Uncertainty Modeling in Earthquake Engineering

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ABSTRACT

Performance evaluation and design of civil facilities against earthquakes is a challenge to engineers because of the large uncertainty in the seismic forces and the system capacity to withstand these forces against damage and collapse. Until recently, treatment of the uncertainty has been limited to selection of design earthquake ground motion parameters based on return period and a series of design factors based on judgment and experience. As a result, the treatment of uncertainty has been incomplete and the reliability of such design against damage and collapse has been unknown and undefined. As risk assessment is an essential part in loss estimate and development of strategy for new design procedures, retrofits, and rehabilitations, a systematic treatment of uncertainty is essential. A critical review of currently available methodologies for uncertainty analysis and application to earthquake engineering has been conducted. Advantages and caveats have been pointed out. The objective is to give a clear and concise description of various methods and procedures that an earthquake-engineering professional can understand and use to treat uncertainty in engineering decision analysis.

The nature of uncertainty and the mathematical foundation for its treatment using statistical (data-based) and non-statistical (judgment and experience-based) information and associated decision making tools are introduced first. Classical and Bayesian theories are compared. Use of expert opinion, role of time and space, and elementary tools for uncertainty analysis are described. The treatment of uncertainty in the chain of events from seismic source, path, and site to structural response then follows. Seismic hazard analysis, efficient excitation measures, SAC ground motion procedure, Monte-Carlo methods including the uniform hazard ground motion procedure, and their applications to probabilistic response demand analysis are briefly described. On the capacity side, material properties, uncertainty in member and system capacity against prescribed limit states including incipient collapse via an incremental dynamic analysis (IDA) are discussed. Reliability analyses with explicit consideration of uncertainties in both demand and capacity based on the first order reliability method (FORM), recently proposed FEMA/SAC procedure, and uniform hazard ground motions are compared. Their applications to development of probabilistic codes and standards are also described. A brief description of performance level identification is also given. The end product of the uncertainty analysis is the damage and loss assessment for risk management purpose, specifically decision on retrofit and rehabilitation. The mapping of limit state to performance objective, loss assessment methodology such as loss accounting, loss categories, development of vulnerability function are described. Finally, several important research issues related to ground motion modeling, facility response, mapping of structural response to damage and loss, risk communication, and supporting databases are raised.

This report is intended to be a living document to be updated and expanded periodically as knowledge and new development of uncertainty modeling and applications unfold in concurrent MAE center projects in the future.

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1. INTRODUCTION, OBJECTIVE AND SCOPE

Introduction

Buildings, bridges and other civil infrastructure must be designed and constructed to withstand the effects of man-made and natural hazards. Recent significant advances in the science of engineering design and the revolution in information technology and computing have made it possible to predict the behavior and performance of complex engineered systems accurately, provided that the demands on the system and the capacity of the system to withstand those demands are known. However, numerous sources of uncertainty arise in this analysis and assessment process and accordingly impact the ensuing technical, economic and social decisions. Some of these uncertainties stem from factors that are inherently random (or aleatory) at the scale of understanding or customary resolution in engineering or scientific analysis (e.g, earthquake occurrence on a known fault; Young's modulus of steel; compression strength of concrete). Others arise from a lack of knowledge, ignorance, or modeling (or epistemic), and are dependent on the model selected (identification of seismotectonic zones in the Eastern US; two-dimensional idealizations of buildings for structural analysis purposes). [A more complete discussion of the nature of aleatory and epistemic uncertainties is found in Chapter 2.] Both sources of uncertainty are equally important and must be considered in decision-making. The inevitable consequence of these uncertainties is risk that the facility will fail to perform as intended or expected by the owner, occupant or user, or society as a whole. It is not feasible to eliminate risk entirely (a society unwilling to accept reasonable risks will become moribund); rather, the risk must be managed in the public interest by engineers, code bodies and other regulatory authorities, urban planners and the political system.

Because of their scope and magnitude, the performance of civil infrastructure systems, unlike that of other common engineered products, is governed by standards and regulations that represent a value judgment by the profession and the regulatory community based on past experience. This judgmental approach to performance assurance generally served society well until quite recently, as the supporting technology evolved relatively slowly in comparison with that in other fields. In recent years, however, increasingly rapid evolution and innovation in technology have allowed less opportunity for learning through trial and error. Standards for public health, safety and environmental protection now are often set in the public arena, and social expectations of civil infrastructure performance have increased and are being voiced more definitively.

In the United States, earthquakes have been viewed during the past three decades as being paramount among the natural hazards impacting civil infrastructure and urban populations. The occurrence of three major earthquakes during that period – San Fernando in 1971, Loma Prieta in 1989, and Northridge in 1994 – has provided the impetus for significant improvements in engineering practices for aseismic design of buildings, bridges, lifelines and other civil infrastructure. These earthquakes have also served to highlight the limitations in scientific and engineering knowledge concerning earthquakes and their socioeconomic impact on urban populations. Notwithstanding the advances in knowledge made possible by recent earthquake engineering research, the uncertainties remaining in seismology and in the response of buildings, bridges, transportation networks and lifelines are among the largest of any of the natural phenomena hazards confronting designers and managers of civil infrastructure.

Engineers and planners traditionally have dealt with risk and uncertainty by making conservative assumptions in analysis and design, postulating worst-case scenarios, and applying factors of safety. Such approaches provide an unknown margin of safety against the failure state, however defined. Often, the decision is so conservative as to be wasteful of public resources; occasionally, it errs in the non-conservative direction. In recent years, there has been a growth in the use of reliability and risk analysis, founded in the mathematics of probabilistic and statistics, to support decision in the presence of uncertainty. Reliability and risk analysis provide the link between the practice of earthquake engineering and its socioeconomic consequences, and the quantitative tools that historically have been missing for the management of risk in the public interest.

Confronted with the need to manage the enormous uncertainties associated with earthquake prediction and infrastructure response, the earthquake community has embraced the notions of reliability and risk analysis. Probabilistic seismic hazard analysis methodologies [discussed in Chapter 3] have provided the basis for the national seismic hazard maps that have been developed and maintained by the U. S. Geological Survey through continuing research and periodic revisions since 1976. The U.S. Nuclear Regulatory Commission has encouraged the use of Probabilistic Risk Analysis (or PRA) since the mid-1970's, and the more recent Independent Plant Examination Program for External Events (IPEEE) required, as a minimum, a seismic margin study involving a probability-based fragility analysis of safety-related plant structures and components [fragility modeling is described subsequently in Chapters 4 and 5]. The recently completed SAC project addressed uncertainties in ground motion intensity and in structural response and capacity of steel special moment frames, and developed relatively simple probability-based procedures for design and condition assessment of such structures.

These and similar programs have led to significant advances in the ability to model and analyze complex civil infrastructure systems, highlighted the importance of uncertainties, and provided extensive databases to support uncertainty modeling. On the other hand, most programs have focused on one type of building technology (e.g., steel frames, Category I structures in NPPs), one source of uncertainty (earthquake occurrence; ground motion attenuation), and one performance limit (generally, life-safety, which is the concern of the building regulatory community). Other states of infrastructure damage, direct economic losses and the indirect costs associated with disruption of the community following an earthquake have proven difficult to assess and have not received as much attention. Moreover, much of the prior research on the performance of civil infrastructure during and after earthquakes has concentrated on areas of the United States with high seismic hazard. While this certainly is understandable, the earthquake hazard on other areas of the United States is non-negligible, when viewed on a competing risk basis with other extreme natural phenomena hazards. The risk to affected communities in these areas (measured in terms of economic or social consequences), where building design and regulatory practices may differ in these areas of the country, may be severe.

Integrated approaches to uncertainty modeling and public or private decision-making, applied on community or regional scales, have rarely been attempted. The state-of-the-art in decision-theoretic methods and computational platforms now has advanced to the point where such integrated approaches can be contemplated.

Objective

This white paper is the first in a series aimed at facilitating understanding and communication among MAE Center researchers of issues regarding the identification, analysis and assessment of uncertainties affecting the performance of civil infrastructure exposed to earthquake hazards. Its short-term objective is to develop efficient methods for systematic treatment of uncertainty in all aspects of damage synthesis modeling, including representations of the seismic source and path, site response, structural and foundation response, damage and loss assessment and social and economic impact. The ultimate goal is to provide guidance to other researchers in the Mid-America Earthquake Center, enabling them to achieve the maximum return on investment in research to quantify and minimize uncertainty.

Scope

General methods for analysis of uncertainty and decision under uncertainty are presented in Chapter 2. Chapters 3 and 4 characterize uncertainty in seismicity, system behavior and response to earthquake ground motions. Chapter 5 identifies limit states of structural performance, and illustrates how uncertainties can be propagated by use of a modern finite element analysis of a reinforced concrete frame. Chapter 6 discusses issues associated with mapping those structural limit states onto a matrix of heuristic performance objectives, such as immediate occupancy, life safety and collapse prevention, and introduces notions of damage and loss assessment. Chapter 7 concludes with a summary of current research issues identified from the review and synthesis of the literature, as well as experience gained in the project to date. In order for each chapter to be reasonably self-contained, a list of references is appended to each chapter.

This research provides a technical basis for the analysis and assessment of uncertainties in consequence-based engineering and its research products are expected to support several concurrent MAE Center projects, including DS-4: Vulnerability Functions, DS-9: Risk Assessment Modeling, and CM-1: Probabilistic Support Tools. As uncertainty modeling ultimately supports developments in seismic risk assessment and consequence-based engineering, we expect that this report will be a living document, and will be revised and updated periodically in the coming years as a result of experience and feedback from other MAE Center research activities.

2. UNCERTAINTY AND DECISION IN EARTHQUAKE ENGINEERING

The threat posed by earthquakes (and other natural disasters) is of concern to legislators, natural security agencies and relief organizations, planners, local administrations, industrial and commercial enterprises, insurance companies and property owners, to name but a few stakeholders. It is therefore not surprising that decisions concerning the reduction of this threat are made at different levels, by different individuals and organizations, and with different specific objectives.

Uncertainty is usually large and weighs importantly in these decisions. One main reason is that large earthquakes are rare events; hence their occurrence during the planned time horizon is uncertain and past experience with their recurrence, characteristics and effects is limited. Another reason is that the economic and social losses from earthquakes are determined by many factors and variables, which are poorly known. Over time, the information basis has expanded, models (of earthquake occurrence, ground motion propagation, local effects, system response etc.) have become more accurate and our ability to limit losses has improved, but uncertainties are bound to remain large for the foreseeable future.

This section discusses various issues that arise in the quantification and use of uncertainty for rational decision-making. Some of the concepts are general and apply whenever decisions under uncertainty are made, while others are of special importance in the earthquake context. More details on the uncertainties one encounters in various areas of earthquake modeling and decision will be given in subsequent sections.

Section 2.1 distinguishes between aleatory and epistemic uncertainty and between statistical and non-statistical information. These distinctions are important to modern engineering risk analysis, especially for earthquake applications. Section 2.2 shows how classical and Bayesian decision theories account for aleatory and epistemic uncertainty and how these theories use statistical and non-statistical information. Section 2.3 discusses the difficult problem of quantifying uncertainty based on expert opinion. Section 2.4 deals with the specific roles of temporal and spatial variation in earthquake loss analysis and decision. These issues are especially relevant for decisions with a long time horizon and spatially extended facilities. Finally, Section 2.5 recalls basic techniques of uncertainty analysis that are often needed in earthquake risk and decision analysis. These include uncertainty propagation, conditional uncertainty, and the Total Probability Theorem.

Compared with the rest of the report, this section has a theoretical flavor. Some of the models considered here may be seen as excessively sophisticated for practical application. For example, time-dependent uncertainty as discussed in Section 2.4 should be formally considered only for decisions involving critical facilities. The reason for presenting these advanced models is to derive important theoretical results, show the effects of factors that in application are frequently neglected, and indicate what is needed for more in depth analysis. The reader should bear in mind that, like in any other engineering field, the best uncertainty and decision models are the simplest ones consistent with the state of knowledge, the computational and human resources available, and the final objective.

2.1 Aleatory and Epistemic Uncertainty and Statistical and Non-statistical Information

2.1.1 Aleatory and Epistemic Uncertainty

It is common in earthquake risk analysis to distinguish between uncertainty that reflects the variability of the outcome of a repeatable experiment (*aleatory* uncertainty) and uncertainty due to ignorance (*epistemic* uncertainty). The variability in the weather and in chance games are examples of the former type, while uncertainty on the age of the universe, the geologic profile of a site, or the earthquake capability of a fault are examples of the latter.

It may thus appear that the labeling of any given uncertainty as aleatory or epistemic is self-evident, but in fact the aleatory/epistemic quality is not an absolute attribute of uncertainty. Rather, it depends on the deterministic or stochastic representation that we make of a phenomenon. To the degree that the representation is compellingly deterministic or stochastic, so is the aleatory/epistemic character of the uncertainty.

To better understand this point, we draw an analogy between nature and a random number generator (RNG). The RNG is an entirely deterministic algorithm, the outcome of which has relative frequency properties that can be described by a probability distribution. Both the deterministic algorithm and the long-term distribution of the outcomes give correct representations of the RNG, but at different levels of detail. The deterministic mechanism is a more fundamental description of the RNG and can be used to predict the exact value of future observations (if one knows the current state of the generator, i.e. the value of the "seed"), whereas the long-term distribution gives only an ensemble property of the sequence.

Like the outcomes of an RNG, earthquakes originate in a fundamentally deterministic way (when a critical physical state is reached for fracture initiation and rapid propagation along a fault). The general physics behind the phenomenon is rather well understood, but fracture initiation and propagation depend on local stresses, strains, geometry and strength properties of the rock that, with current technology, we are unable to observe and model. Hence we resort to statistical representations of the space-time occurrence of earthquakes and their magnitudes, which capture the long-term relative-frequency properties of the earthquake sequence. While correct, this representation is incomplete relative to a detailed deterministic description of the physical process. Similar considerations apply to ground motion attenuation and its effects on exposed facilities.

One difference between the complex earthquake problem and the much simpler random number generator is that in the earthquake case there are physical laws and mathematical theories that one can rely upon in formulating models. For example, one can formulate models of ground motion propagation and site effects in which the physics of the phenomenon is variably resolved. The purely deterministic and purely statistical models are end-members of a near continuum of possibilities. All correct models produce the same long-term earthquake statistics, but the more physical ones can in principle provide more accurate short-term predictions, if the current "state" and other parameters that characterize these models can be determined.

Now we return to the difference between epistemic and aleatory uncertainty in earthquake risk. Having drawn an analogy between the generation of earthquakes by nature and the generation of pseudo-random numbers by the RNG, we may focus on the simpler problem of predicting the next outcome of the RNG rather than the more complex problem of predicting future earthquakes and their effects. We have two models, one completely deterministic (the actual RNG algorithm) and the other purely statistical (a sequence of independent random variables all with the same marginal distribution F). Both models are assumed known, but for the deterministic one the current seed is considered unknown. When the statistical model is used, uncertainty on any outcome, including the next one, is described by F. When using the deterministic model with unknown seed, a reasonable strategy is to make a long simulation of the model, from which the marginal distribution of the seed is obtained. Using this distribution to quantify uncertainty on the present value of the seed, the probability distribution of the next outcome is again calculated to be F. Hence, uncertainty on the next outcome is quantitatively the same for both models. However, in the statistical model F quantifies aleatory uncertainty, whereas in the deterministic model F reflects ignorance of the current seed and hence quantifies epistemic uncertainty. This model-dependence of the epistemic/aleatory quality of uncertainty is captured by the following statement:

Uncertainty that is explicitly recognized by a stochastic model is aleatory. Uncertainty on the model itself and its parameters is epistemic. Hence the aleatory/epistemic split of the total uncertainty is model-dependent.

Interestingly, in the previous example of the RNG, a rational individual betting on the next outcome would make the same decision irrespective of the model used, i.e. in that problem the nature of the uncertainty does not affect the decision.

Is the aleatory/epistemic distinction completely immaterial for decision making? According to Bayesian decision theory, what matters is the evolution of the total (aleatory plus epistemic) uncertainty between the time of the decision and the time when the award is collected; see Section 2.4. This total uncertainty may remain constant over that time period, vary in a predictable way, or vary in a way that cannot be predicted at the time of the decision. Non-predictability is caused by changes in the epistemic uncertainty as new information is acquired and processed. Other than in this limited sense, the epistemic/aleatory distinction is irrelevant to decision making.

The previous analysis downplays the importance of distinguishing between aleatory and epistemic uncertainty for decision making. However, the distinction serves the useful practical purpose of forcing the analyst to consider all sources of uncertainty. For example, it was typical in the early days of seismic hazard analysis to use a single ground motion model with an aleatory regression residual as the only contributor to ground motion uncertainty. Awareness of epistemic uncertainty has forced risk analysts to include the often-large uncertainty on the attenuation equation itself and its parameters.

2.1.2 Statistical and Non-statistical Information

In Section 2.1.1, we have distinguished between epistemic and aleatory uncertainty. In essence, uncertainty about the applicable model (and its parameters) is epistemic, whereas uncertainty given the model (and its parameters) is aleatory.

Classical and Bayesian statistics handle epistemic uncertainty in different ways. In Bayesian statistics one first identifies the set of plausible models and then assigns to each model a probability of being correct based on all available information. In principle, the initial selection of models can be arbitrarily broad, since the implausible models can subsequently be assigned zero probability, without affecting the decision.

By contrast, classical statistics does not assign probabilities to models and deals exclusively with statistical data (with information in the form of outcomes of statistical experiments). Any non-statistical information (for example theoretical arguments, physical constraints, expert opinion) constrains the set of plausible models, but is not subsequently used to quantify uncertainty within the chosen set of models. Therefore, the selection of plausible models is usually a more sensitive operation in classical statistics than in Bayesian statistics.

Classical and Bayesian decision making under uncertainty are compared in Section 2.2. Ways in which Bayesian analysis deals with expert opinion (a frequent form of non-statistical information) are discussed in Section 2.3.

2.2 The Mechanics of Decision Making Under Uncertainty

There are two main approaches to decision making under uncertainty, namely classical decision theory (CDT) and Bayesian decision theory (BDT). Bayesian decision theory is conceptually simpler because it treats all uncertain and unknown quantities as random variables. It is also the more broadly applicable one, because as previously noted it can handle also non-statistical information.

We start in Section 2.2.1 with a presentation of the Bayesian approach, followed in Section 2.2.2 by the classical approach. When all information is statistical, classical and Bayesian analysis often produces identical or very close decisions.

2.2.1 Bayesian Decision Theory

A basic tenant of Bayesian decision theory (see for example Raiffa and Schlaifer, 1961; DeGroot, 1970; or Zellner, 1971) is that a utility can be defined to express the relative desirability of alternative actions (e.g. seismic design decisions) in combination with possible future events (future earthquakes and their consequences). An action is then considered optimal if it maximizes the expected utility relative to all uncertain quantities.

In some instances the above may appear to be a simplistic framework for decision. For example, in the case of several stakeholders one may have to contend with multiple utility functions. Similarly, when different consequences are involved (e.g. economic and social losses), it may be difficult to represent the desirability of a given decision-outcome scenario through a single scalar quantity. However, ultimately the decision-maker will have to formally or informally combine these different utilities and losses to rank decisions on a single scale. This single scale is what we call here utility. With this in mind, maximization of the expected utility may be taken as a reasonable general objective of Bayesian decision making.

Bayesian decision theory involves three basic steps:

1. *Identify all uncertain quantities that affect the utility U(D) for each decision D.* We denote by $\underline{X}$ the vector of such uncertain quantities. For example, in a decision about the retrofit of a building, $\underline{X}$ would include the future seismicity of the area and its effects at the site, the mechanical properties of the system (stiffness, strength, ductility, etc.) given each possible retrofit decision D, the losses for different damage states of the building, etc. Some of the components of $\underline{X}$ may depend on time (for example the pattern of future earthquakes, the future utilization of a building, and future code requirements) or space in the case of spatially extended systems.

2. *Quantify uncertainty on ($\underline{X}$|D).* This is done by separately considering statistical and non-statistical information. Recall that statistical information is in the form of outcomes of statistical experiments and all the other information is non-statistical.

    1. Non-statistical information is used to produce a prior distribution of ($\underline{X}$|D) ("the prior"). In earthquake applications, this operation typically involves the elicitation of expert judgment; see Section 2.3.

    2. Statistical data are subsequently accounted for by modeling the experiment that has generated the data and calculating the likelihood of ($\underline{X}$|D). The posterior distribution of ($\underline{X}$|D) is given by the normalized product of the prior and the likelihood function. This posterior distribution is also called the predictive distribution of ($\underline{X}$|D).

