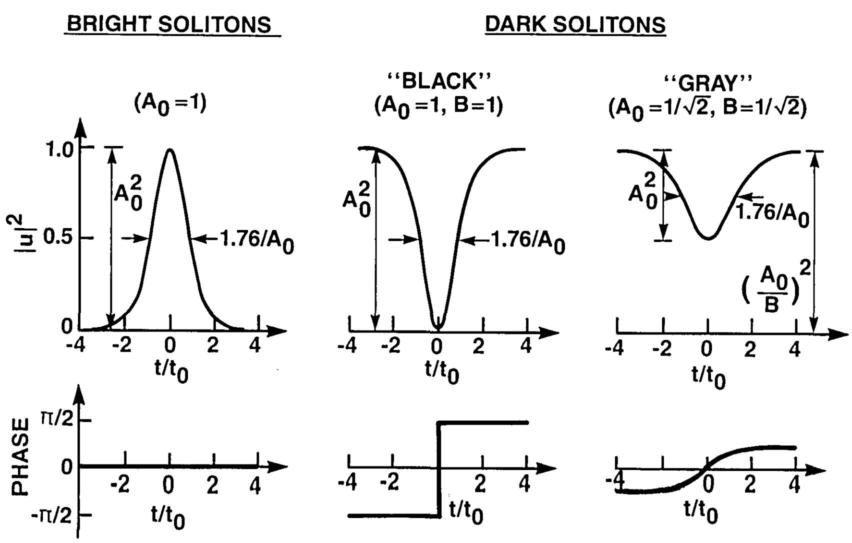

Fig. 1. Intensity and phase as functions of normalized time for bright and dark solitons.

$$u(t, z = 0) = A[1 - \mathrm{sech}(t/t_0)]\exp[-(t/15t_0)^2]. \quad (8)$$

For our numerical simulations, we have solved Eq. (1) (for positive GVD) by using the split-step or beam-propagation method.[12]

The experimental results reported in Ref. 8 are for $\lambda_0 = 620$ nm, $z = 1.4$ m, and input pulses that are approximately of the form of Eq. (7), with $t_0 \approx 110$ fsec. For this case the normalizing fiber length is $z_0 \approx 38$ cm, and thus our simulations correspond to a maximum fiber length of $\approx$5.7 m. For such short fiber lengths the fiber attenuation is negligible, and we have not included it in Eq. (1).

Experimental results are reported in Ref. 6 for significantly longer dark pulses ($\sim$5 psec). In this case the fiber attenuation cannot be neglected because the normalizing fiber length, $z_0 \sim 170$ m, was greater than the fiber attenuation length of $\sim$109 m. Thus those experiments were unable to provide clear evidence for the phenomena we describe in the present paper.

DARK PULSES WITH $A = 1$

Background Pulse

To aid in the interpretation of the dark-pulse propagation, in Figs. 2 and 3 we show solutions of Eq. (1) for a pure background pulse with an amplitude $A = 1$ {i.e., for $u(t, z = 0) = \exp[-(t/15t_0)^2]$}. The perspective plot, Fig. 2, shows clearly that the pulse spreads smoothly as it propagates down the fiber. Detailed profiles of the input pulse and the pulse after propagating a length $z/z_0 = 15$ are given in Figs. 3(a) and 3(b), from which we see that for this length of fiber the pulse has broadened to $\sim$3× its original width, and it has developed a quite flat-topped profile, with a peak intensity that is approximately one third of its original value. Figure 3(c) is a plot of the phase of the pulse at $z/z_0 = 15$ (plotted modulo 2π). The phase is approximately proportional to $(t/t_0)^2$, and, as the time derivative of the phase is the instantaneous frequency, the pulse has an approximately linear frequency chirp. (The linear chirp is a well-known result of the interaction of the GVD and the nonlinear self phase modulation and has been extensively studied and utilized for pulse compression.[9]) The chirp on the pulse at $z/z_0 = 15$ corresponds to a bandwidth $\sim$6× greater than that of the input pulse. Thus the background pulse is clearly quite different from the constant (i.e., cw) background assumed in the analytical theories.

Odd Dark Pulse

In Figs. 4 and 5 we present the results for the propagation of an odd input dark pulse, as defined by Eq. (7), for an amplitude $A = 1$. The perspective plot, Fig. 4, shows that the dark pulse has broadened somewhat but has otherwise retained its basic shape and that the overall background-pulse evolution is close to that illustrated in Fig. 2. The detailed cross sections in Fig. 5 (in the same format as Fig. 3) show that the dark pulse still dips to zero intensity at its center and still has an abrupt phase jump of π at its center.

