Comment on "Time reversal of ultrafast waveforms by wave mixing of spectrally decomposed waves"

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In a recent Letter [Opt. Lett. **25**, 132 (2000)] the properties of two different techniques for time reversal of ultrafast optical waveforms were compared. Although both techniques, spectral phase conjugation and spectral inversion, perform the same function for real pulses, for pulses with complex envelope functions it was asserted that only spectral inversion gives true time reversal. I argue here for a different interpretation.

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Several techniques have been investigated for time reversal of ultrafast optical waveforms. A recent paper[1] reported an investigation of two different spectral nonlinear optics approaches: spectral inversion and spectral phase conjugation (the latter is a real-time version of spectral holography[2,3]). The results were interpreted to show that only spectral inversion (SI) gives true time reversal for waveforms with complex envelope functions. I argue here for a different interpretation.

Consider an input electric field of the form

$$e_{\text{in}}(t) = \frac{1}{2}[s(t)\exp(j\omega_0 t) + s^*(t)\exp(-j\omega_0 t)]. \quad (1)$$

Here $s(t)$ is the envelope or the complex amplitude function, and to save space we consider only the shape of the temporal waveform as it passes a fixed spatial position, which we take as $z = 0$. The time-reversed version of this input field is obtained by replacement of t with $-t$, which yields

$$e_{\text{TR}}(t) = e_{\text{in}}(-t) = \frac{1}{2}[s^*(-t)\exp(j\omega_0 t) + s(-t)\exp(-j\omega_0 t)]. \quad (2)$$

The time-reversed field (2) is obtained by replacement of the original envelope function $s(t)$ with its time-reversed and complex-conjugate version, $s^*(-t)$.

Now let us consider the SI process first demonstrated in Ref. 1. Here $s(t)$ interacts with two short reference pulses (assumed to be infinitely short to simplify the discussion) to produce an output waveform with spectral amplitude $S_{\text{SI}}(\omega) \sim S(-\omega)$, where $S(\omega)$ is the Fourier transform of $s(t)$. The envelope function of the output waveform is given by $s_{\text{SI}}(t) \sim s(-t)$. The envelope function is time reversed, as was pointed out in Ref. 1. However, the actual electric field function,

$$e_{\text{SI}}(t) \sim \frac{1}{2}[s(-t)\exp(j\omega_0 t) + s^*(-t)\exp(-j\omega_0 t)], \quad (3)$$

is not time reversed, since expression (3) is not of the same form as Eq. (2).

Now we consider spectral phase conjugation[1–3] (SPC). The spectral amplitude of the output waveform is given by $S_{\text{SPC}}(\omega) \sim S^*(\omega)$, again for very short reference pulses. The envelope function in the time domain is given by $s_{\text{SPC}}(t) \sim s^*(-t)$, which corresponds to the correct time-reversed electric field in Eq. (2). Time reversal using volume holography or photon-echo processing also gives an output waveform of this form.

Two examples help to illustrate the important differences between SI and SPC:

1. Consider $s(t) = |s(t)|\exp(j\Delta\omega t)$, with $|s(t)|$ assumed to be an even function of t. The actual electric field is written as $e_{\text{in}}(t) = |s(t)|\cos[(\omega_0 + \Delta\omega)t]$, which corresponds to a pulse with center frequency $\omega_0 + \Delta\omega$. For SI processing, we find that $e_{\text{SI}}(t) = |s(t)|\cos[(\omega_0 - \Delta\omega)t]$, which corresponds to a pulse with center frequency $\omega_0 - \Delta\omega$. The SI operation has resulted in a frequency shift, which should not arise in pure time reversal. For the SPC case, we find that $e_{\text{SPC}}(t) = e_{\text{in}}(t)$, which is at the original frequency. This is what we expect: a completely symmetric input pulse should be completely unchanged by time reversal.
2. Consider $s(t) = |s(t)|\exp(j\alpha t^2)$, again with $|s(t)|$ even. The actual electric field is written as $e_{\text{in}}(t) = |s(t)|\cos[(\omega_0 + \alpha t)t]$, which corresponds to a chirped pulse. For SI processing, the output is still $e_{\text{SI}}(t) = |s(t)|\cos[(\omega_0 + \alpha t)t]$; the sign of the chirp is unchanged by SI. The output for SPC is $e_{\text{SPC}}(t) = |s(t)|\cos[(\omega_0 - \alpha t)t]$; the sign of the output chirp is reversed compared with that of the input field. So far this is in agreement with Ref. 1. However, unlike in Ref. 1, we note that only the SPC expression is consistent with inserting $t \rightarrow -t$ into Eq. (1). Intuitively, the SPC behavior is what we expect for time reversal. If a bandwidth-limited input pulse is first chirped by passing through a dispersive medium and then time reversed, we would expect that a second passage through an identical dispersive medium would remove all the chirp. This implies that time reversal must change the sign of the chirp, which occurs with SPC but not SI.

In summary, both spectral inversion and spectral phase conjugation provide interesting transformations of ultrashort pulse signals. In the case of input pulses with real envelope functions, both techniques yield identical, time-reversed output pulses. For pulses with complex envelope functions, however, only SPC (or equivalent techniques such as spectral holography) give correctly time-reversed electric field waveforms.

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