3. *Specify a utility function U(D,$\underline{X}$) to measure the relative desirability of alternative (D,$\underline{X}$) combinations.* Decisions are ranked according to the utility U(D), given by

$$U(D) = E_{\underline{X}|D}[U(D,\underline{X})] = \int_{\text{all } \underline{X}} U(\underline{x},D)\,dF_{\underline{X}|D}(\underline{x}) \qquad (2.1)$$

    where $F_{\underline{X}|D}$ is the predictive distribution of ($\underline{X}$|D). Hence, a decision D* is considered optimal if it maximizes U(D) in Eq. 2.1.

Notice that the predictive distribution accounts for all available information on ($\underline{X}$|D), whether statistical or not, and all uncertainty, whether aleatory or epistemic. Hence neither of these distinctions is influential on the decision and all that ultimately matters is the total uncertainty.

To illustrate this decision procedure and later compare with results from classical decision analysis, we give a simple textbook example. Suppose that a random load X has normal distribution with unknown mean value m and known variance σ^2, i.e. $X \sim N(m,\sigma^2)$ with m unknown. We need to choose a design strength x* such that P[X > x*] = p*, where p* is a given probability (this is a shortcut to maximizing a utility function). Suppose that non-statistical information leads the analyst to a prior on m in the form of a normal distribution with mean m' and variance σ'^2. In addition, statistical information is available in the form of a random sample $\{X_1,...,X_n\}$ from the population of X. The associated likelihood function $l(m|X_1,...,X_n)$ has the form of a normal density with mean value $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and variance σ^2/n. In this case, also the normalized product of the prior density and the likelihood is a normal probability density, with mean value m" and variance σ''^2 given by

$$m' = \frac{m'\frac{\sigma^2}{n} + \bar{X}\sigma'^2}{\frac{\sigma^2}{n} + \sigma'^2}, \qquad \frac{1}{\sigma''^2} = \frac{1}{\sigma'^2} + \frac{n}{\sigma^2} \tag{2.2}$$

Using this posterior distribution of m, the predictive (unconditional) distribution of X is normal, with mean value $\bar{X}$ and variance ($\sigma^2 + \sigma''^2$). Hence x* is given by

$$x^* = \bar{X} + Z_{p*}\sqrt{\sigma^2 + \sigma''^2} \tag{2.3}$$

where Z_{p*} is the (1- p*)-quantile of the standard normal distribution. Note that Eq. 2.3 gives the deviation of x* from $\bar{X}$ as the SRSS (square-root-of-sum-of-squares) combination of two terms: $Z_{p*}\sigma$, which accounts for aleatory uncertainty, and $Z_{p*}\sigma''$, which accounts for epistemic uncertainty on m.

2.2.2 Classical Decision Theory

Classical decision analysis differs from Bayesian decision analysis in that it does not represent quantities with epistemic uncertainty as random variables. While this leads to certain complications and limitations, decisions involving epistemic uncertainty can still be made. First we illustrate the procedure for the previous decision problem, and then give the general principles.

Consider again the problem of choosing x* when $X \sim N(m,\sigma^2)$ and m is unknown. As in the Bayesian case, statistical information on m is provided by a sample $\{X_1,...,X_n\}$. This sample is used to form the estimator $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, which is distributed like $N(m, \frac{\sigma^2}{n})$. Contrary to the Bayesian case, non-statistical information cannot be used and m cannot be regarded as a random variable. To choose x* in a way that reflects both aleatory and epistemic uncertainties, we consider drawing another element X from $N(m,\sigma^2)$, which we regard as the actual random load. While the distribution of X is not completely known due to epistemic uncertainty on m, the distribution of $(X - \overline{X})$ is $N(0,(1+\frac{1}{n})\sigma^2)$ and is known. We conclude that, given $\overline{X}$, the load X has predictive distribution $N(\overline{X},(1+\frac{1}{n})\sigma^2)$. Consequently we choose x* as

$$x^* = \overline{X} + Z_{p^*}\sqrt{\sigma^2 + \frac{\sigma^2}{n}} \qquad (2.4)$$

Note that Eq. 2.4 has the same structure as Eq. 2.3, but with the sample-mean variance σ^2/n in place of the posterior variance σ''^2. The two variances become the same in the special case when $\sigma'^2 \to \infty$. This corresponds to a non-informative prior, which is appropriate when all the available information is of the statistical type. It is interesting that in this case the design values x* produced by the classical and Bayesian approaches coincide. Within classical statistics, σ^2/n in Eq. 2.4 cannot be interpreted as the variance of m given the sample, but still retains the meaning of a penalty term on the variance for epistemic uncertainty.

Generalizing the previous example, suppose that the distribution of X has unknown parameters $\underline{\theta} = \{\theta_1,...,\theta_r\}$. The first and most important step in the classical approach to decision is to find an estimator $\hat{\underline{\theta}} = \{\hat{\theta}_1,...,\hat{\theta}_r\}$ and a function $g(X,\hat{\underline{\theta}})$ such that g has a probability distribution F_g that does not depend on X. In the previous example, $g(X,\hat{\underline{\theta}})$ = $(X - \overline{X})$ and F_g = $N(0,(1+\frac{1}{n})\sigma^2)$. Then one finds x* from the condition $F_g(g(x^*,\hat{\underline{\theta}})) = 1 - p^*$. This formulation can be further extended to problems involving a vector $\underline{X}$ of random variables.

For the example problem considered above, the classical statistics solution is both simple and sensible. However, the classical procedure becomes inadequate for complex decision problems, for two reasons:

1. In cases that involve many random variables and uncertain parameters (for example in earthquake loss problems, which combine earthquake recurrence, attenuation, system response and consequence models) it may be difficult to define functions

$g(X,\hat{\underline{\theta}})$ with distributions that do not depend on the unknown parameters. In these cases a practical way to make decisions is to set the unknown parameters to conservative values such as upper or lower confidence limits $\hat{\underline{\theta}}_{cl} = \{\hat{\theta}_{cl,1},...,\hat{\theta}_{cl,r}\}$. Then x* is chosen as the (1-p*)-quantile of $(X\,|\,\hat{\underline{\theta}}_{cl})$. This "confidence approach" is frequently used in practice, for example in statement like "there is a P% confidence that the annual rate of exceeding interstory drift Δ is below λ", but is not satisfactory. To exemplify, consider again the case when $X \sim N(m,\sigma^2)$ with m unknown. The upper p-confidence level for m based on the estimator $\hat{m} = \overline{X}$ is $\hat{m}_p = \overline{X} + Z_p \frac{1}{\sqrt{n}}\sigma$. Then, setting $m = \hat{m}_p$, one obtains

$$x^* = \hat{m}_p + Z_{p^*}\sigma = \overline{X} + (Z_{p^*} + \frac{1}{\sqrt{n}} Z_p)\sigma \qquad (2.5)$$

Equation 5 is structurally different from Eq. 2.4. For example, Eq. 2.5 combines epistemic and aleatory contributions to x* in an additive way, whereas Eq. 2.4 combines them in a nonlinear way. Also notice that x* in Eq. 2.5 depends not just on p*, but also on the confidence level p. This is undesirable, since there is no objective way to choose p.

2. In the previous example, statistical information on the unknown parameters was assumed to consist of a random sample from the population of X. Alternatively or additionally, one might have a sample or at least estimates of parameters $\underline{\theta}$ from other populations with statistical similarity to the one of interest. For example, in the case of earthquakes, one might have estimates of the slope parameter b of the Gutenberg-Richter law from regions that are tectonically similar to the one of interest. It is then possible to envision a distribution of $\underline{\theta}$ across different populations, from which the parameters of the population of interest are drawn. This distribution is the closest in classical statistics to a prior on $\underline{\theta}$ in the Bayesian approach.

   When information is not in the form of a statistical sample, the classical approach cannot capture it and breaks down. It may be argued that much of the information that is available in earthquake decision problems can be idealized as a sample from a statistical model. One might even argue more generally that this is the way to formally and objectively derive the prior in the Bayesian approach, thus bringing the two decision methods closer together. However the problem is not only conceptual, but practical. The formulation of statistical models for all the available information would be so complex and prone to errors that this is simply an unwise way to proceed. By allowing one to judgmentally assign prior distributions, the Bayesian approach makes it possible to account in a simple way for information that is difficult to model statistically. The use of expert opinion in the Bayesian approach is further elaborated in the next section.

2.3 Use of Expert Opinion

Seismic risk analysis assesses the likelihood of various earthquake losses by synthesizing information from different fields like seismology, geology, engineering, and economics. Information is only partially in the form of samples from simple statistical models. The rest of the information is qualitative, circumstantial, and generally not amenable to simple statistical modeling. In Bayesian approaches, this (non-statistical) information is accounted for by involving domain experts and polling their opinions. While experts may be involved in different capacities, there is a long tradition in seismic hazard analysis (SHA) to ask experts about recurrence and attenuation models (Bernreuter et al., 1989; EPRI, 1989; SSHAC, 1997). Hence, to provide an application context, we shall focus our discussion on the problem of seismic hazard assessment. Similar considerations apply when experts specify system response and consequence models.

Using expert-specified seismicity (recurrence and attenuation) models, the task of SHA is to evaluate the relative frequency F[A] with which an event of interest A will occur during a given period of time. Let $\hat{F}_i[A]$ be the value of F[A] under the i^{th} modeling choice. The question we want to address is how to use the estimates $\hat{F}_i[A]$ from different models (and different experts) to assess the epistemic uncertainty on the true relative frequency F[A]. Since for decision making the most important quantity is the mean value $\langle F[A] \rangle$, we shall limit our analysis to this characteristic of the epistemic distribution.

The proper use of the estimates $\hat{F}_i[A]$ to evaluate $\langle F[A] \rangle$ depends on the interpretation of the models suggested by the experts:

1. The most general and conceptually satisfying view of seismicity models is that they are <u>tools to produce imprecise estimates of the unknown relative frequency</u> F[A]; see for example Winkler (1993). Then uncertainty on F[A] given the estimates $\hat{F}_i[A]$ should be quantified through Bayes' theorem. However, this theorem requires knowledge of the likelihood function (the probability of the estimates $\hat{F}_i[A]$ given the true value F[A]). Given the complex nature of the models, specification of the likelihood function is often a very difficult task and for this reason, formal Bayesian analysis is rarely used. It is nevertheless interesting that different likelihood models produce different expressions for the mean $\langle F[A] \rangle$ in terms of the estimates $\hat{F}_i[A]$ (they produce different "model-combination rules"). Two idealized cases, which produce the mean and median combination rules, are mentioned below.

   Example 1. Suppose that $\hat{F}_i[A] = F[A] + \varepsilon_i$, where the ε_i are independent and identically distributed (iid) normal variables, independent of F[A], with zero mean. Then, for a flat prior on F[A], one obtains from Bayesian analysis that

$$\langle F[A] \rangle = \frac{1}{n}\sum_{i=1}^{n}\hat{F}_i[A] \qquad \textbf{(mean rule)} \qquad (2.6)$$

Example 2. Suppose now that $\ln\hat{F}_i[A] = \ln F[A] + \varepsilon_i$, where the ε_i are iid normal, independent of F[A], with zero mean. For a flat prior on lnF[A], one obtains

$$\langle \ln F[A] \rangle = \frac{1}{n}\sum_{i=1}^{n} \ln\hat{F}_i[A] \qquad (2.7)$$

For large n, this implies

$$\langle F[A] \rangle = \text{median}\{\hat{F}_i[A]\} \qquad \textbf{(median rule)} \qquad (2.8)$$

The mean and median rules can in some cases produce very different values for the mean of F[A].

The above are just two examples. Different likelihood models from those postulated above would produce other combination rules. The important conceptual result is that <u>the mean value of</u> F[A] <u>is not necessarily the average of the model estimates</u> $\hat{F}_i[A]$.

An alternative to Bayesian analysis is to ask the experts or a "super-expert" to judgmentally assess the epistemic uncertainty on F[A] given the estimates $\hat{F}_i[A]$. These methods are less objective but more practical than formal Bayesian analysis. The human mind is a very effective processor of complex information, such as results from best-estimate and bounding models, theories affected by biases and approximations, previous experience with different methods, dependencies of various types, etc. Therefore, the results of judgmental methods may actually be more accurate than those produced by formal but necessarily simplified mathematical rules.

2. A different viewpoint, which is frequently used in SHA, is that <u>the seismicity models specified by the experts form a set of mutually exclusive and collectively exhaustive hypotheses about the real world</u>. It follows from this interpretation that one of the models must be correct. In this case the distribution of F[A] is obtained by assigning probabilities P_i to the models, such that $\sum_i P_i = 1$. The same probabilities apply to the model estimates, i.e. $P\{F[A] = \hat{F}_i[A]\} = P_i$ and the mean value of F[A] is

$$\langle F[A] \rangle = \sum_{i=1}^{n} P_i \cdot \hat{F}_i[A] \qquad \textbf{(mean rule)} \qquad (2.9)$$

Requirements for this approach to be correct are that the models must be exact (no approximation allowed) and form an exhaustive set (no other interpretation of seismicity is possible). In practice, these requirements are rarely verified or enforced.

The above applies when information is provided by a single expert or a group of likely thinking experts. In reality experts often disagree. This fact of life introduces yet another level of complexity (selection of experts, expert interaction rules in formulating models, expert dependencies and degree of expertise, etc.). However, the previous discussion still applies. In particular, formal Bayesian methods are again exact in principle (but even more complicated to implement) and viewing discrete expert/model estimates as mutually exclusive and collectively exhaustive hypotheses remains an expedient but possibly inaccurate method. As the complexity of the inference problem increases, judgmental procedures become increasingly attractive. A state-of-the-art methodology to handle expert opinion in seismic hazard analysis has been recently proposed by SSHAC (1997).

Irrespective of how information is processed and uncertainty is assessed, a key requirement is that the processing and assessment procedures be fully documented, so that an independent reviewer can evaluate their reasonableness and duplicate the results.

2.4 The Role of Time and Space in Earthquake Decisions

Time and space feature prominently in earthquake loss analysis and decision. Time is involved not only because earthquakes and their consequences are obviously distributed over time, but more fundamentally because decisions may be delayed and many factors that influence the assessment of hazard and risk evolve over time. The latter include modeling and computational capabilities, knowledge about seismicity, retrofitting methods, resource availability, economic and social variables, and the regulatory environment.

Geographical variability (of ground motion, construction practices, behavior and strength parameters, etc.) is important when analyzing systems that have significant spatial extent. This is the case in the assessment of regional losses, earthquake risk to transportation networks and other lifeline, and the vulnerability of portfolios of facilities. Spatial variability is also important in updating uncertainty on earthquake losses immediately following an earthquake, based on incomplete information. Some issues related to space and time variability are discussed next.

2.4.1 Time

The fact that earthquake losses occur over time obviously requires actualization of such losses in the utility function. However, as indicated above, time may enter the decision problem also in other, more complex ways. We examine these other effects of time by analyzing four decision cases of increasing complexity and realism. In all cases, A is an event of interest, for example the event "due to earthquakes a system of interest fails sometime during [0,T]", and T_f is the time when A occurs. We set $T_f = \infty$ if no failure takes place prior to T. The decision D might consist of choosing a design strength for the system. The first case we consider is when T is very short; hence no significant change in the state of uncertainty or the regulatory environment occurs during [0,T] and the reward or penalty may be assumed to be collected immediately following the decision. In the other three cases T is long and different assumptions are made on the regulatory constraints and the acquisition of information during [0,T]. Case 4 is the most realistic one, since it allows both the regulatory limit on the hazard and the assessed hazard to vary randomly in time.

Using a Bayesian approach, we view relative frequencies as random variables due to epistemic uncertainty. Specific issues we want to address are: Are the expected values of these relative frequencies sufficient for decision or one needs other characteristics of their epistemic distribution? (The reason why the mean might suffice is that, from application of the Total Probability Theorem, the mean value of the relative frequency of an event is the probability of that event.) We also want to know what happens when epistemic uncertainty varies in time and when a regulatory limit is imposed on the hazard.

Case 1: Instantaneous Reward

Suppose that, immediately after the decision D is made, either A or its complement A^C occurs, with consequent utility U(D,A) or $U(D,A^C)$. Due to epistemic uncertainty, the relative frequency F[A|D] is uncertain with mean value $\langle F(A\,|\,D)\rangle$. Conditional on F[A|D], the utility of decision D is given by

$$U(D|F[A|D]) = F[A|D]\; U(D,A) + (1-F[A|D])\; U(D,A^C) \qquad (2.10)$$

The unconditional utility U(D) is obtained by taking expectation of Eq. 2.10 with respect to F[A|D]. This gives

$$U(D) = \langle F(A\,|\,D)\rangle\; U(D,A) + (1-\langle F(A\,|\,D)\rangle)\; U(D,A^C) \qquad (2.11)$$

Notice that U(D) depends only on $\langle F(A\,|\,D)\rangle$ and not also on other characteristics of the distribution of F[A|D], such as the variance. The reason is that the conditional utility in Eq. 2.10 is a linear function of F[A|D]. This result is often used by "decision theorists" to support the assertion that all that matters for decision is the (epistemic) mean of any uncertain relative frequency.

Case 2: Long Time Horizon - No Regulatory Constraint on the Hazard and no Acquisition of Information over Time

Suppose now that T is a long period of time, but no new information is collected during [0,T]. If A does not occur in [0,T], the utility of D is $U(D,A^C)$ as before, and if A occurs at time t < T, the utility is U(D,t). Conditional on the distribution F_{T_f} of the time of failure T_f (recall that if failure does not occur, T_f is set to ∞), the utility of D is

$$U(D\,|\,F_{T_f}) = U(D,A^C)[1-F_{T_f|D}(T)] + \int_0^T U(D,t)\; f_{T_f|D}(t)\,dt \qquad (2.12)$$

where $f_{T_f|D}(t) = \dfrac{\partial F_{T_f|D}(t)}{\partial t}$ is the probability density function of T_f under decision D. An important feature of Eq. 2.12 is that $U(D|F_{T_f})$ is linear in $F_{T_f|D}(t)$. Therefore, the unconditional utility U(D) is obtained by replacing $F_{T_f|D}(t)$ in Eq. 2.12 with its mean value $\left\langle F_{T_f|D}(t) \right\rangle$:

$$U(D) = U(D, A^C)[1 - \left\langle F_{T_f|D}(t) \right\rangle] + \int_0^T U(D,t) \left\langle f_{T_f|D}(t) \right\rangle dt \tag{2.13}$$

Like in Case 1, the utility depends only on the expected value of the relative frequencies involved and is the same irrespective of the amount of epistemic uncertainty around such values.

Case 3: Long Time Horizon - Regulatory Constraint on the Hazard and no Acquisition of Information over Time

We now modify Case 2 by introducing a regulatory constraint on the hazard function h(t). For any given decision D, the hazard function is given by

$$h(t \mid D) = \frac{f_{T_f|D}(t)}{1 - F_{T_f|D}(t)} \tag{2.14}$$

The function h(t) has the property that h(t)dt is the probability of failure during [t,t+dt], given survival to time t. Therefore, it makes sense for regulations to require that h(t) does not exceed some maximum acceptable level h*. If h(t|D) is uncertain, then the probability that A occurs in [t,t+dt] given non-occurrence up to t is $\langle h(t \mid D) \rangle dt$ and the regulatory constraint should be imposed on $\langle h(t \mid D) \rangle$. Finally, in problems of the type of interest here, 1-$F_{T_f|D}(t)$ is close to 1 for t < T; hence one may set h(t|D) = $f_{T_f|D}(t)$ and consider the regulatory constraint to be of the type $\left\langle f_{T_f|D}(t) \right\rangle \leq h^*$. Whenever this constraint is violated, the mean hazard must be reduced through costly retrofits.

The utility must now include a penalty for possible violations of the regulatory constraint. Since no new information is acquired over time, the function $\left\langle f_{T_f|D}(t) \right\rangle$ is known at the time of the decision. Consequently, it is known whether and when the regulatory limit h* will be exceeded. The utility function has still the form in Eq. 2.13, where U(D,A^C) and U(D,t) include the retrofit penalties if $\left\langle f_{T_f|D}(t) \right\rangle$ exceeds h* at some time t < T. Once again, we find that for decision the expected hazard function $\left\langle f_{T_f|D}(t) \right\rangle$ is sufficient and other characteristics of the epistemic distribution of $f_{T_f|D}(t)$ are immaterial.

Case 4: Long Time Horizon – Random Regulatory Constraint and Acquisition of Information over Time

In this final case we allow the state of information on the distribution $F_{T_f|D}$ (on the relative frequency of event A and its time of occurrence T_f) to vary in an uncertain way during [0,T], due to information that will be collected during that period. We further allow the regulatory limit h* to vary randomly in time, to reflect the changing attitude of society towards risk. Hence, the regulatory constraint has the form $\left\langle f_{T_f|D,I_t}(t) \right\rangle \leq h^*(t)$, where I_t is the information available at time t.