To follow the evolution of the dark pulse in more detail, we show plots of the intensity difference between the total pulse with the dark pulse and the pure background pulse in Fig. 6. Note that as the dark pulse propagates it not only drops in intensity and spreads, but it also develops a background

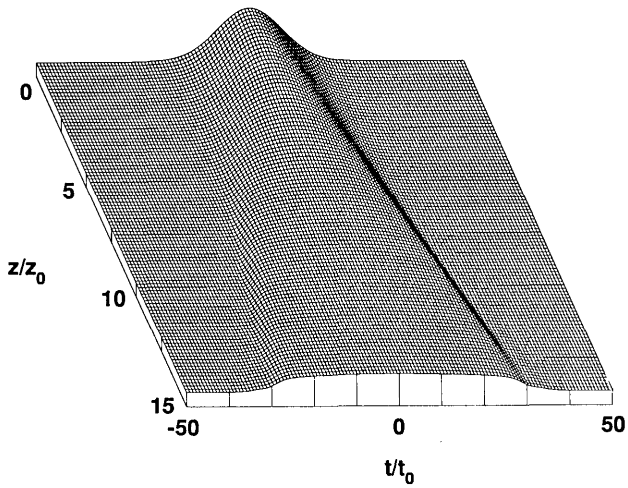

Fig. 2. Perspective plot showing pulse intensity versus time as a function of distance along the fiber. This plot is for the pure background pulse, with amplitude $A = 1$.

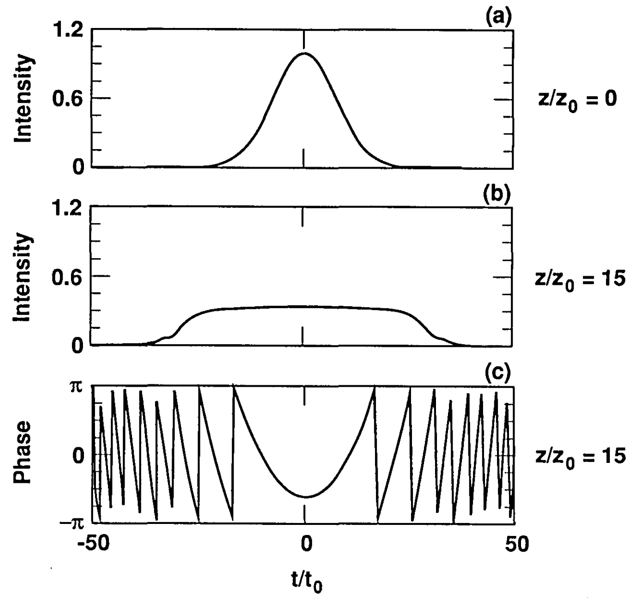

Fig. 3. Intensity profiles of the data in Fig. 2: (a) for the input pulse and (b) for the pulse at $z/z_0 = 15$. (c) Gives the phase of the pulse in (b).

plateau. This behavior suggests that the dark pulse is maintaining its soliton character and is shedding its excess energy (actually negative energy because it is a dark pulse) into the plateau. (If $I_p\tau^2$ stays constant while I_p decreases, then the energy in the pulse, which is proportional to $I_p\tau$, must vary inversely with τ.) To test for the soliton nature of the dark pulse, we have estimated the plateau level for each curve in Fig. 6 and then measured the peak intensity and pulse width of the dark pulse relative to that plateau level. The solid curves in Fig. 7 show the results. We see that the product $I_p\tau^2$, although not quite constant, is approximately oscillating about unity (to within the accuracy with which we estimated the plateau level). This oscillation is similar to that observed in the propagation of a bright soliton with a small nonsoliton background[10] and is presumably a result of interference between the soliton and the background. The dark-

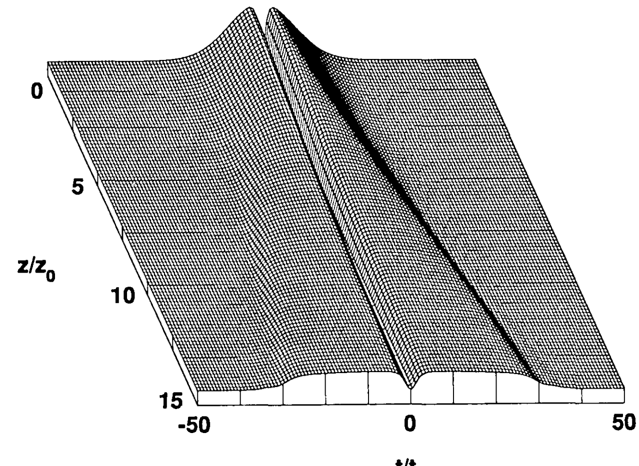

Fig. 4. Perspective plot, as in Fig. 2, for an input odd dark pulse given by Eq. (7), with amplitude $A = 1$.