At the time when the decision D is made (t = 0), one is uncertain about the true hazard $f_{T_f|D}(t)$, the mean hazard that will be calculated at future times t, $\left\langle f_{T_f|D,I_t}(t) \right\rangle$, and the regulatory limit h*(t) at those future times. Conditional on these quantities, the utility of decision D is

$$U\left[D \middle| f_{T_r|D}(t), \left\langle f_{T_f|D,I_t}(t) \right\rangle, h^*(t)\right] = U\left[D, A^C, \left\langle f_{T_f|D,I_t}(t) \right\rangle, h^*(t)\right][1 - F_{T_f|D}(T)] + \int_0^T U\left[D, t, \left\langle f_{T_f|D,I_t}(t) \right\rangle, h^*(t)\right] f_{T_f|D}(t)\, dt \tag{2.15}$$

where the U terms on the right hand side are the utilities of decision D given the temporal evolution of the mean risk $\left\langle f_{T_f|D,I_t}(t) \right\rangle$, its regulatory limit h*(t), and either non-occurrence of A (first term) or occurrence of A at time t < T (term in the integrand). It is important that these terms are nonlinear in the f and h* functions. As we shall show later, this nonlinearity is the root cause of non-sufficiency of the mean hazard and of conservatism in decision making in this case.

Both terms on the right hand side of Eq. 2.15 can be simplified. In the first term, the probability $[1 - F_{T_f|D}(T)]$ may often be set to 1 in good approximation. In the second term, the utility $U\left[D, t, \left\langle f_{T_f|D,I_t}(t) \right\rangle, h^*(t)\right]$ is usually dominated by the adverse consequences of the event A occurring at time t. Hence one may neglect the contribution from possible regulatory violations and write this term as $U(D,t)$, the utility of decision D when failure occurs at time t. After these simplifications, Eq. 2.15 gives the conditional utility of D as the sum of two terms: one accounts only for possible regulatory violations and the other accounts only for possible system failure.

We now obtain the unconditional utility of D by taking expectation of Eq. 2.15 with respect to the functions $f_{T_f|D}(t)$, $\left\langle f_{T_f|D,I_t}(t) \right\rangle$ and h*(t). Taking advantage of the above simplifications of Eq. 2.15, one obtains

$$U(D) = E_0\left\{U\left[D, A^C, \left\langle f_{T_f|D,I_t}(t) \right\rangle, h^*(t)\right]\right\} + \int_0^T U(D,t) \left\langle f_{T_f|D,I_0}(t) \right\rangle dt \tag{2.16}$$

where E_0 denotes expectation relative to epistemic uncertainty at time t = 0. The function $\left\langle f_{T_f|D,I_t}(t)\right\rangle$ in the first term is the mean hazard at time t, evaluated with the knowledge available at time t. This function will become known as time evolves, but at time t = 0 it is random, with epistemic uncertainty. The mean value of $\left\langle f_{T_f|D,I_t}(t)\right\rangle$ based on information I_0 available at t = 0 is the deterministic function $\left\langle f_{T_f|D,I_0}(t)\right\rangle$ that appears in the integral term.

In the special case with no regulatory constraint, the first term on the right hand side of Eq. 2.16 becomes $U(D,A^C)$ and U(D) depends only on $\left\langle f_{T_f|D,I_0}(t)\right\rangle$, the mean hazard based on knowledge at the time of decision. Hence, in the absence of regulatory constraints one should be risk-neutral and decide based on the expected hazard. The introduction of hazard regulations causes the utility to become nonlinear in $\left\langle f_{T_f|D,I_t}(t)\right\rangle$ (first term on the right hand side of Eq. 2.16) and makes the decision-maker appear risk-averse. Notice that here risk aversion is not the result of a psychological tendency to avoid risk; rather, it is the logical consequence of possible future infringements of the regulatory constraints.

The previous analysis resolves a frequent controversy between risk theorists and practitioners. Based on simple decision models, the theorists have often contended that in the presence of epistemic uncertainty all that matters is the expected hazard at the time of decision making. However, intuition often suggests a more prudent course of action, such as using mean-plus-sigma or some high-quantile hazard value. The "theorists" are right in Cases 1-3, whereas in Case 4 the "practitioners" are correct in being prudent. Case 4 further shows that the degree of conservatism depends on the random fluctuation of the calculated mean hazard during the lifetime of the project (this fluctuation is nil in Cases 1-3). This way of introducing conservatism is quite different from using rules such as the "mean-plus-sigma" value at the time of decision. For example, everything else being equal (including the mean-plus-sigma hazard), conservatism should increase with the lifetime of the project T and the volatility of the future mean hazard. The volatility of the mean hazard becomes zero if no new information becomes available over time. This corresponds to Case 3 and calls for risk-neutral decisions.

Again notice that these results diffuse the importance of exactly classifying sources of uncertainty as being aleatory or epistemic. We have seen in Section 2.1 that uncertainties may be of one type or the other depending on the chosen model. Changing their interpretation modifies all the characteristics of the hazard processes $f_{T_f|D,I_0}(t)$ and $f_{T_f|D,I_t}(t)$, except their mean values. The fact that the utility depends on the hazard through only these two mean functions shows that the correct classification of uncertainty is unimportant. What is important is that the deterministic function $\left\langle f_{T_f|D,I_0}(t)\right\rangle$ and the random function $\left\langle f_{T_f|D,I_t}(t)\right\rangle$ are correctly quantified.

In concluding this section on temporal fluctuations, we observe that sometime it is best to make decisions sequentially over time. For example, a delay tactic may be dictated by the limited availability of resources or by the fact that additional information should be collected before the final decision is made. This extended framework complicates the decision problem by including the times when actions are to be taken and the information to be collected as additional decision variables.

Techniques for decision making in time-dependent contexts include Kalman filtering (Sage and Melsa, 1971), dynamic programming (Bellman, 1957), and Markov decision theory (Howard, 1971; Ross, 1970). All these techniques apply to systems whose state evolves over time. For more statistical approaches, which emphasize decision strategies that involve the sequential collection of information, see for example Schmitz (1993).

2.4.2 Space

Further complications arise in the uncertainty analysis of risk and losses from spatially extended systems. Such systems include networks, lifelines and populations of buildings, as well as individual facilities for which the ground motion cannot be considered identical at all support points (for example long bridges).

In some cases the need to consider spatial variability is due not to the spatial extent of the system of interest, but to the fact that the available information is spatially distributed. For example, the relief effort immediately following an earthquake would be facilitated by knowing how much damage was suffered by different facilities. Geographically distributed damage information can be generated synthetically (by running numerical simulations), but to improve accuracy these theoretical estimates must be updated using field information. Updating requires a probabilistic model of how damages vary in space and how they depend on other covariates like soil and topographic conditions, facility type, occupancy class, etc.

Modeling spatial uncertainty for earthquake risk analysis and decision is a difficult problem because of the complex spatial variation of quantities like ground motion characteristics, behavior and resistance variables, economic parameters and human factors. In quantifying spatial variability, one must first consider the factors that influence these quantities and the spatial variability of those factors (energy radiation pattern and seismic wave propagation parameters, local geology and topographic conditions, type and quality of construction, use and economic value, etc.). Each of these factors has its own way of varying in space, with fluctuations at different scales. Since we cannot account in our models for all the physical and socio-economic factors that affect damages and losses, the effects of the unidentified factors are lumped together into "error terms", which themselves have spatial dependencies.

Consider for example the reduction in flow capacity of a highway segment due to landslides or other ground failures. This being a series system (a highway segment will be largely unusable if a severe disruption occurs anywhere along the segment), the flow capacity following an earthquake is well approximated by the minimum flow capacity anywhere along the segment. Two extreme modeling hypotheses are:

1. *Maximum dependence among the disruptive events along the highway.* If, as realistic, the risk of disruption varies with location along the highway, maximum dependence corresponds to the assumption that lower-risk locations do not fail unless higher-risk locations fail. In this extreme case the usability of the highway depends exclusively on what happens at the highest risk location and failure at any other location can be neglected (of course, such failures would have to be considered if interest is in the economic losses rather than the usability of the highway).

2. *No dependence among the failure/no-failure events at different locations.* Provided that failure has a nonzero probability of occurring over a continuum of spatial locations, the probability of highway closure is in this case 1, irrespective of the size of the earthquake.

The previous two assumptions usually produce very different results. Interestingly, the condition of no spatial dependence does not necessarily produce the most conservative results. For example, the total claim for an insurance company is additive over the individual property losses. While the mean value of a sum is the sum of the mean values irrespective of the dependence among the variables, the variance increases with increasing correlation. A larger variance means an increase in the likelihood of extreme losses, which are of primary concern in the insurance business.

These examples illustrate different ways in which spatial variation enters earthquake decisions and the need to explicitly model spatial dependence.

2.5 Some Elements of Uncertainty Analysis

We conclude this methodology section by recalling results from probability theory that are especially useful in the analysis of earthquake risk and losses.

2.5.1 Uncertainty Propagation

Uncertainty propagation refers to the problem of quantifying uncertainty on a set of dependent variables $\underline{Y} = [Y_1, \ldots, Y_n]$, given the joint distribution of another set of variables $\underline{X} = [X_1, \ldots, X_k]$ and a functional relation $\underline{Y} = g(\underline{X})$. For example, the interstory drift induced in a structure by an earthquake of given magnitude M and epicentral distance R depends on various uncertain characteristics of the ground motion at the site (for example the ordinates of the response spectrum at the modal periods) and various uncertain behavioral parameters of the structure. In this case Y is a scalar (the interstory drift), whereas $\underline{X}$ is the vector of all relevant and uncertain ground motion/response characteristics, plus random model errors if applicable.

Different methods are available to characterize uncertainty on $\underline{Y}$. The most frequently used ones are: 1. exact analysis, which is practically feasible only in simple cases, 2.

second-moment (SM) and first-order second-moment (FOSM) analysis, which aim at obtaining only the mean values, variances and covariances of the components of $\underline{Y}$, and 3. Monte Carlo simulation, which produces artificial random samples from the distribution of $\underline{Y}$.

1. Exact Analysis

A general theoretical solution can be given for all uncertainty propagation problems. Let $f_{\underline{X}}(\underline{x})$ be the joint probability density function (pdf) of $\underline{X}$ (assumed here to be a continuous random vector) and $F_{\underline{Y}}(\underline{y})$ be the joint cumulative distribution function (cdf) of $\underline{Y}$. Also, let $\Omega_{\underline{y}} = \{\underline{x}: g_i(\underline{x}) \le y_i\}$ be the set of values of $\underline{X}$ such that $Y_i \le y_i$ for all i. Then

$$F_{\underline{Y}}(\underline{y}) = P[\underline{X} \in \Omega_{\underline{y}}] = \int_{\Omega_{\underline{y}}} f_{\underline{X}}(\underline{x}) d\underline{x} \tag{2.17}$$

While general and exact, Eq. 2.17 has the limitation of being often impractical, because the set $\Omega_{\underline{y}}$ may be difficult to obtain, the integral may not have an analytic solution, or the high dimensionality of $\underline{X}$ may make numerical integration impossible. Some cases however exist in which Eq. 2.17 produces useful results. An important one is when $Y = \sum_{i=1}^{k} X_i$ and the variables X_i are independent. Then the integral in Eq. 2.17 reduces to a series of "convolution integrals" that can be easily evaluated in the frequency domain (using characteristic functions instead of probability density or cumulative distribution functions).

2. SM/FOSM Analysis

Let $\underline{X}$ be a random vector with n components and $\underline{Y}$ a vector of k components. Suppose that, possibly after linearization, $\underline{Y}$ is related to $\underline{X}$ as

$$\underline{Y} = \underline{a} + \underline{BX} \tag{2.18}$$

where $\underline{a}$ is a given vector and $\underline{B}$ is a given matrix. If $\underline{X}$ has mean value vector $\underline{m}_X$ and covariance matrix $\underline{\Sigma}_X$, written $\underline{X} \sim (\underline{m}_X, \underline{\Sigma}_X)$, then $\underline{Y} \sim (\underline{m}_Y, \underline{\Sigma}_Y)$ where

$$\begin{aligned} \underline{m}_Y &= \underline{a} + \underline{B}\underline{m}_X \\ \underline{\Sigma}_Y &= \underline{B}\underline{\Sigma}_X\underline{B}^T \end{aligned} \tag{2.19}$$

When Eq. 2.18 holds exactly, use of the results in Eq. 2.19 is referred to as second-moment (SM) analysis. When Eq. 2.18 is the result of linearization of a nonlinear relation, this procedure is called first-order second-moment (FOSM) analysis.

2.5.2 Conditional Uncertainty

A basic problem of conditional uncertainty or uncertainty updating is to quantify uncertainty on a random vector $\underline{Y}$ given observation of another random vector $\underline{X}$. For example, during inspection of a building for seismic qualification, certain observations are made on the type of construction, age and integrity of the building, and analyses are possibly made to assess its performance under earthquake loading. These observations, arranged into a vector $\underline{X}$, are then used to update the initial uncertainty on earthquake resistance characteristics $\underline{Y}$.

Like for uncertainty propagation, there is an exact and general solution to the problem of conditional uncertainty, which however can be implemented only in special cases. An alternative is to use second-moment techniques. These methods are reviewed next.

The exact solution uses the joint distribution of $\underline{X}$ and $\underline{Y}$, say the joint density $f_{\underline{X},\underline{Y}}(\underline{x},\underline{y})$ if $\underline{X}$ and $\underline{Y}$ are continuous. Given $\underline{X} = \underline{x}^*$, where $\underline{x}^*$ is the vector of the observations, the conditional distribution of $(\underline{Y}|\,\underline{X} = \underline{x}^*)$ satisfies:

$$
\begin{aligned}
f_{\underline{Y}|\underline{X}=\underline{x}^*}(\underline{y}) &\propto f_{\underline{X},\underline{Y}}(\underline{x}^*,\underline{y}) \\
&\propto f_{\underline{Y}}(\underline{y}) \times f_{\underline{X}|\underline{Y}=\underline{y}}(\underline{x}^*) \\
&\propto f_{\underline{Y}}(\underline{y}) \times l(\underline{y}\,|\,\underline{x}^*)
\end{aligned}
\tag{2.20}
$$

The second expression in Eq. 2.20 follows from using $f_{\underline{X},\underline{Y}}(\underline{x},\underline{y}) = f_{\underline{Y}}(\underline{y}) \times f_{\underline{X}|\underline{Y}=\underline{y}}(\underline{x})$ and $l(\underline{y}\,|\,\underline{x}^*)$ in the last expression is the likelihood function, defined as any function proportional to $f_{\underline{X}|\underline{Y}=\underline{y}}(\underline{x}^*)$. The last expression in Eq. 2.20 also corresponds to Bayes' theorem, with $f_{\underline{Y}}(\underline{y})$ that represents prior uncertainty on $\underline{Y}$ (before observation of $\underline{X}$) and $f_{\underline{Y}|\underline{X}=\underline{x}^*}(\underline{y})$ that represents posterior uncertainty.

Equation 20 is completely general. However, its implementation requires knowledge of the joint distribution of $\underline{X}$ and $\underline{Y}$, which may not be known. Also in the case when $f_{\underline{X},\underline{Y}}(\underline{x},\underline{y})$ is known, numerical evaluation of $f_{\underline{Y}|\underline{X}=\underline{x}^*}(\underline{y})$ may be prohibitive it $\underline{Y}$ is a large vector. An important special case for which Eq. 2.20 produces simple analytical results is when $\underline{X}$ and $\underline{Y}$ have joint normal distribution,

$$
\begin{bmatrix} \underline{X} \\ \underline{Y} \end{bmatrix} \sim N\left(\begin{bmatrix} \underline{m}_X \\ \underline{m}_Y \end{bmatrix}, \begin{bmatrix} \underline{\Sigma}_{XX} & \underline{\Sigma}_{XY} \\ \underline{\Sigma}_{YX} & \underline{\Sigma}_{YY} \end{bmatrix} \right)
\tag{2.21}
$$

where the mean value vector $\underline{m}$ and the covariance matrix $\underline{\Sigma}$ are shown in partitioned form. In this case one finds from Eq. 2.20 that $(\underline{Y}|\,\underline{X} = \underline{x}^*)$ has multivariate normal distribution with mean vector and covariance matrix given by:

$$
\begin{cases}
\underline{m}_{Y|\underline{X}=\underline{x}^*} = \underline{m}_Y + \underline{\Sigma}_{YX}\underline{\Sigma}_{XX}^{-1}(\underline{x}^* - \underline{m}_X) \\
\underline{\Sigma}_{Y|\underline{X}=\underline{x}^*} = \underline{\Sigma}_{YY} - \underline{\Sigma}_{YX}\underline{\Sigma}_{XX}^{-1}\underline{\Sigma}_{YX}^{T}
\end{cases}
\tag{2.22}
$$

Notice that the conditional mean of $\underline{Y}$ in Eq. 2.22 depends (linearly) on the observations $\underline{x}$*, whereas the conditional covariance matrix is independent of $\underline{x}$*. Hence the posterior covariance matrix can be calculated before $\underline{X}$ is observed. This feature of Eq. 2.22 allows one to compare alternative data collection strategies in terms of the information they provide on $\underline{Y}$, before committing to any such strategy. Hence Eq. 2.22 is often used for planning data gathering experiments.

In the scalar case (only one X and only one Y), Eq. 2.22 reduces to

$$\begin{cases} m_{Y|X=x^*} = m_Y + \rho \dfrac{\sigma_Y}{\sigma_X}(x^* - m_X) \\ \sigma^2_{Y|X=x^*} = \sigma^2_Y(1-\rho^2) \end{cases} \tag{2.23}$$

where ρ is the correlation coefficient between X and Y. Notice that the reduction factor on the a priori standard deviation is $\dfrac{\sigma_{Y|X=x^*}}{\sigma_Y} = \sqrt{1-\rho^2}$. In order for this factor to be significantly smaller than 1, the correlation coefficient ρ must be close to 1. For example, for ρ = 0.8 one obtains $\dfrac{\sigma_{Y|X=x^*}}{\sigma_Y}$ = 0.6.

Another observation is that, since the joint normal distribution is completely characterized by the first and second moments of $\underline{X}$ and $\underline{Y}$, the conditional results in Eq. 2.22 depend only on these first and second moments (in addition of course to $\underline{x}$*). One might wonder whether Eq. 2.22 holds in general, irrespective of the joint distribution of $\underline{X}$ and $\underline{Y}$. This is not the case; however,

1. Equation 22 holds for a class of joint distributions of $\underline{X}$ and $\underline{Y}$, not just the normal distribution. Characterization of this class of distributions is simple, but is beyond the scope of this short review;

2. Equation 22 holds for all joint distributions if the expressions for the conditional mean vector and conditional covariance matrix are given a different interpretation. The interpretation comes from estimation theory: Suppose that all that is known about the joint distribution of $\underline{X}$ and $\underline{Y}$ are the mean values, variances and covariances. Rather than asking for the mean vector and covariance matrix of ($\underline{Y}|\underline{X} = \underline{x}$*), which under this condition of limited information are not defined, we look for an estimator $\underline{\acute{Y}}$ of $\underline{Y}$, with the following characteristics: 1. $\underline{\acute{Y}}$ should be linear in $\underline{x}$*, 2. $\underline{\acute{Y}}$ should be unbiased in the sense that $E[\underline{\acute{Y}}] = \underline{m}_Y$, and 3. $\underline{\acute{Y}}$ should be optimal, in the sense that the variance of the estimation errors $e_i = \acute{Y}_i - Y_i$ is minimum among all linear unbiased estimators. The estimator that satisfies these three conditions is called the BLUE (Best Linear Unbiased) Estimator, $\underline{\acute{Y}}_{BLUE}$. It can be shown that $\underline{\acute{Y}}_{BLUE}$ is the same as the conditional mean in Eq. 2.22 and the covariance matrix of the estimation error $\underline{e}_{BLUE} = \underline{\acute{Y}}_{BLUE} - \underline{Y}$ is the same as the conditional covariance matrix in Eq. 2.22.

For these reasons, the results in Eq. 2.22 are very often used for uncertainty updating, also with non-normal distributions.

2.5.3 The Total Probability Theorem

The Total Probability Theorem (TPT) is one of the most useful results in earthquake risk analysis. Technically, the TPT relates the probability of an event A to the conditional probability of A given each of several events B_i and the marginal probabilities of the events B_i, as

$$P[A] = \sum_i P[A \mid B_i] P[B_i] \tag{2.24}$$

Equation 24 holds under the condition that, with probability 1, one and only one of the B_i events occurs. A continuous version of this theorem allows one to find the marginal density of a random variable Y from the conditional density of (Y|X) and the marginal density of X, where X is any other random variable:

$$f_Y(y) = \int_{-\infty}^{\infty} f_{Y|X=x}(y) f_X(x) dx \tag{2.25}$$

Notice that the integrand in Eq. 2.25 is the joint density of X and Y. Hence Eq. 2.25 is the familiar formula that gives a marginal density from the joint density through integration. A relation completely analogous to Eq. 2.25 holds when X and Y are vectors.

To exemplify the use of the TPT in earthquake risk, suppose that A is the event "a building will be severely damaged by the next large earthquake in the region". Direct assessment of P[A] is difficult because the occurrence of A depends on the magnitude M and epicentral distance R of the earthquake, the resulting ground motion intensity (say the PGA value) at the site of the building, and the resistance of the building (say the value pga* of PGA beyond which severe damage occurs). All of these quantities are usually uncertain. The TPT allows one to parcel the problem into simpler sub-problems: The distribution of PGA at the site is first obtained by using Eq. 2.25, as

$$f_{PGA}(pga) = \int_{\text{all }(m,r)} f_{PGA|m,r}(pga) f_{M,R}(m,r) dm dr \tag{2.26}$$

Then P[A] is calculated by using the TPT a second time, in the form

$$P[A] = \int_{\text{all pga}} F_{PGA*}(pga) f_{PGA}(pga) d(pga) \tag{2.27}$$

This way of obtaining P[A] is computationally efficient. A second desirable feature of the TPT approach is that the probabilities or probability distributions that are required to calculate P[A] are conveniently confined each to a different disciplinary area (seismicity for $f_{M,R}$, ground motion attenuation for $f_{PGA|m,r}$, structural analysis for F_{PGA*}). Hence the TPT decomposes an intrinsically multi-disciplinary problem into sub-problems, each in the domain of a different expert community.

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3. SEISMICITY, GROUND MOTIONS, AND DEMANDS ON THE SYTEMS

In structural performance evaluation, it is convenient to describe the system performance in terms of demand and capacity. The demand can be the force (shear, bending moment, axial forces, overturning moment) or the response (displacement, velocity, acceleration, drift, ductility, energy dissipation) in the system caused by the ground excitation. The capacity of the system is the maximum forces or response that the system can withstand without member or system failure. The member or system failure in turn can be described by various limit states of interest to the engineers. For example, commonly used limit states in terms of response demand are drift limits corresponding to various performance requirements from immediate occupancy to collapse prevention. In theory, both the capacity and demand depend on the excitation and the structural property. Consider the performance of a structural system for a given period of time. The demand as described above such as system global drift is clearly a quantity that fluctuates in time and highly uncertain depending on the seismic excitation during the period. The capacity is primarily a property of the system. It is common practice to use the maximum response or force over a given time period (annual or per 50 years) as the demand variable. The uncertainty in the demand so defined can be traced back to the chain of events that cause the response or force as shown in Fig. 3.1. They are briefly described as follows with emphasis on the uncertainty modeling and treatment. Details of course can be found in the extensive literature on these subjects. For simplicity, from this point on the aleatory uncertainty will be referred to as randomness and the epistemic uncertainty as uncertainty.

3.1 Source

Over a specified period of time, the threat of seismic excitation to a given system at a given site can come from events at different times and of different magnitudes, distances, focal depths and rupture surface geometries and features. The randomness and uncertainty of these major source parameters are briefly discussed in the following.

3.1.1 Occurrence Time

The random occurrence in time can be modeled by random processes, such as from the simple Bernoulli sequence, its limiting form the simple Poisson process, to more involved renewal and Markov processes. These models allow one to calculate the probability of number of occurrences over a given period. The Bernoulli and Poisson models are time-independent or models with no memory such that the probability of number of occurrences depends only on the time interval considered and independent on the calendar time and history of what happens before. The only parameter in the model is the annual probability of occurrence, p, for the Bernoulli sequence or mean occurrence rate, ν, for the Poisson process. In spite of the rather restrictive assumption, these two models are quite robust and have been widely used and from which we derive the concept of return period. The return period has been used often without question about the time-independence assumption associated with the underlying models. Based on the Poisson model, the number of occurrence, N, over an interval (0,t) is given by

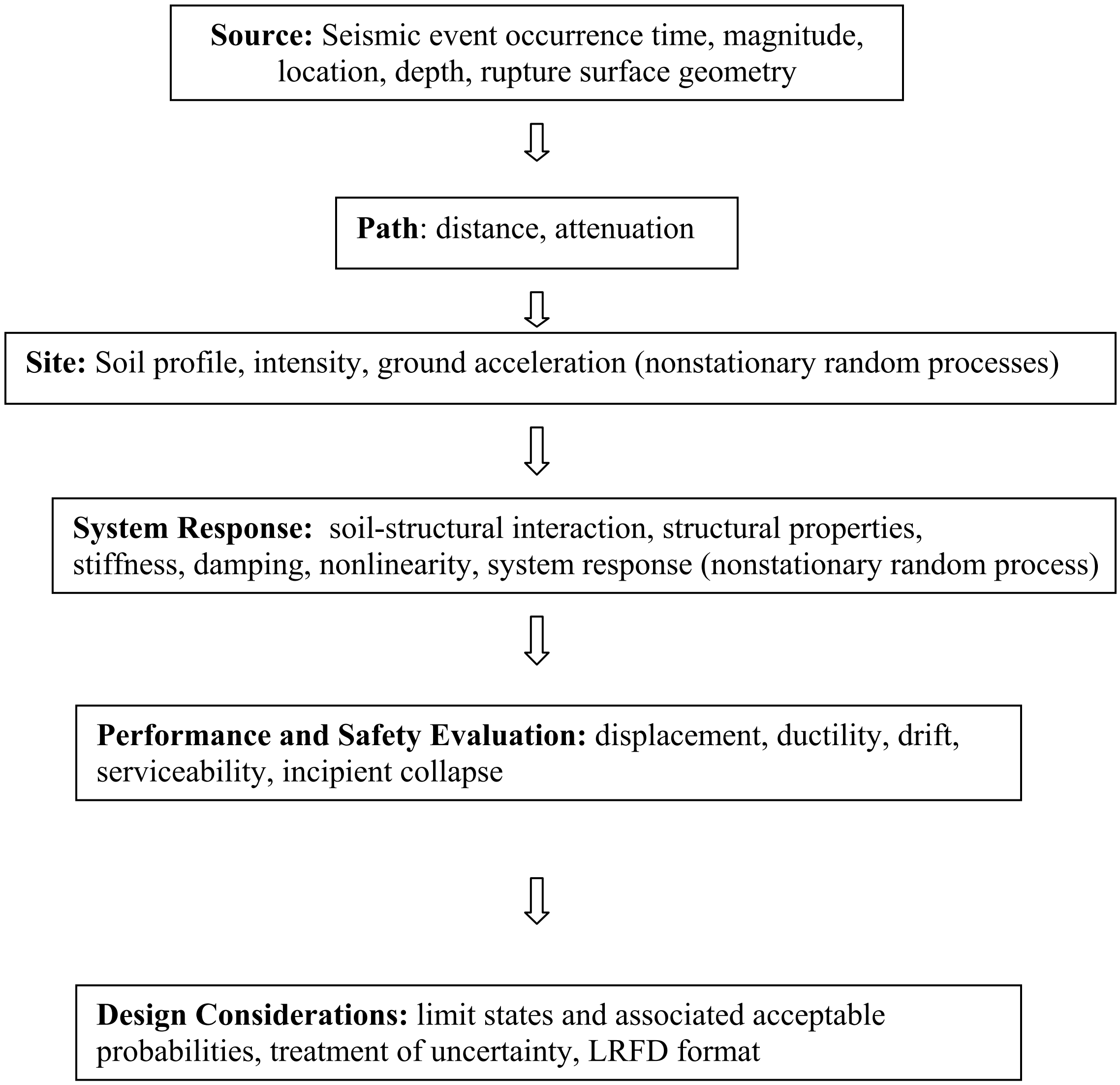

Figure 3.1. Probabilistic Performance Evaluation and Design for Earthquakes

$$P_t(N = r) = \frac{(\nu t)^r}{r!} e^{-\nu t} \tag{3.1}$$

The mean waiting time till the next occurrence is $1/\nu$, the return period. If a Bernoulli sequence is used, the return period is 1/p. When ν is calculated based on a short record, the uncertainty could be significant. The Bayesian method outlined in Chapter 2 can be used to incorporate the effect of such uncertainty. If n_0 earthquakes were recorded in the last t_0 years, the Bayesian prediction is (e.g. Ang and Tang 1975)

$$P_t(N=r) = \frac{(r+n_0)!}{r!n_0!}\frac{(t/t_0)^r}{(1+t/t_0)^{r+n_0+1}} \tag{3.2}$$

The difference could be significant when n_0 is small. For example, for the case of $n_0 = 2$, $t_0 = 10$, $t = 1$, the probability of at least one event ($N \geq 1$) in a given year is 0.181 according to Eq. 3.1 and 0.249 according to Eq.3.2. In general, the Poisson model works well when there are a number of sources. Individually their occurrences may show time dependence but collectively the occurrences over time tend to become less dependent and can be approximated by a Poisson process. When dealing with individual and particularly large events such as characteristic earthquakes along well-defined faults with good records of the past occurrences, the latter two models are often used to incorporate the time dependence. For example, a renewal process has been used in the probabilistic prediction of large events in the western United States (Working Group 1995) in which the random time intervals between occurrences are assumed to be a lognormal random variable. The probability of number of occurrences of a renewal process for a given period would depend on past history and the mathematics is more involved. Often the interest is on the next occurrence within t years from now knowing the last event occurred t_0 years before. Counting the time from the last event, the probability is given by

$$P(T < t_0 + t \mid T > t_0) = P(t_0 < T < t_0 + t)/P(T > t_0) \tag{3.3}$$

in which T is the inter-occurrence time, e.g., a lognormal random variable. The above probability can be easily calculated when the first two moments of T are known. Note that the probability depends not only on the time interval (0, t) but also on past occurrence time history given by t_0. Note also that the Poisson model would yield

$$P(T < t) = P_t(N \geq 1) = 1 - e^{-\nu t} \tag{3.4}$$

since the independence assumption implies that past history has no bearing on future occurrences. The difference between the two models could be significant.

3.1.2 Epicenter Location

The exact location of future earthquake epicenter is unknown. Random spatial distribution models can be used for this purpose. For example, line and areal source models where the epicenter is assumed to follow certain distribution on the line or within a well-defined region have been used for this purpose in the past. Such distributions can be obtained based on occurrence statistics collected from past earthquakes. For example, in the context of a Poisson occurrence model, one can express the mean occurrence rate of future events per unit area as function of the location $\nu(x, y)$ for an areal source and $\nu(\ell)$ as function along the line source. One can then evaluate the probability of occurrence of various events within the area or along the line. The occurrence rate of events in a given region, the random magnitude, and spatial distribution of epicenter given the occurrence in time can be used to model the temporal and spatial randomness of future events as will be shown in Section 3.3.2.

3.1.3 Magnitude

The magnitude variability is generally described by the Gutenberg-Richter equation, which is a linear equation of logarithmic frequency N and magnitude M for certain range of the magnitude

$$\text{Log}N = a - b\,\text{Log}\,M \qquad \text{for } m_L < M < m_U \qquad (3.5)$$

The implication is that the variability of magnitude given the occurrence of an earthquake can be modeled by a truncated exponential probability density of the following form

$$f_M(m) = Cb\,e^{-b(m-m_L)}) \qquad \text{for } m_L < M < m_U \qquad (3.6)$$

The randomness in magnitude is therefore captured by the above distribution. Depending on the range, the variability in magnitude described by the above distribution in terms of coefficient of variation is quite large and close to one. When data are limited the uncertainty in parameters a, b, m_U, and m_L could be also important. (Note that in the above equations and equations that follow, the capital letter refers to the random variable such as M and the lower case letter refers to the numerical value of the random variable such as m).

3.1.4 Rupture Surface

There are many other random parameters of the source such as the size and geometry of the rupture surface, stress drop, and slip variation within the surface that could be also important factors for consideration. The effects of the randomness of these parameters are to certain extent absorbed in the attenuation equation and seldom explicitly considered in seismic risk analysis. The exception is in simulation of individual large events. For example, random field models have been developed for the slip variation within the rupture surface and used in simulation of ground motions (e.g., Somerville et al 1997, Wen and Wu 2001).

3.2 Path and Site

As the seismic waves propagate from the source through the rock and soil media to the ground surface at the site, they are attenuated or amplified and many factors contribute to the uncertainty in the attenuation and amplification processes. As mentioned above, the effects of many other random parameters associated with the source are also included in the attenuation model. As a result, the randomness in the attenuation model is usually very large as can be seen from the large scatter of attenuation of various ground motion intensity measures such as spectral ground acceleration and velocity based on observations during past earthquakes. As shown in previous chapters, the forms of the attenuation equations are usually a result of wave propagation theory modified by observational results. The most important independent variables in the attenuation equations are the magnitude (M), distance (R), and site soil classification (S). In view of the large uncertainty, the attenuation equation A(M, R, S) generally describes the central value and the scatter is modeled by a random variable. When the intensity measures are plotted on a logarithmic graph, the scatter generally follows approximately a normal distribution. Therefore given M, R, and S, the intensity measure, e.g. spectral acceleration S_a at the site is approximately a lognormal random variable with expected (mean) value E[log S_a] described by the attenuation equation; i.e.

$$E[\log S_a(M,R,S)] = A(M,R,S) \tag{3.7}$$

The scatter is given by $\sigma_{\log S_a}$, in which σ denotes standard deviation. σ in general is also a function of M and S (e.g. Boore and Joyner, 1994) but usually is regarded as a constant as an approximation. Therefore, in such a formulation, all the randomness in wave propagation from the source to the site and some randomness associated with the source is captured by $\sigma_{\log S_a}$. Note that the mean and standard deviation in these equations are in terms of log S_a not S_a. After proper conversion, the mean and coefficient of variation of S_a can be shown to be

$$E(S_a) = \exp[2.3A+0.5(2.3\sigma)^2] \tag{3.8}$$

$$\delta_{S_a} = \sqrt{e^{(2.3\sigma)^2} - 1} \tag{3.9}$$

in which A and σ are the attenuation equation prediction and scatter in log scale. For example, a scatter of $\sigma_{\log S_a}$ = 0.3, a value commonly seen in attenuation equations, actually means a coefficient of variation of 78% in S_a. The probability that S_a exceeds a given limit of a_0 is therefore given by

$$P(S_a > a_0 \mid M,R,S) = 1 - \Phi[\frac{\ln a_0 - 2.3A(M,R,S)}{2.3\sigma}] \tag{3.10}$$

in which Φ is the standard normal cumulative distribution. Note that the above equation describes the randomness in attenuation alone when M, R, and S are known. M, R, and S are also random variables, which would influence the demand on the structural system. Also, the uncertainty in attenuation equation itself (modeling errors) is evident from the various forms of attenuation equations for the same region, which give different results. Again such modeling errors are not generally considered explicitly.

3.3 Ground Excitation and Structural Response

The demand on the structure over a given time period in the future is the ground motions and structural responses that they produce. They are unpredictable and random functions of time. In theory, they can be modeled by a continuous random process whose parameters depend on the source, path, site, and structural properties. The ground excitation given the occurrence of an earthquake in the future is therefore a continuous random process of time that depends on magnitude m, distance r, and site condition, i.e. $a(t|m,r,s)$. The structural response in turn is also a random process depending on the excitation and the structural properties and the excitation parameters. Although such random process models have been developed for both the excitation and structural responses based on random process theory and method of random vibration (e.g. Wen 1989, 1990), the nonstationarity in the excitation and quite often nonlinear and inelastic dynamic response of the system render the theoretical treatment difficult for real structural systems.

3.3.1 Excitation Intensity Measures

In performance evaluation, the structural response demands are often described in terms of the maximum responses such as maximum global displacement, maximum interstory drift or energy dissipation over the duration of the excitation. These demand variables are random and the annual maximum or maximum value over 50 years is customarily used. The uncertainty in these demand variables can be traced back to those in the structural properties as well as source, path, and site parameters. The propagation of uncertainty along the chain of events that lead to the demand variable as shown in Fig. 3.1 is a rather complicated process involving random variables and random processes and linear and nonlinear input-output relationship. To simplify the problem, engineers have been trying to find some key ground excitation intensity measures that correlate well with the structural demand variable. The peak ground acceleration, velocity, and displacement have been traditionally used for this purpose. These measures generally show poor correlation with the structural response since the structural properties are not considered.

Luco and Cornell (2001) recently examined a number of intensity measures that reflect the structural properties such as fundamental period and damping ratio based on extensive regression analyses of steel structural systems of different design and configurations under excitation of recorded ground motions. The results showed that the spectral acceleration or displacement at the structure’s fundamental period corresponding to a damping ratio of 5 % generally give good results. To incorporate effects of higher modes and inelastic response, intensity measures consist of the combined first and second mode spectral acceleration or displacement and first mode elastic and inelastic spectral accelerations were examined. They give even better results as indicated by the smaller scatter in the regression relationship compared with using only the fundamental period elastic response. This is achieved, however, at the expense of more complicated form of the intensity measure. One advantage of using spectral response variable is that these quantities can be related directly to M, R, and S via the attenuation equation and additional dependence of structural response on M and R are small and can be ignored in approximation (Shome et al, 1998). To consider the effect of bi-axial excitation, Wang and Wen (2001) also proposed a bi-directional spectral displacement defined as the maximum of the vector sum of the displacements in two principal directions at the two fundamental periods of the structure in the two principal directions. It can be used to better correlate with the bi-axial structural response measure such as bi-axial drift ratio which is defined in the same way.

3.3.2 Seismic Hazard Analysis

The uncertainty in the seismic excitation can be therefore approximately described in terms of a random variable of the above intensity measure such as the maximum spectral acceleration over a given period of one year or fifty years. The probability of exceedance of such a random variable is generally referred to as the seismic hazard curve. For example, if the spectral acceleration S_a is used, the probability of exceedance in t (e.g. 50) years is given by

$$P_t(S_a > a) = H_t(a) \tag{3.11}$$

$H_t(a)$ is the hazard curve, which can be constructed from the probabilistic models of the source, path, and site as described above based on available regional seismicity information. For example, consider a region in which there is a well-defined fault of characteristic earthquakes of known magnitude. The probabilistic distribution of the inter-occurrence time and date of last occurrence are also known. There is also an areal source of smaller events whose occurrences can be modeled by a Poisson process with an occurrence rate which is a function of the location, $\nu(x, y)$; and whose magnitude can be modeled by an exponential distribution based on a Gutenberg-Richter equation. In addition, there is also a line source along which the occurrence rate $\nu(\ell)$ and magnitude distribution of future events are known. The attenuation equations for events from these sources have also been established. Assuming the events from these three sources are statistically independent, the seismic hazard over the next t years can be evaluated as follows:

$$P_t(S_a>a) = 1- [P_C(S_a<a \mid C)P(C)][P_A(S_a < a)][P_L(S_a<a)] \tag{3.12}$$

In which C denotes occurrence of characteristic events modeled by Eq. 3.3; the conditional probability of spectral acceleration given the occurrence of the event can be estimated from the attenuation equation model as given in Eqs 3.8 to 3.10. A and L refer to the areal and line sources. The last two terms in Eq. 3.12 are obtained by considering contribution from all future events within the areal and line sources and the occurrence as a Poisson process as follows

$$P_A(S_a<a) = e^{-t\iiint \nu(x,y)P(S_a>a \mid m,r,s)f_{MA}(m)dxdydm} \tag{3.13}$$

$$P_L(S_a<a) = e^{-t\iint \nu(\ell)P(S_a>a \mid m,r,s)f_{ML}(m)d\ell dm} \tag{3.14}$$

in which Eqs.3.6 and 3.10 are used and subscript MA and ML refer to magnitude of events in the areal and line sources. The above procedure allows one to evaluate the spectral acceleration of different periods corresponding to a given probability of exceedance. The resulting response spectra are called uniform-hazard response spectra (UHRS). The commonly used probability of exceedance is 50 %, 10 %, and 2 % in 50 years such as in the USGS National Earthquake Hazard Mapping Project (Frankel et al

1996). The UHRS therefore are an efficient way of describing the seismic hazard and ground motion demand on the structure since the response of a linear structure corresponding to the above probability of exceedance can be easily predicted using the well-known modal superposition method.

For nonlinear systems, the UHRS cannot be directly used since modal superposition method can no longer be applied. There have been large efforts in the past on extension of the concept of UHRS to nonlinear inelastic systems. Based on investigation of large number of single-degree-of-freedom (SDOF) systems under recorded ground motions, uniform-hazard inelastic response spectra have been established by researchers (Nassar and Krawinkler 1992, Miranda and Bertero 1994, Collins et al 1996). Empirical rules have been developed so that the uniform-hazard inelastic response spectra (UHIRS) can be constructed from the linear UHRS. The spectra give the ductility ratio of a SDOF system of given period and yield strength corresponding to a given probability of exceedance. The UHRS therefore describes the demand on an SDOF inelastic system. It is mentioned that most real structural systems cannot be adequately described by a SDOF system since the effect of higher modes cannot be included; hence the application of UHIRS is limited.