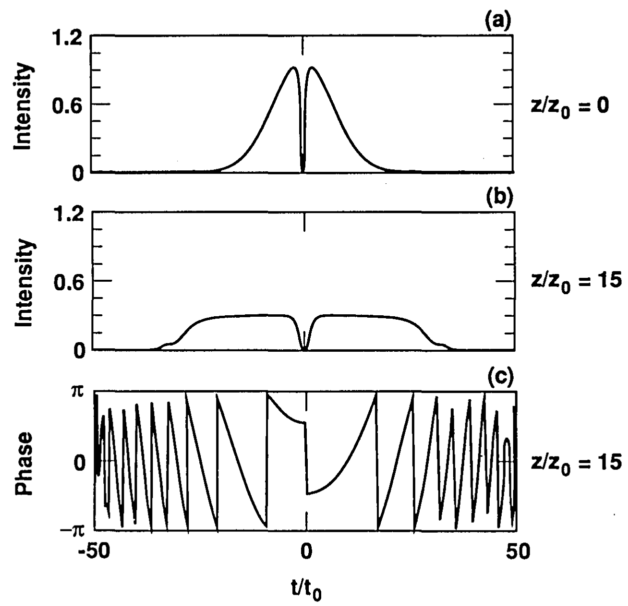

Fig. 5. Intensity and phase plots of the data in Fig. 4 in the same format as Fig. 3.

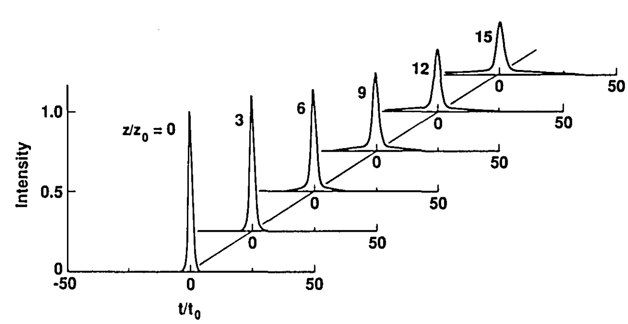

Fig. 6. Intensity difference of the data in Figs. 2 and 4 for selected fiber lengths.

pulse evolution is also similar to the evolution of bright solitons in fibers with loss, in which case the pulse evolves adiabatically to maintain its soliton character, provided that the attenuation length of the fiber is much greater than the soliton period (z_0).[13] Therefore we conclude that despite the chirp and decreasing intensity of the background pulse, the odd dark pulse does propagate as a dark soliton. The results in Ref. 8 provide quantitative experimental confirmation of the present simulations.

Even Dark Pulse

As noted above, numerical simulations and experiments show that an even dark pulse (i.e., with a constant phase) will split into a complementary pair of dark solitons.[4,5] For our study of the effects of finite-width background pulses on even dark pulses, we used input pulses of the form of Eq. (8) with amplitude $A = 1$. This dark pulse has a width that is 2.15× that of the odd dark pulse of Eq. (7), and thus for the same value of A it has approximately twice the (negative) energy. We chose this input pulse in the expectation that it would have approximately the appropriate energy to split into two high-contrast dark solitons. (It is also the same functional form as used in the numerical simulations in Ref. 5.)

From the perspective plot, Fig. 8, and the detailed cross sections in Fig. 9, we see that this input even dark pulse does split into a pair of reasonably high-contrast odd dark pulses, with complementary phase functions, which move toward the edges of the background pulse. The intensity difference