3.3.3 Modeling of Epistemic Uncertainty by Logic Tree

When dealing with uncertainty in the selection of magnitude, recurrence model, and attenuation equation, etc., in seismic hazard analysis, a logic tree is frequency used with branches for different models or values, each with assigned likelihood based on judgment/experience (e.g. Frankel et al 1996, Frankel 1995). It is therefore a method for treating the epistemic uncertainty. At each branch of the tree, further characteristics and uncertainty can be assigned in accordance with the expert's opinion (e.g., SSHAC 1995). For example, referring to Fig.3.2, going from the site to the source, possible attenuation equations are first identified. The occurrence model is either a memory-less Poisson process according to the Gutenburg-Richter equation, a renewal process for characteristic events with a specified recurrence time distribution, or a Markov process with memory specified by a transition matrix. At each branch, candidate models or equations are assigned with a relative likelihood reflecting the judgment/experience of the experts. For example, if it is a characteristic event, the possible choices of magnitude are M_1, M_2, or M_3 with given relative likelihood. In the seismic hazard analysis these relative likelihood of the magnitude is then converted into discrete probability mass function and incorporated into the risk analysis. For example, because of the modeling uncertainty, the result of the seismic hazard analysis as given in Eq.3.12 becomes a random variable. A common practice is to determine the expected (mean) value of the seismic risk by integration (or summation) over all possible combinations of these values weighted by their likelihood (or probability mass). A simpler and more convenient way is to approximate the mean risk estimate by using the mean value at each branch when possible (such as the a and b values in the Gutenburg-Richter equation or the magnitude of a characteristic earthquake) and reduce the number of possible combinations and hence the required numerical effort. A more general approach is to consider the likelihood of each branch and evaluate the risk corresponding to a percentile value (or confidence level) as given in Section 2.2.2. It can be done easily via a Monte-Carlo method as will be covered in a later section. The implications of using the mean value versus the percentile value will be illustrated in an example in the following sections.

3.3.4 Probabilistic Structural Response Demand Analysis

To establish the probabilistic relationship between the ground motion intensity measure and the response of MDOF nonlinear systems, one can use method of random vibration

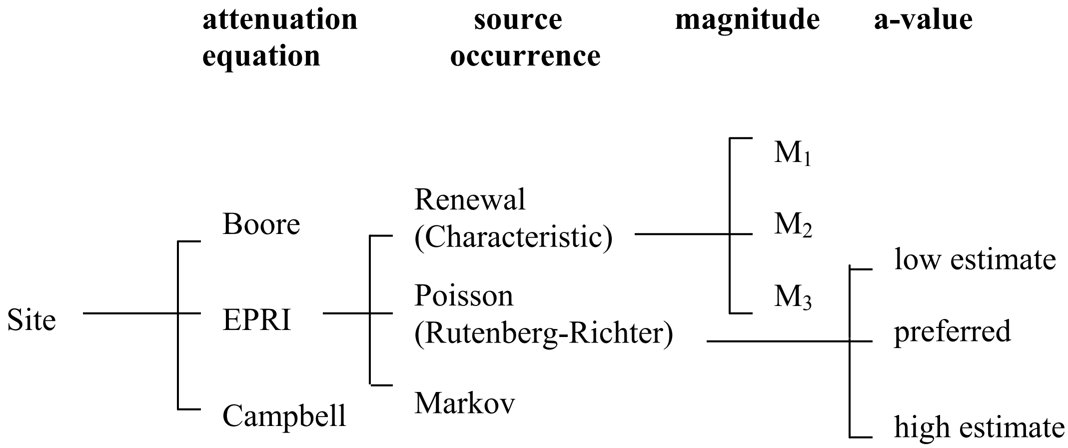

Figure 3.2. Logic tree in seismic hazard analysis

with the ground motion modeled as a random process. Alternatively one can use a regression analysis of nonlinear time history responses under recorded ground motions. Because of the inherent nonstationary nature of the random excitations in earthquakes and the analytical difficulty in modeling complex nonlinear member response behaviors such as brittle fracture in random vibration, the latter approach is more practical. To cover a wide range of excitation and structural response in the inelastic range, such ground motions are frequency scaled. Although the frequency content and duration of ground motions due to events of different magnitudes and distances are different and scaling may violate the basic physics of ground motions, results based on extensive investigations (Shome et al 1998) show that when intensity measure such as spectral acceleration is used in the scaling, the errors are small. The regression analyses therefore allow one to establish the functional relationship between the intensity measure and the structural demand variables such as global (roof) and local (interstory) drifts and energy dissipation (cumulative damage). In the following, the relationship between maximum interstory drift and spectral acceleration is used as an example. The method can be applied to other structural demands under a different intensity measure such as spectral displacement or bi-directional spectral displacement (Wang and Wen 2000).

Based on extensive regression analyses of response of steel structures, Cornell et al (2002) proposed that the maximum interstory drift can be expressed as a simple power function of the spectral acceleration:

$$D = c\,(S_a)^b \qquad (3.15)$$

Such relationship is necessarily approximate and there are large scatter around the regression line. The regression prediction is therefore the estimate of the mean demand conditional on a given value of the excitation intensity measure, $E(D|S_a = a)$. The scatter in terms of the coefficient of variation, $\beta_{D|S_a=a}$, also depends on the intensity but again often regarded as constant as an approximation. The structural response demand given the excitation intensity therefore can be described by a random variable of a given distribution. The lognormal distribution generally gives a good fit, which can be used to describe the randomness in structural demand variable due to ground motion record-to-record variation even though these ground motions are of the same S_a. The probability of the structural demand being exceeded in t years can therefore be evaluated by the total probability theorem to incorporate the contribution from all values of S_a.

$$P_t(D>d) = \int P(D > d|S_a = a)[-\frac{dH_t(a)}{da}]da \quad (3.16)$$

in which

$$P(D > d|S_a = a) = 1 - \Phi(\frac{\ln d - \lambda}{\beta_{D|S_a=a}})$$

$$\lambda = \ln[c(S_a)^b] - 0.5\beta^2_{D|S_a=a} = \ln\tilde{D}, \quad \tilde{D} = \text{median value of D}$$

Note that the calculation as shown in the above general analytical procedure could be quite involved and has to be carried out numerically. In code procedures and for a fast and approximate evaluation, closed form solution is desirable. It has been shown (Cornell et al 2002) that if the result of the seismic hazard analysis as given above can be approximately described by a power law

$$H_t(a) = k_0 a^{-k} \quad (3.17)$$

in which k_0 and k are the scale and decay coefficients. The above lognormal distribution assumption for the demand given excitation allows Eq. 3.16 to be evaluated in a closed form

$$P_t(D > d) = H_t(a^d) \exp[\frac{1}{2}\frac{k^2}{b^2}\beta^2_{D|S_a=a}] \quad (3.18)$$

in which a^d is the spectral acceleration level corresponding to the demand d according to Eq. 3.15. Eq. 3.16 or in more concise form Eq. 3.18 therefore describes a probabilistic structural response demand curve in which all the important randomness in excitation and structural response is considered. The first term is the demand curve without consideration of the randomness in the response-excitation relationship. The exponent function is the correction for this randomness. Note that the correction factor involves both the structural (b) and hazard (k) parameters. Note that Eq.3.17 is intended for approximating the tail distribution. It is no longer valid when the spectral acceleration is very small. The method is demonstrated by a simple numerical example as follows.

Consider a 3-story steel structural building with a fundamental period of 1 sec at a location where the 50-year seismic hazard and maximum interstory drift ratio as function of the spectral acceleration can be described by

$$H_{50}(a) = 0.0068\ a^{-3} \quad (3.19)$$

$$D = 0.06\ a^{1.2} \quad (3.20)$$

The hazard is such that the spectral acceleration at 1 sec is 0.4 g corresponding to an exceedance probability of 10% in 50 years and 0.7 g corresponding to 2% in 50 years, typical values for a site in the Los Angeles area. The building response is such that the maximum interstory drift ratio is 2% at a spectral acceleration of 0.4g and 4% at 0.7g, reasonable values for such steel building. Assuming the randomness in the drift ratio-spectral acceleration regression analysis $\beta_{D|S_a}$ = 0.3, the 50-year probabilistic maximum interstory drift demand curve is then according to Eq. 3.18,

$$P_{50}(D > d) = 0.0068\ [(\frac{d}{0.06})^{\frac{1}{1.2}}]^{-3} \exp[0.5(\frac{3}{1.2})^{2} 0.3^{2}] \quad (3.21)$$

The correction factor to account for the randomness in the demand as given in the exponential function is 1.33 and the 50-year demand curve is simplified to $0.009904(d/0.06)^{-2.5}$ and shown in Fig.3.3.

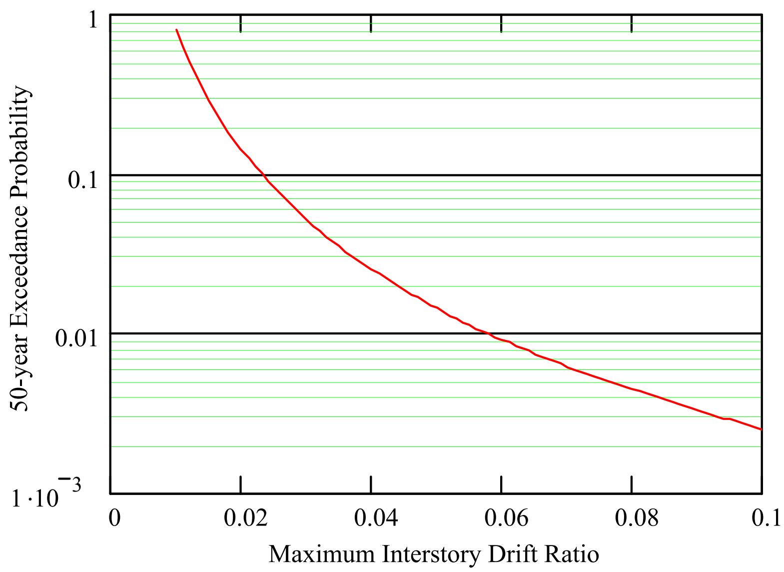

Figure 3.3. 50-year maximum interstory drift ratio demand curve

As demonstrated in previous sections, uncertainties either statistical in nature such as sampling errors or empirical in nature based on judgment could be important and have not been accounted for in the above formulation. These include uncertainties in the structural response analysis methods, choice of probability distributions and parameters and assumptions and approximations used in the source, path, and site parameters. Treatment and impact of these uncertainties will be covered more in details in Section 4.4 on probabilistic performance evaluation.

3.4 Simulation and Monte-Carlo Methods

The above formulation relies on accurate prediction of structural response by the excitation intensity measure. This may not be true for all structural systems under all possible future excitations. For example, the higher mode effects, near-source effects, and many detailed structural response behaviors cannot be predicted satisfactorily with any simple intensity measure since in reality the structure response is a function of the whole ground excitation time history. Any scalar intensity measure would fall short in predicting accurately the detailed structural response behavior. An entirely different approach to the evaluation of the probabilistic structural demand that uses the full ground motion time history is the Monte-Carlo method.

3.4.1 Monte-Carlo Method

Referring back to Figure 1, instead of using analytical method to tract the propagation of uncertainty as shown in the previous sections, one can imitate nature by generating random variables or random processes required in the chain of events according to the underlying models. It allows one to simulate the future ground excitations at the site and perform time history analyses of the structure to evaluate the response demand. In other words, one simulates the whole process from the source to structural response on computer and repeats the process for a large number of times. The probabilistic description of the structural demand is therefore done by statistical analysis of the structural responses from the large samples of simulation results. The Monte-Carlo (MC) method has been widely used in many disciplines and by many researchers in earthquake engineering. For example, Collins et al (1996) and Wu and Wen (2000) used available seismicity information in the region surrounding the Los Angeles and Santa Barbara areas for simulation of a large number of ground motions.

The advantages of the MC method are clearly conceptually straightforward and can be applied to systems that are complex and show nonlinear response behavior. It bypasses the need for identifying and justifying an intermediate intensity measure. All the propagation of uncertainty is captured automatically in the process including the epistemic uncertainty. For this purpose, the different source models and attenuation equations can be mixed in the simulation according to the logic-tree by selecting the model or equation with frequency according to their assigned likelihood (or probability). The main disadvantage is obviously the large numerical effort required, in particular the large number of ground motions and structural response time histories that need to be performed for an accurate estimate of the very low probability level of structural demand. Also, the uncertainties are all mixed together and it would be difficult to examine the effect of different sources of uncertainty. To keep the sampling error in terms of the coefficient of variation of the estimated demand exceedance probability, δ_{P_f}, within a given limit, δ_0, the sample size required is

$$N \approx \frac{1}{P_f \delta^2{}_0} \tag{3.22}$$

For example, if the annual P_f is 10^{-3} and the desired accuracy is $\delta_0 = 20\%$, the required sample size N is 25,000 simulated years of the structural performance. The computation problem therefore can easily get out of hand. This problem is especially serious if nonlinear and inelastic structural response behaviors including brittle member failure are to be considered in the structural systems. Another difficulty is a truly physical model for simulation of ground excitation should be based on wave propagation models, e.g. the broadband simulation method (Saikia and Somerville 1997) which would add significantly to the numerical effort. The validity of such method is another issue. The simulation method is therefore appropriate for special important systems where detailed description of the structural demand is needed and such large numerical effort can be justified.

3.4.2 Variance Reduction Techniques

The numerical burden in the Monte-Carlo method can be lessened via a smart simulation scheme. There have been earnest efforts of research in this area in the past decade and many numerical schemes have been developed by which the convergence of the MC method can be improved, sometime dramatically. Examples are various variance reduction techniques such as importance sampling and the adaptive sampling methods. A summary of some of the recent developments, for example, can be found in Schueller and Spanos (2001). In the importance sampling method, random variable realizations (sampling) are artificially concentrated at certain region that is most productive, i.e. causing structural limit states to occur. In the adaptive sampling method, simulation is conducted in stages where samplings in later stages are concentrated in more productive regions depending on the results of previous stages therefore is much more efficient. The problems with these methods are that either prior knowledge on the system behavior is needed in pinpointing the most productive sampling region which may not be available or reliable enough to prevent gross errors of sampling at the wrong location or that additional extensive analytical or numerical efforts are required that make the method unattractive in practical applications. For these reasons, these methods have not been widely used in probabilistic modeling in earthquake engineering.

3.4.3 Method of De-aggregation

An extreme form of the importance-sampling which has been used in earthquake engineering is the "de-aggregation" concept where only ground motions due to the earthquake of given M and R that contributes the most to the "event" under consideration are simulated. The event is generally the spectral acceleration corresponding to a given exceedance probability. Referring to Eqs. 3.12 to 3.14, it is seen that the probability of a given spectral acceleration being exceeded such as 2% or 10% in 50 years is a result of integration or aggregation of contribution from sources of different M and R, and with different attenuation A. If we de-aggregate the integral and identify the combination of M, R and A that contributes the most to the integral, then we can use the ground motion time histories of such an event to "represent" the seismic environment for performance evaluation. It is clear that ground motions produced by such method are only a very approximate representation of all possible ground motions that contribute to the event of interest. The approximation could be poor if there is no clearly dominant event. Also the de-aggregation result is dependent on the event of interest and the process has to be repeated each time a spectral accelerations of a different period and a different exceedance probability is considered.

3.4.4 SAC Ground Motion Procedure

In the recent SAC/FEMA effort (Somerville et al 1997), the selection of the ground excitations to match the spectral acceleration with a given probability of exceedance is extended for all periods. The starting point in this procedure is the UHRS corresponding to a given probability of exceedance such as 50%, 10 %, and 2 % in 50 years. Recorded and simulated ground motions based on a broadband procedure (Saikia and Somerville 1998) were then selected and scaled to obtain suits of ten ground motions whose median response spectra match the 5% damping UHRS approximately for a wide period range and a given probability level. This is done for Los Angeles, Seattle, and Boston. Since the selected ground motions are from events of different magnitudes and distances, even with properly chosen scaling factors, the scatter of the response spectra of the ground motion suite compared with the UHRS in terms of coefficient of variation is generally of the order of 30%. Such ground motion suites represent the excitation demand corresponding to a given probability of exceedance. They can be used for evaluation of demand on nonlinear and inelastic systems.

The median value of the structural response under the suite of ten ground motions can be used as the response demand on the structure corresponding to a probability of exceedance of 50%, 10%, or 2% in 50 years. Note that unlike the seismic hazard analysis described in Eqs. 3.12 to 3.14 where regional seismicity is used, the ground motions selected in this procedure may come from a seismicity environment that is totally different from that of the site. Therefore strictly speaking, the ground motions generated in this procedure may not represent any possible future events in this region. However, since after the scaling, their response spectra match those which were obtained using regional seismicity and spectral acceleration is a good predictor of structural response there are good reasons to believe that these ground motions would produce structural response close to those caused by future ground motions at the site. The SAC ground motions have been used extensively in the SAC Steel Project in performance evaluation and recommendation of reliability-based design procedures.

3.4.5 Smart Simulation and Uniform-Hazard Ground Motions (UHGM)

A smart simulation procedure for generating uniform-hazard ground motions (UHGM) similar to the SAC/FEMA procedure was recently proposed by Wen and Wu (2001) for mid-America cities. Although moderate to large events occurred in the past in mid-America including the three large events in 1812 to 1813 in New Madrid, Tennessee, records that can be of engineering interest are scarce therefore simulation is the only means for producing ground motion time histories for performance evaluation and design. The procedure consists of three stages, the first stage is the same as the Monte-Carlo method described previously and used in Collins et al (1996). Future events were generated in the region of three cities (Memphis, TN, St Louis, MI, and Carbondale IL) based on available regional seismicity information, latest attenuation and random-vibration based ground motion models for the region. The point source model by Atkinson and Boore (1995) was used for non-characteristic events and finite-fault model by Beresnev and Atkinson (1998) was used for characteristic events in the New Madrid seismic zone. A Poisson process is assumed for all events. The effect of site soil condition was modeled by the quarter-wavelength model of Boore and Joyner (1991). A large number of events and ground motions equivalent to 90,000 years of records were generated. The second stage is processing the simulated ground motions to construct the UHRS for each city for both soil and rock sites corresponding to exceedance probabilities of 10% and 2 % in 50 years. It essentially achieves the same goal of the seismic hazard analysis in the previous section from an entirely different approach. The results of the UHRS compared well with those of USGS National Earthquake Hazard Mapping project. The third stage is selection of uniform-hazard ground motions from the pool of large simulated ground motions such that the response spectra of the selected ground motions match UHRS for all period range in a least square sense. These ground motions represent the future ground excitation corresponding to probabilities of exceedance of 10% and 2 % in 50 years. They come from events of different magnitudes and distances with different attenuations. Conceptually, the procedure is similar to the de-aggregation method except the matching is done for all periods and the there are ten contributing events of various magnitudes and distances. The matching is similar to that in SAC/FEMA ground motions procedure except the selected events are all possible future events in the region. Fig. 3.4 shows the contributing events for the three cities for different hazard levels. Note that at the 10/50 level, future contributing events are mostly close and small or distant and larges. At the 2/50 level, almost all are characteristic events from the New Madrid seismic zone.

It has been shown in Wen and Wu (2001) that the median value of the response spectra of the uniform hazard ground motions match the target UHRS closely (within 10% for a wide period range), and the median inelastic response spectra using the ten uniform-hazard ground motions also match those based on 9000 ground motion record closely. It indicates that the median value of the structural response to the ten uniform hazard ground motions is an accurate estimate of the structural response demand for both linear and nonlinear systems. Fig. 3.5 shows a structural response (column drift ratio %) demand curve for a two-story steel moment frame at Carbondale, Illinois before and after retrofit using the uniform-hazard ground motions. A lognormal assumption is used for the demand curve. One therefore achieves the same goal of determining the probabilistic demand curve such as that shown in Fig. 3.3 based on Eq. 3.21, but from an entirely different approach.

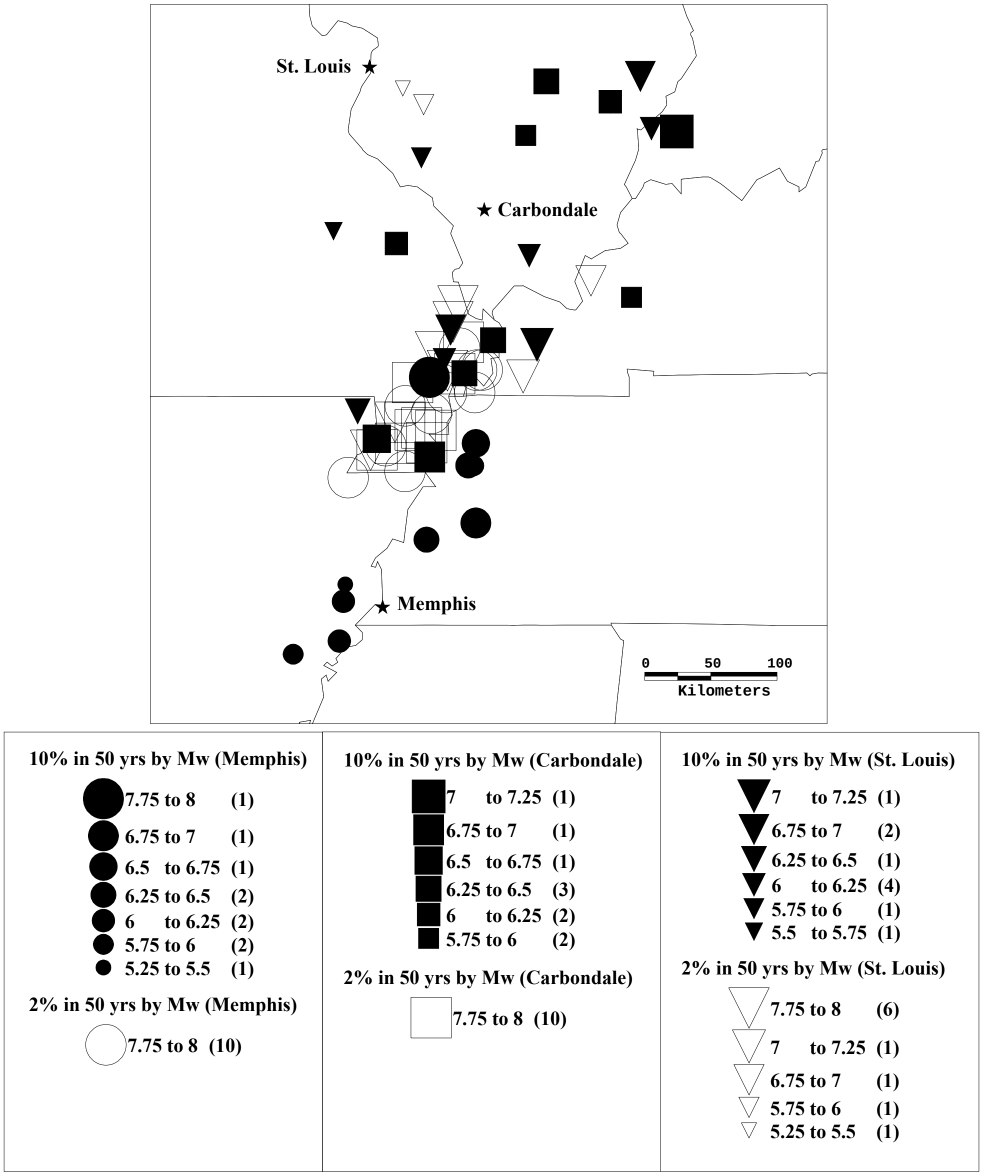

Figure 3.4. Epicenters and magnitudes of events contributing to uniform hazard ground motions for Memphis, Carbondale and St. Louis (number of events in the magnitude range is shown in the parentheses) (Wen and Wu 2001).

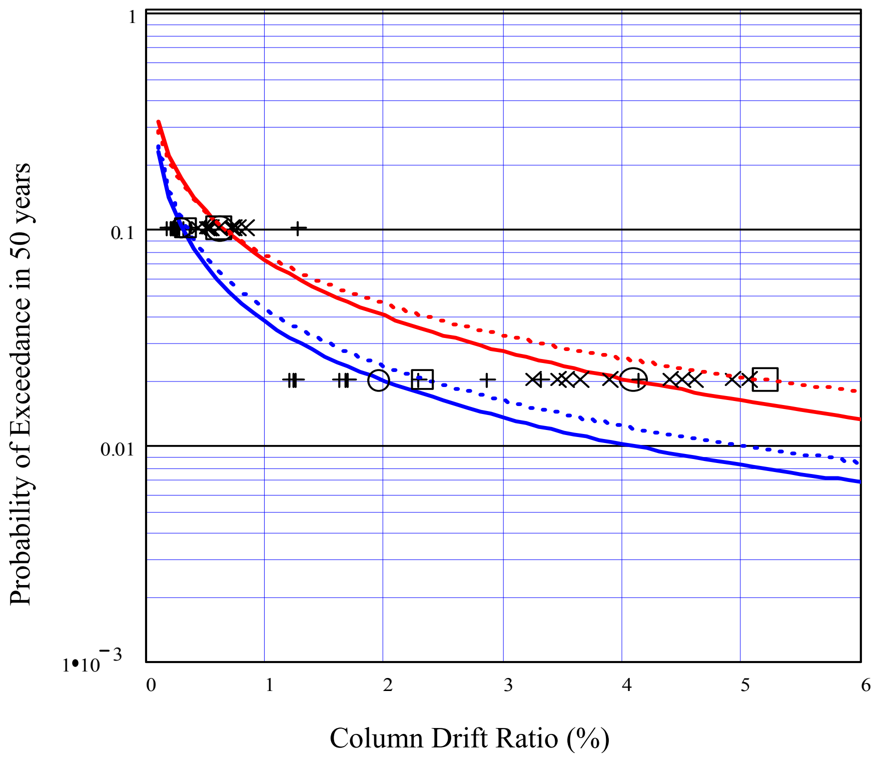

Figure 3.5. Probabilistic column drift ratio demand curve of a two-story steel moment frame building at Carbondale, IL before and after retrofit with shear walls (data points before retrofit [×], after retrofit [+], median value (o), and median including structural capacity uncertainty [□]; dashed and solid lines indicate performance curve with and without consideration of capacity uncertainty) (Wen and Wu 2001).

3.5 Demand on Spatially Extended Systems

The treatment of demand uncertainty as mentioned in the foregoing is suitable for structures or systems that do not occupy a large area and can be approximated as a point system. For spatially extended systems such as long bridges, building stock and facilities of a community or a city, and transportation networks, additional considerations on the seismic demand are needed. The demand on the systems obviously depends on the spatial correlation of the seismic excitation. As indicated in Section 2.4.2, an independence assumption here could lead to serious underestimate of the uncertainty of the demand since the positive correlation contributes significantly when taking summation of random quantities which is basically the situation we face here.

Under the assumption that the excitation can be modeled by a stationary random process, the spatial correlation of random seismic waves at two different points during a given event can be described by a coherence function. The coherence function describes the correlation or dependence between two random processes, somewhat like the correlation coefficient between two random variables. It is a function of the separation of two points, apparent wave propagation velocity, and frequency. It can be determined from array records of past earthquakes. It allows one to model the correlation of excitations at any two points of the system and hence response of a spatially extended system via a random vibration analysis or simulation (e.g. Samaras et al 1983). As seismic excitation frequency content and intensity clearly vary with time a stationary process treatment is obviously an approximation. Another correlation that needs to be carefully considered is the event-dependent intensity correlation. In other words, during an large event, the excitation intensity would be high for an extended area even the excitations may not be highly correlated in time when the separation is large. Similarly, during a small event, the intensity would be low for an extended area. This correlation would play a significant role in total demand uncertainty.

To include these correlations of demand on spatially extended systems, an event-based or scenario-type approach seems to be most suitable. The demand on the system can be described in terms of events of given magnitudes and distances. The deterministic scenario earthquake approach such as the worst-case scenario has been commonly used in the past. Such an event of course has extremely small probability of occurrence. It does not give a true picture of the future seismic demand on the system and provides little information for long-term planning based on cost versus benefit. For example, it may be the moderate and frequent events that cause the most damage/ cost to the systems.

To consider all possible demands, simulation of future events can be used but is computationally impractical. To reduce computation cost, a method similar to the smart simulation method used in generating the uniform hazard ground motions (see Section 3.4) looks promising. Events of various magnitudes and distances are first generated according to the regional seismicity and screened for each hazard level using the uniform hazard response spectra for the general location. The response spectra may be the averaged value of different sites if the system occupies a very large area such as a transportation network. As the response spectra are a good measure of demand on systems of wide range of frequency, the limited number of uniform hazard events after the screening (such as those given in Fig 3.4) would be representative of the future seismic demand on the spatially extended systems in a particular location. Given the magnitude and distance of such an event, the system response to the event can be then evaluated using the most suitable method to incorporate the effect of spatially correlation. For example, a coherence function matrix may be used in simulation of ground motions at different points. Alternatively, a physical wave propagation type of approach such as the broadband procedure (Saikia and Somerville 1998) can be used for this purpose. The intensity spatial correlation may be accounted for via a simple procedure such as an intensity map for the given event. The median value of the system response to all events of the same hazard level will have a probability corresponding to the hazard level, from which the system demand curve similar to Fig 3.5 can be obtained. The spatial correlation can be therefore properly accounted for.

3.6 List of Symbols and Notations

Aleatory uncertainty (or randomness): inherent variability that is irreducible by additional knowledge, information, or data.

Epistemic uncertainty (or uncertainty): modeling errors that are reducible with additional knowledge, information, or data.

Fragility function: conditional probability of limit state given the excitation or displacement demand.

Standard normal variate: a random variable with a standard normal distribution with zero mean and unit standard deviation.

Uniform hazard response spectrum (UHRS): response spectrum corresponding to a prescribed probability of exceedance such as 10% in 50 years.

Uniform hazard ground motions (UHGM): ground motions whose response spectra match the uniform hazard response spectra.

β : dispersion parameter in a lognormal distribution = $\sqrt{\ln(1+\delta^2)}$

β : reliability (or safety) index

C: structural member or system capacity against a prescribed limit state

D: displacement demand variable, e.g., interstory drift ratio, global drift ratio

δ : coefficient of variation

$f_X(x)$: probability density function of random variable X

Φ : cumulative distribution of a standard normal variate

Φ^{-1}: inverse function of Φ.

$H_t(a)$: seismic hazard function; probability of spectral acceleration exceeding “a” in “t” years

λ : scale parameter in a lognormal distribution = $\ln\mu - 0.5\beta^2$

M: earthquake magnitude

μ : mean value

ν : mean occurrence rate of a Poisson process

R: epicentral distance

σ : standard deviation

S_a: spectral acceleration

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4. PERFORMANCE LEVEL IDENTIFICATION

Performance levels or limit states for both structural and nonstructural systems are defined in this document as the point in which the system is no longer capable of satisfying a desired function. There are many types of performance levels in the field of earthquake engineering. In addition, performance levels can be identified by qualitative and quantitative approaches. Below, both methods are summarized.

4.1 Traditional Qualitative Approaches

Qualitative approaches for identification of system performance levels have traditionally been used in building codes. In particular, most building codes require designers to ensure life safety of the occupants during factored loading (ultimate strength design or load and resistance factored design) and serviceability or functionality during unfactored loading. Some additional examples of performance levels or damage levels are given in: SEAOC (1995) such as fully operational (negligible damage), operational (light damage), life safety (moderate damage), near collapse (severe damage), and collapse (complete damage); and in FEMA/SAC (2000) such as immediate occupancy and collapse prevention.

FEMA 273 (1997), and its update FEMA 356 (2000), has probably the most comprehensive documentation on performance levels that are defined qualitatively. Performance levels for structural systems, nonstructural systems, and overall building systems are briefly summarized below:

FEMA 273 (1997)/356 (2000) define discrete *structural performance levels* as:

(1) ***Immediate Occupancy*** - occupants are allowed immediate access into the structure following the earthquake and the pre-earthquake design strength and stiffness are retained;
(2) ***Life Safety*** - building occupants are protected from loss of life with some margin against the onset of partial or total structural collapse; and
(3) ***Collapse Prevention*** – building continues to support gravity loading, but retains no margin against collapse.

In addition to the discrete structural performance levels, FEMA 273 (1997)/356 (2002) also define *structural performance ranges* such as:

(1) ***Damage Control Range*** – Range of structural damage between immediate occupancy and life safety; and
(2) ***Limited Safety Range*** – Range of structural damage between life safety and collapse prevention;

FEMA 273 (1997)/356 (2000) also defines *nonstructural performance levels* as:

(1) ***Operational*** - nonstructural components are able to function as prior to the earthquake;
(2) ***Immediate Occupancy*** – building access and life safety systems generally remain available and operable;
(3) ***Life Safety*** – nonstructural damage that is not life threatening; and
(4) ***Hazards Reduced*** - damage that includes potentially falling hazards, but high hazard components are secured and will not fall. Preservation of egress, fire suppression systems, and other life safety issues are not ensured;

In terms of identifying the overall *building performance level* FEMA 273 (1997)/356 (2000) combines both the structural and nonstructural performance levels. Several possible combinations are provided in a matrix format, in addition to identifying those that are likely to be selected as a basis for design. Several of the typical *building performance levels* are:

(1) ***Operational*** – sustain minimal or no damage to the structural and nonstructural components, and the building is immediately suitable for normal use;
(2) ***Immediate Occupancy*** - sustain minimal or no damage to the structural elements and only minor damage to the nonstructural components. Although immediate reoccupancy may be possible, some clean-up, repair, and restoration of service utilities may be necessary before the building can function as normal;
(3) ***Life Safety*** – sustain extensive damage to the structural and nonstructural components, and be in need of repairs before reoccupancy. Repairs may also be deemed economically impractical;
(4) ***Collapse Prevention*** – consists of the structural collapse prevention level with no consideration of nonstructural vulnerabilities. The building may pose a significant hazard to life safety and be deemed as a complete economic loss.

It is important to note that these traditional performance level definitions are based on ***qualitative*** definitions. Engineers and researchers have long utilized the maximum inter-story deformations (drifts) during earthquakes to correlate with the previously described levels of structural performance. For illustration purposes, FEMA 273 (1997)/356 (2000) presents inter-story drift values that are typical for each structural performance level for the different types of structural systems. For example in reinforced concrete frame structures, inter-story deformations of 1%, 2%, and 4% of the story height may be acceptable for immediate occupancy, life safety, and collapse prevention, respectively. However, it is clear that these deformation limits will depend on a variety of variables that include: degree of section confinement and detailing in potential plastic hinge zones; level of column axial load and second-order P-delta effects; nonstructural participation; redistribution of forces; and pre-existing damage. For nonstructural performance levels, story accelerations or forces may be more representative since many of these components have brittle behavior and are sensitive to the applied force level.

4.2 Quantitative Approaches

Although current building codes and state-of-the-art publications have attempted to define the various performance levels for structural and nonstructural systems, performance levels have only been identified qualitatively. Therefore, designers have to determine quantitative response limits (either displacements or forces) for both the members of the system and the overall system that correspond to the qualitative code descriptions. Another approach for defining structural performance levels might be based on quantitative procedures using nonlinear pushover techniques and incremental dynamic analyses (described later in Section 5.3.2). These quantitative performance levels can be utilized by the designer and judged to supersede the qualitative performance levels in current building codes.

Example structural system performance levels for framed structures that can be identified analytically using nonlinear pushover procedures (Dooley and Bracci, 2001) are:

(1) ***First Yield (FY)*** – Inter-story deformation at which a member of a story or of a structures initiates yielding under an imposed lateral loading;

(2) ***Plastic Mechanism Initiation (PMI)*** – Inter-story deformation at which a story mechanism (typical of a column sidesway mechanism), an overall beam sidesway mechanism, or a hybrid mechanism initiates under an imposed lateral loading; **and**

(3) ***Strength Degradation (SD)*** – Inter-story deformation at which the story strength (resistance) has degraded by more than a certain percent of the maximum strength (usually about 20 percent). Note that strength degradation can occur due to material nonlinearities in the analytical models and also due to geometric nonlinearities from P-delta effect.

For example, consider the portal frame in Fig. 4.1. Under imposed lateral force F, the story shear force versus inter-story deformation Δ can be calculated using nonlinear pushover techniques. A hypothetical representation is shown in Fig. 4.1b. The FY performance level corresponds to an inter-story deformation at first member section yielding, shown at the base of the columns in Fig. 4.1a, which in this case formed during the same analysis step. The PMI performance level subsequently occurs after both ends of the beam yield. It is important to note that the sequence and form of member yielding during applied loading prior to the mechanism formation. Both can have significant effects on the levels of structural deformability and overstrength (capacity) in building structures. Also important to note is that the PMI performance level may or may not be the point at which collapse is eminent. This will obviously depend on the degree of section confinement and detailing, second-order P-delta effects, nonstructural damage, redistribution of forces, and pre-existing damage. Finally, the SD performance level corresponds to an inter-story deformation when the story strength has degraded by more than 20 percent.

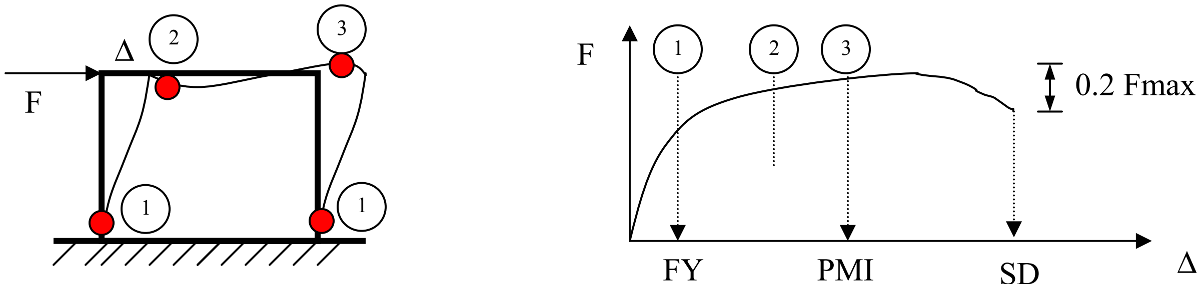
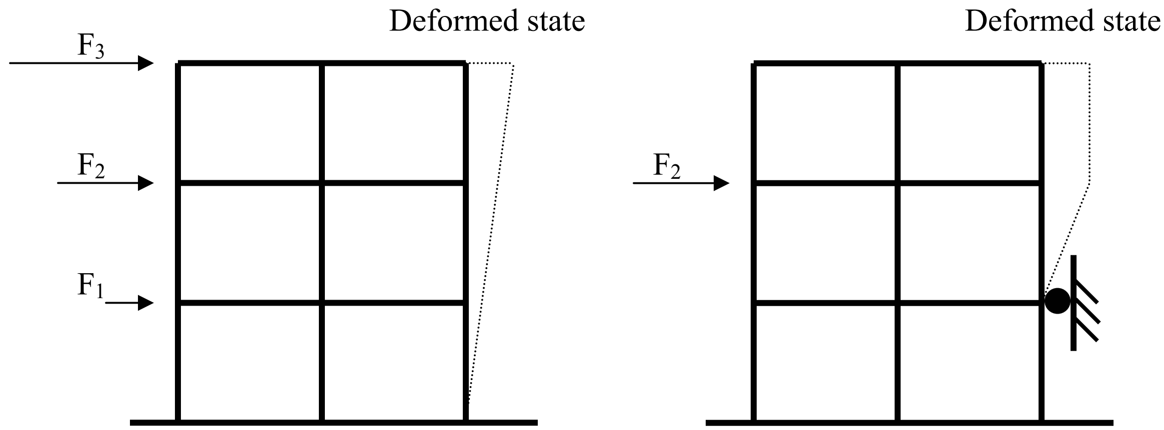


Figure 4.1. Pushover Analysis and Yield Formation

The previous example demonstrates the identification of various quantitative performance levels, such as First Yield, Plastic Mechanism Initiation, and Strength Degradation. They may be judged by an engineer to be representative of the qualitative definitions presented previously, such as Immediate Occupancy, Life Safety, and Collapse Prevention, respectively.

A key input parameter required in identifying such quantitative performance levels is the imposed lateral loading or deformations. The previous example was a simple portal frame, which is primarily a single-degree-of-freedom system. For multi-story frame structures with many degrees-of-freedom, building codes have traditionally required lateral loading distributions for earthquake analysis that are primarily proportional to the fundamental model shape of the structure (or similar to the inverted triangular loading). Since multi-story buildings are susceptible to higher mode response and impulse-type loading scenarios during earthquakes, loading patterns that have been typically used for determining structural demands as in current building codes may not be appropriate for identifying quantitative performance levels, which are ***capacities***. As such, the imposed lateral loading or deformation should be consistent with those that have a critical consequence. Fig. 2 shows the deformation pattern in a framed structure during inverted triangular lateral loading and during loading that might be critical for the second story of the building. Fig. 2a shows that the story deformations are uniformly distributed throughout the building, where Fig. 2b shows that the deformations on the second floor would impose a more critical story mechanism for the second floor.