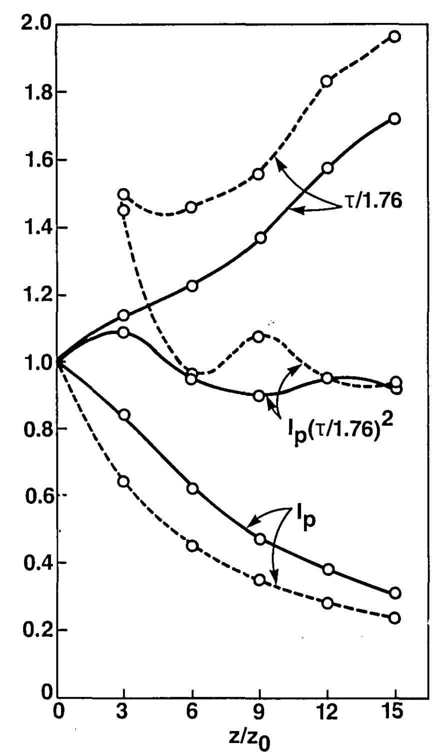

Fig. 7. Dark-pulse peak intensity I_p, pulse width τ, and soliton constant $I_p\tau^2$, approximately corrected for the nonsoliton background level, as functions of fiber length. The solid curves are for an input odd dark pulse [Eq. (7) with $A = 1$], and the dashed curves are for an input even dark pulse [Eq. (8) with $A = 1$].

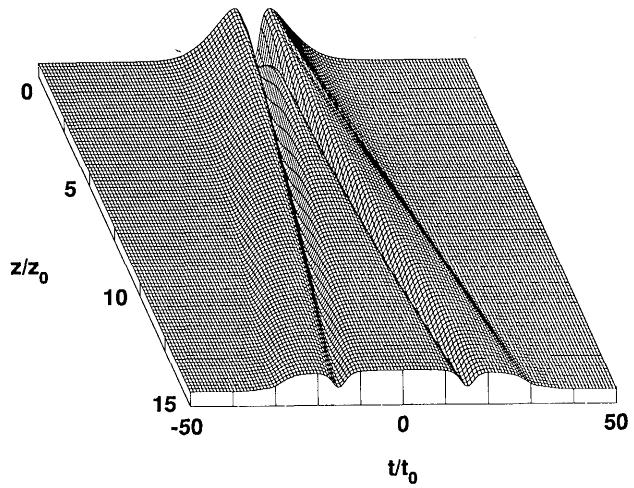

Fig. 8. Perspective plot, as in Fig. 2, for an input even dark pulse, given by Eq. (8), with amplitude $A = 1$.

between the total pulse and the pure background pulse is plotted in Fig. 10. From this figure we see that once the input dark pulse has split the resulting two dark pulses evolve much like the single dark-pulse case of Fig. 6. The dark pulses decrease in intensity, increase in width, and develop a broad background plateau. Estimating the plateau level as we did for the data in Fig. 6 and measuring the peak intensity and pulse width relative to that plateau, we obtain the results plotted as dashed curves in Fig. 7. The product $I_p\tau^2$ oscillates about a value close to unity, behavior very similar to that for the input odd dark pulse. Correcting for the plateau level, the blackness parameter B increases slowly from ≈ 0.89 at $z/z_0 = 3$ to ≈ 0.91 at $z/z_0 = 15$, and the net phase shifts across the dark pulses are consistent with Eq. (5b). We thus conclude that these dark pulses are also propagating as dark (gray) solitons despite the decreasing intensity and the frequency chirp of the background pulse. The results in Ref. 8 provide quantitative experimental confirmation of the present simulations.

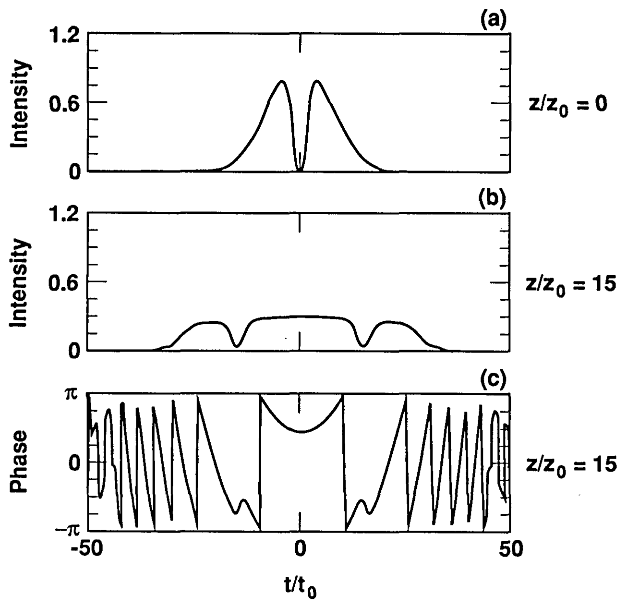

Fig. 9. Intensity and phase profiles of the data in Fig. 8 in the same format as Fig. 3.

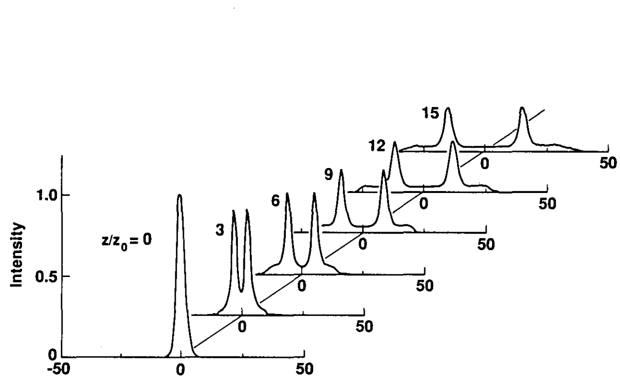

Fig. 10. Intensity difference of the data in Figs. 2 and 8 for selected fiber lengths.

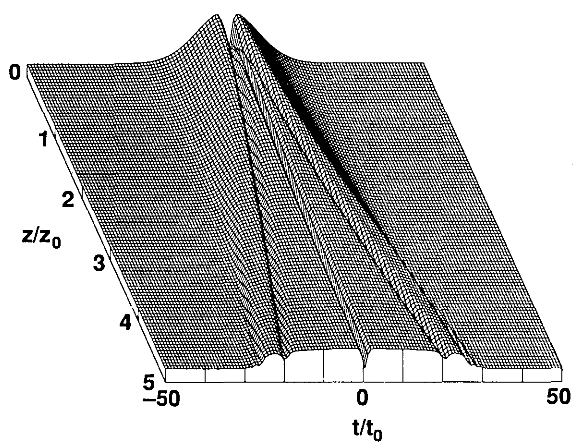

Fig. 11. Perspective plot, as in Fig. 2, for an input odd dark pulse, given by Eq. (7), with amplitude $A = 2.5$. Note that this plot is for a total fiber length of only $z/z_0 = 5$.

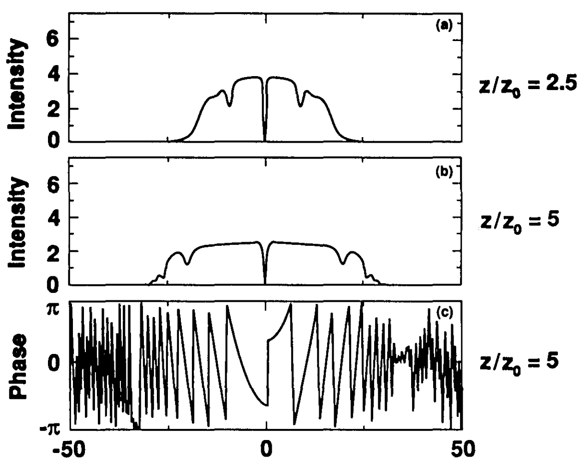

Fig. 12. Intensity and phase profiles of the data in Fig. 11. (a), (b) Give the intensity profiles at $z/z_0 = 2.5$ and $z/z_0 = 5.0$, respectively, and (c) gives the phase profile for $z/z_0 = 5.0$.

HIGHER-INTENSITY DARK PULSES

For higher-intensity input pulses, the pulse evolution is obviously more complicated and more so than we can cover in the present paper. We therefore present only a single example that illustrates a typical behavior of higher-intensity pulses. The example we have chosen, with the results plotted in Figs. 11 and 12, is an input odd dark pulse of the form given in Eq. (7) with an amplitude $A = 2.5$. (Note that in this case we have terminated our simulation at a fiber length of $z/z_0 = 5$.) The figures show that the input odd dark pulse evolves into a single black soliton and a pair of gray solitons, in qualitative agreement with the experimental results presented in Ref. 8. The velocity and contrast of the gray solitons are increasing functions of the input intensity. For still higher input intensities, we find that additional pairs of gray solitons are generated. This is quite different from the case of bright solitons, for which at high intensities higher-order solitons are generated. It has been pointed out[4] that this difference results from the fact that for positive GVD the nonlinear term in Eq. (1) corresponds to a repulsive potential, and thus there are no multiple-dark-soliton bound states. Further details of this case will be described in a subsequent publication.

SUMMARY AND CONCLUSIONS

We have shown that the nonlinear Schrödinger equation [Eq. (1)] predicts that for positive GVD optical dark pulses, superimposed upon bright background pulses as little as ~10× wider, will evolve into dark optical solitons, despite the rapid evolution of the background pulse. The excellent agreement between the experimental results of Ref. 8 and similar numerical simulations of Eq. (1) demonstrates, once more, that Eq. (1) accurately describes many of the phenomena of nonlinear pulse propagation in optical fibers.

ACKNOWLEDGMENT

This research was performed, in part, under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under contract W-7405-ENG-48.

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