From nonlinear pushover analyses, a hypothetical comparison of the story shear vs. inter-story drift responses for the two different loading scenarios is shown in Fig. 3. The figure shows that the previously described FY and PMI performance levels may depend on the imposed lateral loading or deformation patterns.



(a) Inverted Triangular Loading /1st Mode Response (b) Critical 2nd Story Response

Figure 4.2 Example Loading Patterns for Pushover Analysis

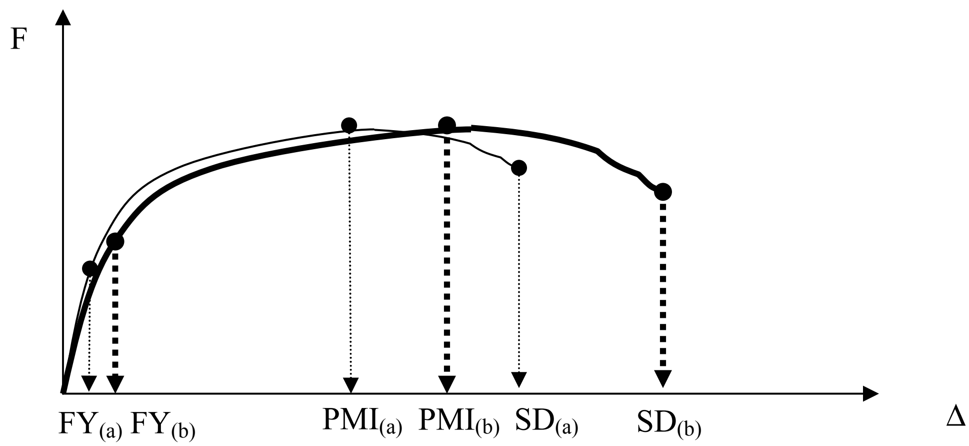


Figure 4.3. Story shear vs. inter-story drift in pushover analysis

4.3 Uncertainty Identification and Quantitative

Identifying the sources of and quantifying the magnitude of the uncertainty for the various performance levels, whether using the qualitatively or quantitative methods described previously, is briefly described in this section and is described in more detail in Section 5.3.

The sources of both aleatory and epistemic uncertainty in identifying the various performance levels for the structural system, nonstructural system, and the overall building system are many. In terms of the quantitative methods for determining performance limits, some of the sources of aleatory uncertainty include the variability in material properties for both structural strength and stiffness, degree of section confinement and detailing for ductility, construction errors, etc. Some of the sources of epistemic uncertainty include the nonlinear models for the variety of structural materials and components, imposed loading distribution to identify critical response, effects of using monotonic response to represent capacity when actual earthquake demand is cyclic in nature, differences in analytical programs, etc.

4.4 List of Symbols and Notations

FY: Inter-story deformation at which a member of a story or of a structures initiates yielding under an imposed lateral loading.

PMI: Inter-story deformation at which a story mechanism (typical of a column sidesway mechanism), an overall beam sidesway mechanism, or a hybrid mechanism initiates under an imposed lateral loading.

SD: Inter-story deformation at which the story strength (resistance) has degraded by more than a certain percent of the maximum strength (usually about 20 percent).

4.5 References

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5. SYSTEM CAPACITY AND PERFORMANCE EVALUATION

Structural capacity, as defined in Section 3.3, is the maximum force, displacement, velocity, or acceleration that a member or a system can withstand without failure, or more specifically, without reaching a prescribed limit state. The capacity is therefore dependent on the material properties, member dimensions, system configuration, the limit state under consideration, and methods and models used in describing the capacity. As in the case of demand, both (aleatory) randomness and (epistemic) uncertainty are important elements in the evaluation of capacity and need to be carefully considered. In the following the capacity uncertainty and probabilistic treatments are described of first the construction materials, and then structural members and finally structural systems. Since capacity is always related to the limit state under consideration, some of the more frequently used limit states in performance evaluation will also be discussed. Again there is a large body of literature on this subject; emphasis here is on the uncertainty treatment.

5.1 Material Properties

The member and system capacity depend directly on the material strength which is inherently random. The randomness can be modeled by random variable based on test data. It is common to use the first two moments, i.e. the mean and standard deviation (or coefficient of variation), to describe the central value and the variability. Normal or lognormal distribution is commonly used for convenience. The actual strength of the material of a given member may differ significantly from the nominal values used in member capacity calculation. The correspondence between the nominal value and the actual value therefore needs to be established to estimate the real member capacity. The strength variability obviously depends on the material, manufacturing process, and sometimes the testing protocol. In general, the variability in masonry and timber construction material is larger than those in reinforced concrete and steel. Material property variability and test data up to 1980 can be found in the report by Ellingwood et al. (1980). For example, the coefficient of variation of strength of timber varies in the range from 10% to 30% depending on species and in flexure or compression; and that of masonry walls from 10% to 26% depending on configuration and in compression or flexure. The coefficient of variation of compressive and tensile strength of concrete is around 18% and that of the yielding strength of steel reinforcement and steel member elements is around 10% or less. Properties of construction material such as concrete and structural steel evolve over time. Strength statistics of newer material such as high-strength steel and concrete may be found in more recent literature. For example, statistics on yield and ultimate strength of structural steel under various environmental conditions can be found in the recent FEMA/SAC report (2000).

5.2 Uncertainty in Member Capacity

5.2.1 Member Capacity under Monotonic Load

The inherent randomness in the material property carries over to the structural members made of these construction materials. In addition, there is randomness in the dimensions of the members and also the capacity refers to a particular limit state such as shear, bending, or buckling failure under monotonic or cyclic loading condition. The randomness in terms of the bias (mean capacity/nominal capacity) and coefficient of variation of steel, reinforced concrete, masonry and glulam structural members (beams, columns, and walls) of various configurations and for various limit states can be found in Ellingwood et al. (1980). The majority of the bias factor is between 1.0 and 1.2 and the coefficient of variation under 20%. Normal or lognormal distribution has been used to model the capacity randomness. The difference between the two models is small when the coefficient of variation is small.

5.2.2 Members Capacity under Cyclic Load- Damage Index

For seismic loading, one is specially interested in the member capacity under cyclic loading since members in a structural system generally undergo stress reversals of various amplitudes and the member may reach a limit state under combined action of large deflection and cumulative damage. To account for both effects, various damage indices have been proposed. The most widely used is the index developed by Park and Ang (1985) based on test results of 403 reinforced concrete members. The index is a linear function of maximum displacement δ_m and total hysteretic energy dissipation normalized by member ultimate displacement and monotonic loading δ_u, and yield force Q_y.

$$D = \frac{\delta_m}{\delta_u} + \frac{\beta}{Q_y \delta_u} \int dE \qquad (5.1)$$

Different value of the index corresponds to different limit states such as 0.4 for serious damage and 1 corresponding to complete damage (collapse). Test data show that the damage index capacity of reinforced concreted member can be modeled by a lognormal random variable with a mean value equal to 1.0 and a coefficient of variation of 0.53. It indicates that the randomness in the reinforced concrete member capacity is quite large. The index has been used in damage evaluation of buildings and other structures, e.g. Park et al. (1985).

5.2.3 Rotation Capacity of Steel Connection Members

An important structural member in steel buildings is the connections between beams and columns. After the large number of brittle fracture failures found in many buildings due to1994 Northridge earthquake, the capacity of connections against rotation demand under cyclic loading during earthquake excitations has attracted much attention of the structural profession. In the FEMA/SAC project (SAC 2000), a comprehensive testing program of a large number (120) of welded and bolted connections of various configurations has been carried out according to pre-Northridge practice and for post-Northridge design in which many different improvements were incorporated. Test results of hundreds of experiments prior to 1994 were also analyzed. The connection rotation capacities for both pre- and post-Northridge connections were obtained. The capacity is defined in accordance with two limit states; θ_p the rotation limit when plastic deformation occurs and θ_g, the rotation limit corresponding to severe damage that the gravity load carrying capacity of the member is compromised. Test data generally show the dependence on the depth of the beam or the depth of the connection element of these capacities and large scatter. The mean values and standard deviations as linear functions of the depth of the beams were established from regression analyses of test results. Depending on the specific connection type and the depth of the beam, the rotation capacity and variability in terms of these two statistics shows large variation. For example, the capacity of the post-Northridge welded-flange-bolted-web connections has the following means and standard deviations,

$$\mu_{\theta_p} = 0.021 - 0.0003 d_b \tag{5.2}$$

$$\sigma_{\theta_p} = 0.012 - 0.0004 d_b$$

and

$$\mu_{\theta_g} = 0.050 - 0.0006 d_b \tag{5.3}$$

$$\sigma_{\theta_g} = 0.011 + 0.0004 d_b$$

For such a connection with a beam depth of 24 inches, $\mu_{\theta_p} = 0.0138$, and a standard deviation $\sigma_{\theta_p} = 0.0024$ (or a coefficient of variation $\delta_{\theta_p} = 17.4\%$); and $\mu_{\theta_g} = 0.0256$ and a standard deviation $\sigma_{\theta_g} = 0.00206$ (δ_{θ_g} = 8%). The variability is moderate. For a free-flange and welded web connection with a beam depth of 36 inches, the regression results give $\mu_{\theta_p} = 0.0238$ and $\sigma_{\theta_p} = 0.0032$ ($\delta_{\theta_p} = 134\%$); and $\mu_{\theta_g} = 0.0364$ and $\sigma_{\theta_g} = 0.0604$ (δ_{θ_g} =166%). The variability is very large. Such large variation of coefficient of variation for different connections could partly due to the small number of samples used in the regression analysis. No distribution models were recommended for the capacity. In view of the small sample size and large coefficient of the variation, selection of the distribution model should be done with care. Note that with a distribution model, say a normal distribution, one can predict the probability of limit state of plastic deformation or loss of gravity load carrying capacity of the connection member when the rotation demand θ_d is known

$$\text{P(plastic deformation)} = P(\theta_p < \theta_d) = \Phi\left(\frac{\theta_d - \mu_{\theta_p}}{\sigma_{\theta_p}}\right) \tag{5.4}$$

$$\text{P(loss of gravity load carry capacity)} = P(\theta_g < \theta_d) = \Phi\left(\frac{\theta_d - \mu_{\theta_g}}{\sigma_{\theta_g}}\right) \tag{5.5}$$

Probability of capacity being exceeded given the demand is also generally referred to as the fragility function. Since under earthquake excitations the demand is also a random variable, the limit state probability can be evaluated based on a reliability analysis as shown in the next section.

5.2.4 Bayesian Models of Member Capacity

When calculating member capacity against a prescribed limit state, mathematical models based on mechanics are used. In all mathematical models, there are errors associated with the assumptions and approximations of such model that it needs to be calibrated against experimental results or field observations. Rigorous tracking of the uncertainty in the mathematical model based on our prior knowledge of the mechanical behavior of the components and calibrating the model against experimental data can be done via the Bayesian statistical method (Section 2.2.1). Such a models have been recently developed for structural members by researchers (e.g., Gardoni et al. 2001, Sasani et al. 2001). The basic concept behind this method can be illustrated by a simple example as follows. Consider a structural member model predicting the member capacity, y, against a prescribed limit state by the following equation

$$y = g\,(\boldsymbol{\theta}; \mathbf{x}\,) + \varepsilon \qquad (5.6)$$

in which $\boldsymbol{\theta} = \theta_1, \theta_2, \ldots.\theta_k$ denotes the set of the parameters for the mathematical model; $\mathbf{x} = x_1, x_2, \ldots. x_n$ represents the sample values of y from experimental or field observations; ε is a random variable representing the unknown errors in the model assumed to follow a normal distribution. Within the context of such formulation, given the parameters $\boldsymbol{\theta}$, y is a normal random variable. Calibrating of the model parameters in view of observational evidence is formulated by regarding the model parameters as random variables governed by distributions based on prior knowledge (such as mechanics principles, structural analysis methods, and engineering judgment/experience). The parameters are calibrated (or updated in Bayesian terminology) in view of sample evidence of y as follows:

$$f''(\boldsymbol{\theta}) = k\, L(\boldsymbol{\theta})\, f'(\boldsymbol{\theta}) \qquad (5.7)$$

in which $f'(\boldsymbol{\theta})$ = the prior distribution of the model parameters; $L(\boldsymbol{\theta})$ is the sample likelihood function or the conditional probability of observing $\mathbf{x}$ given $\boldsymbol{\theta}$; and k is the normalizing constant. Note that the epistemic uncertainty such as those associated with knowledge and modeling errors including those due to small sampling (small n) is included in the formulation. In general, for small n and sharp $f'(\boldsymbol{\theta})$, i.e., strong knowledge based information and weak observational information, the prior distribution dominates. On the other hand, if $f'(\boldsymbol{\theta})$ is flat or diffuse and n is large, then $L(\boldsymbol{\theta})$ dominates. The overall uncertainty in the posterior distribution $f''(\boldsymbol{\theta})$ is less than either $f'(\boldsymbol{\theta})$ or $L(\boldsymbol{\theta})$. One of the advantage of the Bayesian method is that even incomplete data of $\mathbf{x}$ such as those in the form of upper or lower bound due to certainty in the data collecting process or experimental procedure can be incorporated into $L(\boldsymbol{\theta})$ without difficulty. The method has been applied to evaluation of the capacity of circular reinforced concrete bridge columns and shear walls against deformation and shear demand due to cyclic loads. The advantage of this model compared with deterministic models was also shown. Fig.5.1 shows the result of the probabilistic prediction of the capacity of RC column against drift ratio demand, i.e. the conditional probability of failure of the column given that it reaches a drift ratio (or fragility curve).

5.3 Uncertainty in System Capacity

The description of uncertainty in system capacity is more involved since a structural system consists of many components and the system behavior is complex under dynamic excitation, especially when the system goes into nonlinear range. The system capacity can be therefore more conveniently described in terms of the system limit states of interest.

5.3.1 System Capacity against Damage

Commonly used system limit states are those corresponding to different damage states and performance levels. For example in SEOAC Vision 2000 (1995), there are five performance (damage) levels, fully operational (negligible), operational (light), life safe (moderate), near collapse (severe), and collapse (complete) and each level is related to a structural response level indicated by a transient and a permanent drift limit. In the FEMA/SAC project for steel buildings, the performance/damage levels were reduced to a more manageable two, immediate occupancy and collapse prevention. The system capacity is again described in terms of inter-story drift angles. The uncertainty in the

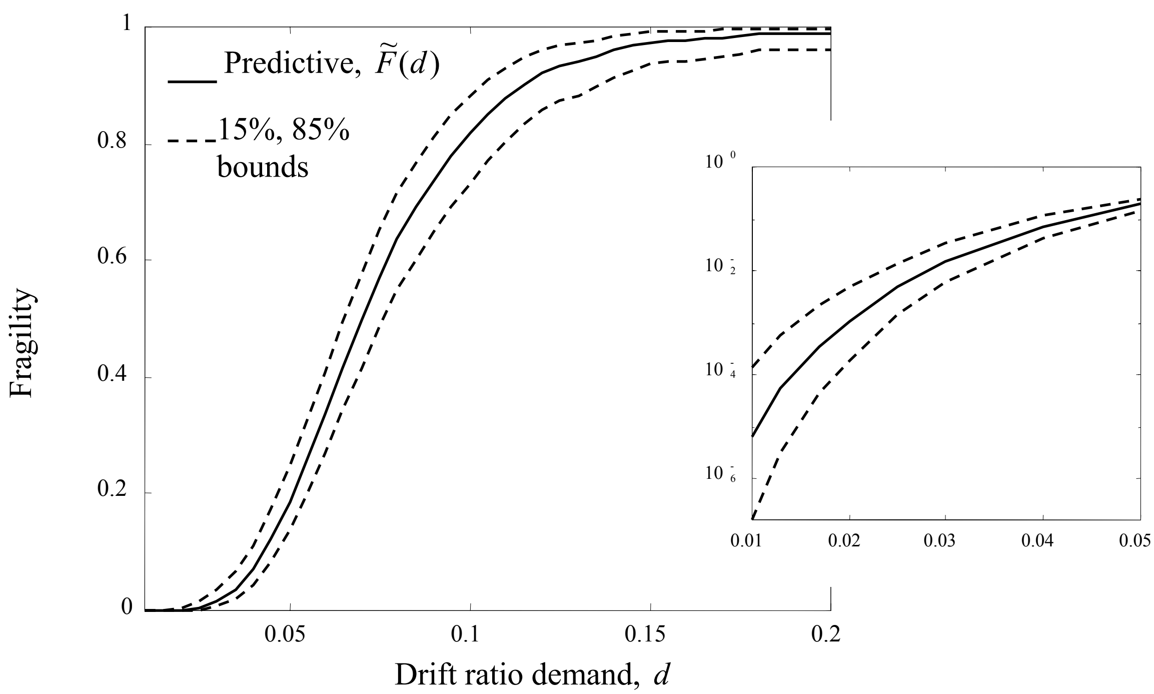

Figure 5.1. Probabilistic Capacity of RC Column against Drift (Gardoni et al. 2002) (The Inserted Small Figure Shows the Fragility for d < 0.05 in log scale)

system capacity therefore can be described in terms of the drift capacity for different performance levels, such as the median drift capacity and its coefficient of variation. The commonly accepted distribution for the capacity is the lognormal distribution for its convenience in reliability analysis and reliability-based design as will be seen in the next section. Structures of different construction material, configurations, and designs would have different drift thresholds. Determination of drift capacities for different performance levels is largely a process of combination of analysis and judgment/experience. The determination of system collapse prevention capacity is discussed further in the following.

5.3.2 System Capacity against Collapse-Incremental Dynamic Analysis

Of all the limit states and the corresponding system capacities, system collapse is the most difficult to determine. The reason is obvious that the structural dynamics close to collapse is extremely complex and is still largely an unsolved problem due to nonlinear member and system response behaviors. The large record-to-record variation of ground motions and structural response behaviors further complicate the matter. Collapse of structures under random excitations is a difficult mathematical problem of stochastic stability. Solutions can be obtained only for simple idealized systems under excitations of simple stochastic processes such as white noise. Engineers have used an inelastic static pushover analysis in the past to estimate this capacity. It provides insight into the structural response behavior at large displacement but considers the first mode static response only, which is basically different from dynamic response. As a result, such analysis generally over-predicts the response and underestimates the capacity. Improvements can be made by considering higher modes via modal pushover analysis as shown by Goel and Chopra (2000). Vamvatsikos and Cornell (2001) extended the concept of pushover analysis to dynamic response in the form of incremental dynamic analysis (IDA). The system capacity against collapse is evaluated by dynamic response analyses of the system under a suite of ground motion time histories such as the SAC ground motions. Each time history is scaled according to the spectral acceleration such that the structural response goes from linear elastic range to nonlinear inelastic and finally becomes unstable, i.e. a large increase in response with a small increase in the spectral acceleration. The displacement at the transition point is defined as the system displacement capacity against collapse. As mentioned earlier, due to the large record-to-record variation of the ground motions and extremely complex structural nonlinear behavior, the transition point is not always easy to pinpoint. Engineering judgments are often necessary and there are large scatters for different excitations with the same spectral acceleration. Fig. 5.2 shows an example of the inter-story drift using IDA of a 9-story steel frame under SAC ground motions. The uncertainty in capacity against collapse can be described in terms of the mean and standard deviation of the inter-story drift capacity under multiple recorded ground motions from IDA. The coefficient of variation of this displacement capacity is generally of the order of 30%. Such a procedure has been used in the FEMA/SAC procedure.

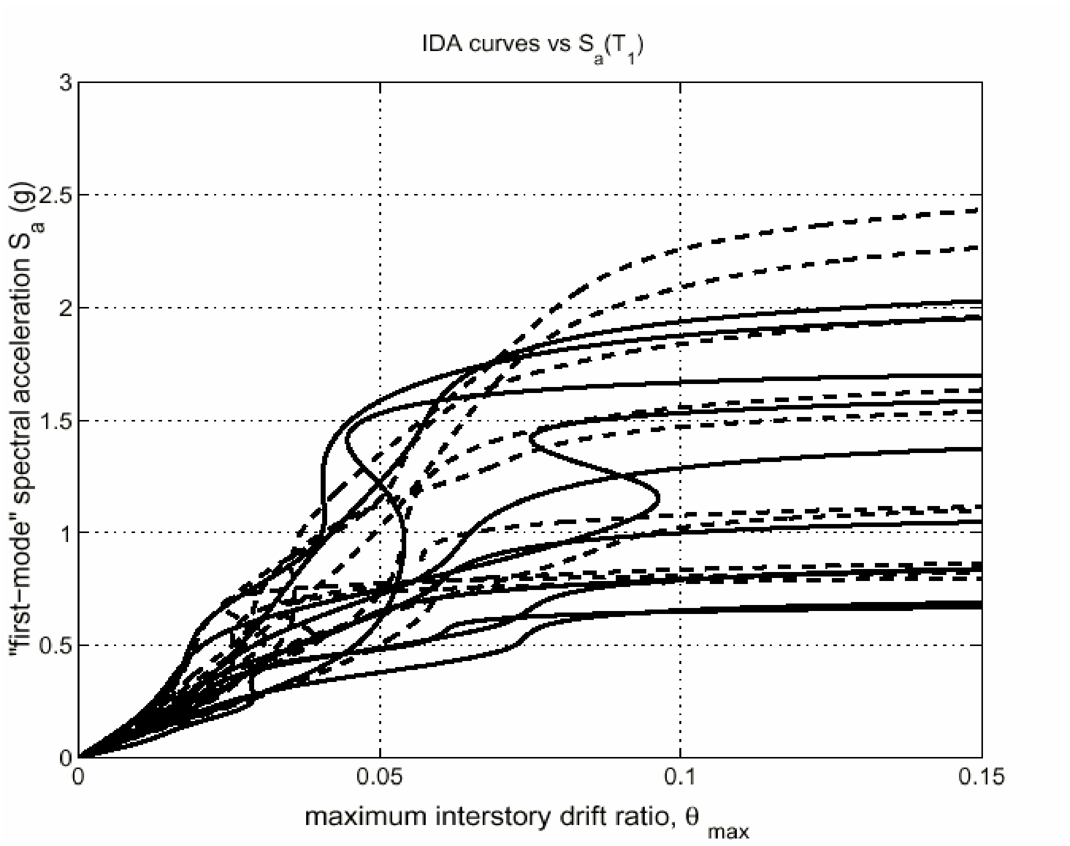

Figure 5.2. Results of Incremental Dynamic Analysis of 9-story Steel Moment-Resisting Frame with Fracturing Connection under SAC Ground Motions (Vamvasitkos and Cornell 2002)

5.4 Reliability of Structural Systems

In view of the large uncertainties in both demand and capacity as shown in the previous sections, the performance of the structural systems can be described meaningfully only when these uncertainties are taken into consideration explicitly. In other words, evaluation of the performance needs to be described in terms of reliability of the structural system against various limit states over a given period of time. Since the earthquake occurrence and the ground excitations and structural responses are random functions of time, the reliability problem is therefore a problem of a vector random process of time passing from a prescribed safe domain to an unsafe domain defined by the limit state. A rigorous mathematical solution of the so-called “first passage” problem is generally difficult. In reliability analysis, the random process first passage problem is often replaced by a more tractable formulation in which a structural performance function corresponding to a given limit state is constructed in terms of a set of random variables representing the uncertainty in the problems. The reliability problem is then solved using the first two moments of the random variable and a first order or second order approximation of the performance function. They are commonly referred to as the first order reliability method (FORM) or the second order reliability method (SORM). In earthquake engineering, an even simpler formulation of the problem is used in terms of two variables, demand versus capacity, as described in the previous sections, for a given limit state. Alternatively, depending on the problem, a simulation method may be more suitable to evaluate the reliability. These methods, their advantages and disadvantages in application to earthquake engineering are briefly described in the following.

5.4.1 FORM and SORM

Given a limit state, the performance of a structure can be described by a performance function, g(**X**), of all the excitation and structural property variables, or basic random variables **X** : $X_1, X_2, .. X_n$. The reliability problem is then formulated as follows:

$$g(X_1, X_2, \ldots X_n) \quad \begin{cases} < 0 & \text{safe domain (no failure)} \\ > 0 & \text{unsafe domain (failure)} \\ = 0 & \text{limit state boundary or surface} \end{cases} \qquad (5.8)$$

Knowing the probability distribution of the basic random variables, one can evaluate the probability of limit state by integration of the joint density function over the unsafe domain. When the number of basic random variables is large and when the performance function g(**X**) is nonlinear, the integration is generally difficult. The FORM/SORM method essentially replaces g(**X**) by its first-order or second-order Taylor series expansion at a point $\mathbf{X_0}$ where the contribution to the failure probability is the highest (or the most likely failure point). The reliability problem can be then solved in closed form in terms of the first two moments of the basic random variables.

The accuracy of the method is generally very good, especially when the limit state probability is small. Other advantages of the FORM/SORM is that at the most likely failure point, $\mathbf{X_0}$, the linearized performance function can be used as a performance or safety checking equation which can be easily cast into a load resistance factor design (LRFD) format familiar to engineers. $\mathbf{X_0}$ is therefore called the "design point". Also, sensitivity analysis of reliability to change in any design variables can be easily carried out using the results of the FORM/SORM analysis. Structural system limit states often cannot be expressed by single performance function. Under such circumstance, a system reliability analysis is needed involving unions and intersections of multiple failure modes each described by a performance function [g_k(**X**); k = 1 to n]. Computer software (e.g., CALREL,1989) can be used for such analysis. The FORM/SORM is a robust, well-developed methodology. It has been successfully used in formulation of reliability-based design in several codes and standards such as AISC (1985) and ASCE-7 (1998). For earthquake engineering problems, when the structure becomes very nonlinear and especially when brittle member failures occur that construction of the performance functions for structural members and systems may be difficult. Recent development of FORM/SORM and application to structural reliability analysis can be found in Der-Kiereghian et al. (2002).

5.4.2 Demand versus Capacity Formulation in FEMA/SAC Procedure

The reliability problem is simplified considerably if the limit state can be stated in terms of the demand exceeding the capacity. Although it may be an over-simplification in that the capacity and demand may not always easily defined for certain limit states such as the case of system collapse discussed in the previous section. In earthquake engineering applications, simplicity nevertheless offers some advantages, and especially in code procedure formulation. This is the method used in the reliability based, performance oriented design procedure proposed in the SAC/FEMA Steel Project (Cornell et al. 2002), which is described in the following.

Considering now the limit state described in terms of only two random variables; C (capacity) and D (demand), the performance function in Eq. 4.8 is now simplified to g(**X**) = C-D a linear function. The probability of limit state over a given period of time, t , is then given by the probability integral

$$P_t = P_t(C < D) = \int P_t(C \le D | D = d) f_D(d) dd \qquad (5.9)$$

Simple closed form solutions of the integration can be obtained when both C and D can be modeled by normal or lognormal random variables and C and D are independent.

$$P_t = 1 - \Phi(r) \qquad (5.10)$$

$$r = \frac{\mu_C - \mu_D}{\sqrt{\sigma^2_C + \sigma^2_D}} \text{, when both C and D are normal}$$

$$= \frac{\lambda_C - \lambda_D}{\sqrt{\beta^2_C + \beta^2_D}} \text{, when both C and D are lognormal}$$

in which, $\beta_X = \sqrt{\ln(1 + \delta_X{}^2)}$, and $\lambda = \ln \mu_X - 0.5\beta_X{}^2 = \ln \tilde{x}$. $\tilde{x}$ = median value of X

This is not the case when the demand described by Eq. 3.18 is not a simple normal or lognormal variable. The closed form solution, however, can still be obtained when the capacity variable C can be modeled by a lognormal random variable and the seismic hazard can be described by a power function as given in Eq. 3.17.

Limit State Probability Considering Randomness Only

Assume that the capacity randomness can be modeled with a lognormal variate with a median value $\tilde{C}$ and dispersion coefficient β_{CR}. Referring to Eq. 3.18, it can be shown that the limit state probability is given by

$$P_t = H_t(a^{\tilde{C}}) \exp[\frac{1}{2}\frac{k^2}{b^2}(\beta^2{}_{D|S_a=a} + \beta^2{}_{CR})] \qquad (5.11)$$

Note that the limit state probability consists of the probability of the spectral acceleration exceeding the median structural capacity multiplied by a correction factor depending on the randomness in the demand and the capacity, as well as the hazard characteristics (k) and demand/capacity relationship (b). The limit state probability as given in Eq. 5.11 therefore accounts for all randomness in the problem.

Continuing with the example problem in Section 3.3, and assuming that the median drift ratio capacity of this 3-story building against incipient collapse is 0.05 with a dispersion parameter β_{CR} = 0.35, the 50-year incipient collapse probability of the building is given by

$$P_{50} = H_{50}(a^{\tilde{C}}) \exp[\frac{1}{2}\frac{3^2}{1.2^2}(0.30^2+0.35^2] = H_{50}(0.86)\, e^{0.664} = 0.0107\times 1.94$$

$$= 0.0208 \qquad (5.12)$$

Which corresponds to an annual probability of 0.42×10^{-3}, or a return period of 2380 years

Impact of Uncertainty

Referring to Chapter 2, one can see that when the uncertainty is considered the limit state probability becomes a random variable and needs to be treated as such. Depending on the application, for example, one may want to evaluate the mean value of the limit state probability to perform a risk/benefit analysis or evaluate the percentile value for a confidence interval estimate. The uncertainty can be conveniently grouped into those in the hazard analysis, excitation/demand relationship, and structural capacity estimate. For example, the parameters k_0 and k in the seismic hazard model (Eq. 3.17), a and b in the regression equation for structural response (Eq. 3.15), and the parameters in structural capacity models (Eqs. 5.2 to 5.5) may all have uncertainty due to either modeling (e.g. incorrect functional form) or sampling (small number of test results) errors. For simplicity and tractability in analysis, the uncertainties in the seismic hazard, structural demand, and structural capacity models are assumed to be lognormal variables with a median values given by the model predictions and dispersion parameters β_{HU}, β_{DU}, and β_{CU}. The subscript H, D, and C denote hazard, demand, and capacity respectively and U denotes uncertainty. Similarly, the dispersion parameters of the randomness in the demand and capacity are denoted by β_{DR}, and β_{CR}. Incorporating the uncertainty as defined above into Eq. 5.11, one can obtain the mean estimate of the limit state probability as follows:

$$E[P_t] = E[H(a^{\tilde{C}})]\exp[\frac{1}{2}\frac{k^2}{b^2}(\beta^2{}_{DR} + \beta^2{}_{DU} + \beta^2{}_{CR} + \beta^2{}_{CU})] \qquad (5.13)$$

in which $E[H(a^{\tilde{C}})] = H(a^{\tilde{C}})\exp[\frac{1}{2}\beta_{HU}]$ (5.14)

In other words, effects of randomness and uncertainty are now combined. Note that the expected limit state probability is equal to the mean estimate of the hazard exceeding the median structural capacity multiplied by a correction factor that increases exponentially with the total uncertainty in the demand and capacity, and depends on the hazard and regression analysis parameters (k and b). The seismic hazard given in the USGS National Earthquake Hazard Maps is that of the mean hazard (Frankel et al. 1997) with regard to modeling uncertainty, Eq. 5.13 is therefore compatible with the USGS hazard maps.

To estimate the percentile values for a confidence interval estimate, in principal all uncertainty dispersions need to be considered. In the FEMA/SAC procedure, it is assumed that the uncertainty in seismic hazard has been incorporated through the mean hazard curve in Eq. 5.14. The confidence interval estimate is then obtained as function of the demand and capacity uncertainty using the mean hazard curve. The limit state probability corresponding to a percentile level q (probability of q not being exceeded) is given by

$$P_{q,t} = \tilde{P}_t \exp[K_q \beta_L] \tag{5.15}$$

in which, $$\tilde{P}_t = E[H(a^{\tilde{C}})]\exp[\frac{1}{2}\frac{k^2}{b^2}(\beta^2{}_{DR} + \beta^2{}_{CR})] \tag{5.16}$$

$$\beta_L = \sqrt{\frac{k^2}{b^2}(\beta^2{}_{DU} + \beta^2{}_{CU})} \tag{5.17}$$

$$K_q = \Phi^{-1}(q) \tag{5.18}$$

$\tilde{P}_t$ is the median (50%) value of P_t . Note that the median estimate is the same as that considering only the randomness in the demand and capacity. The q percentile limit state probability is equal to $\tilde{P}_t$ multiplied by a factor depending on the uncertainty dispersion parameters and the percentile value. K_q is the standard normal variate value corresponding to this percentile value.

Continue with the example given in Eq. 5.12 and now consider also the uncertainty. It is assumed that:

(1) β_{HU} = 0.30 in the hazard analysis (Eq. 3.17),
(2) β_{DU} = 0. 25 describes the errors in the demand regression analysis as function of the spectral acceleration (Eq. 3.15), and
(3) β_{CU} =0.28 indicates the uncertainty in the estimate of capacity against collapse by the IDA procedure in Section 5.3.

The impact of different uncertainties on the incipient collapse probability can be illustrated as follows. The median incipient collapse probability of this steel building considering randomness only is 0.0208 (Eq. 5.12). According to Eqs. 5.14 and 5.16 when the seismic hazard analysis uncertainty is included in the form of mean hazard, this median value increases to

$$\tilde{P}_{50} = H_{50}(0.86)\exp[\frac{1}{2}\times 0.3]e^{0.664} = 0.0241 \qquad (5.19)$$

The mean estimate of the 50-year incipient collapse probability considering all uncertainties according to Eq. 5.13 is

$$E[P_{50}] = H_{50}(0.86)\exp[\frac{1}{2}\times 0.3]\exp[\frac{1}{2}\frac{3^2}{1.2^2}(0.30^2 + 0.25^2 + 0.35^2 + 0.28^2)]$$
$$= 0.0107\times 1.161\times 3.01 = 0.0374. \qquad (5.20)$$

It is seen that the mean estimate is always higher than the median, reflecting the additional effect of uncertainties in demand and capacity. Finally, when these uncertainties are accounted for in the form of a confidence level, then the probability is dependent on the confidence level. For example, the incipient collapse probability will not exceed the following value with 85% confidence (Eq. 5.15)

$$P_{85\%,\,50} = 0.0241\exp[1.03\times\sqrt{\frac{3^2}{1.2^2}(0.25^2 + 0.28^2)}\,] = 0.0616 \qquad (5.21)$$

The above performance evaluation procedure has been applied to both pre- and post-Northridge steel moment frame buildings (Foutch 2000,Yun et al. 2002, Lee and Foutch 2002). For example, buildings designed in accordance with the 1997 NEHRP provisions and constructed with SAC pre-qualified connections have a confidence level of 90% of meeting the requirement of probability of incipient collapse being less than 0.02 in 50 years.

5.4.3 Method of Simulation

Randomization of Capacity

The simulation method can be applied to reliability evaluation when the structural capacity uncertainty is considered. In a direct Monte-Carlo method, one needs only to randomize the capacity of the structure according to the randomness and uncertainty models as mentioned above in the time history analysis of the structure. The limit state probability can be then calculated from the response statistics of the repeated time history analyses. It is conceptually simple. The difficulty is obvious that the randomization needs to be done for each element. Also, as in the structural demand analysis, the computational effort may become excessive

Uncertainty Correction Factors

To incorporate the effect of capacity uncertainty into the smart simulation procedure and at the same time avoid the difficulty of detailed modeling at the component level as mentioned above, one can use a hybrid procedure. The structure is first regarded as deterministic and the smart simulation is performed at a given hazard level, e.g. 50%, 10%, or 2% in 50 years to obtain the probabilistic structural demand curve, e.g., the median response to the set of uniform hazard ground motions (UHGM) corresponding to a hazard level as shown in Fig. 3.5. At a given hazard level, the effect of the uncertainty can be then incorporated by multiplying the median estimate by a correction factor similar to that given in Eq. 5.13. As shown in Fig. 3.5, at a given hazard level such as 10% in 50 years or 2% in 50 years, the demand described by the median response to the UHGM corresponds to the probability of exceedance considering the randomness and uncertainty (if included in the simulation in Section 3.4) in the excitation only. To account for the randomness and uncertainty in the capacity and demand, the limit state probability is multiplied by a correction factor (Wen and Foutch 1997).

$$C_F = 1 + \frac{1}{2} S^2 \delta^2{}_T \tag{5.22}$$

in which S is the sensitivity coefficient to the change in structural capacity depending on the seismic hazard and the median structural capacity; δ_T is the coefficient of variation of the total randomness and uncertainty in the demand and capacity.

$$\delta_T \approx \sqrt{\delta^2{}_{RC} + \delta^2{}_{UC} + \delta^2{}_{RD} + \delta^2{}_{UD}} \tag{5.23}$$

in which RC, UC, RD, and UD denote randomness and uncertainty in capacity and demand. Alternatively, if the limit-state probability is kept the same, the median value can be multiplied by a correction factor C_D to reflect the effect of total uncertainty as follows:

$$C_D = 1 + \frac{1}{2} S \delta^2{}_T \tag{5.24}$$

The seismic hazard, e.g., in terms of 50-year probability of exceedance of the spectral acceleration, can be generally modeled by a lognormal distribution. If the hazard curve is not available, it can be determined directly from the UHGM. The median values of the spectral acceleration of the UHGM at 10% and 2% in 50 years allow one to determine the two lognormal distribution parameters λ and ζ. The sensitivity coefficient S is then given by

$$S = \frac{\ln a_C - \lambda}{\zeta^2} \tag{5.25}$$

in which a_C is the median capacity of the system in terms of spectral acceleration S_a. Note that when the hazard dispersion measure ζ is large, S is small indicating that the the uncertainty in hazard dominates and the result is not sensitive to the structural capacity uncertainty.

For example, in Fig. 3.5, the 50-year spectral acceleration hazard parameters for Carbondale Illinois are λ = -6.85 and $\zeta = 2.07$(Wen and Wu 2001), for the building with a fundamental natural period of T= 0.15 sec after retrofit. The sensitivity coefficient S at 10/50 and 2/50 hazard levels are calculated to be 0.618 and 1.0 respectively according to Eq. 5.24. Assuming a total uncertainty of δ_T = 50% for the structural drift capacity against all limit states, the correction factor C_F and C_D are respectively 1.05 and 1.07 at 10/50 hazard level 1.13 and 1.13 at 2/50 hazard level. It is seen that the effect of the demand and capacity total uncertainty is small since it is overshadowed by the large uncertainty of the seismic excitation (large value of the dispersion parameter ζ) in eastern United States. The median values can be then modified by the correction factors and fitted by a lognormal (dashed) curve as the risk curve of column drift capacity being exceeded as shown in Fig. 3.5.

5.5 Probabilistic Codes and Standards

Although the uncertainty of seismic loadings has been well recognized by the profession, the incorporation of uncertainty in most building code procedures has been limited to the selection of a design earthquake based on probability, such as 10% in 50 years or a return period of 475 years. This design earthquake is then used in conjunction with a series of factors reflecting the influence of structural period, soil condition, structural inelastic behavior, importance of the structure, etc. These factors are determined based on analysis, judgment, and experience and often calibrated in such a way that the resultant designs do not deviate significantly from the acceptable practice at the time. Therefore, despite their simplicity and ease of use, a significant shortcoming of the current design procedures is that the reliability of the final design is undefined and difficult to quantify.

The large losses during recent earthquakes bring to focus what reliability current buildings have against future earthquakes and as a result the performance-based design (PBD) concept began to receive serious attention of the profession. In SEAOC (1995), PBD is described by a performance matrix for various response (drift) limits that the structure has to satisfy under earthquakes of different probabilities of exceedance,i.e., 50% in 30 years (frequent), 50% in 50 years (occasional), 10% in 50 years (rare), and 10% in 100 years (very rare). Adding more levels of design earthquakes is equivalent to putting more constraints on the structural performance. The selection of these additional design earthquakes and corresponding performance limits, however, need to be carefully done to ensure internal consistency. Also, since the probability is prescribed on the seismic hazard uncertainties in the structural capacity and demand have not been considered, the reliability of the structure against specific limit state is still unknown.

In a reliability-based design, the limit-state reliability analysis is reversed. In other words, the problem is to determine the required structural capacity for a given target reliabilities against a set of structural limit states. Such design procedures have been recently developed. These procedures represent a big step forward in accounting for uncertainty in demand and capacity in codes and standards. They are described in the following. There are in addition a few open questions related to reliability-based design. One is the age-old question of how safe is safe enough or how one should set the target probabilistic performance curve as shown in Figure 5.3. It is also addressed briefly in the following.

5.5.1 LRFD based on FORM

If the building response is largely static or can be satisfactorily described by an equivalent static procedure, then the uncertainty in both loading and resistance can be described by a set of random variables. The limit state can be described in terms of these basic random variables and the reliability analysis can be carried out in a straightforward manner using the well-known and well tested First Order Reliability Method (FORM) (see Section 5.4.1). In FORM the reliability-based design is formulated at the most likely failure point (or design point) of the basic random variables, which satisfies the limit state function. This method has been used as basis for reliability-based design and cast in a Load and Resistance Factor Design (LRFD) format familiar to design engineers,

$$\sum_{i=1}^{j} \phi_i R_i \geq \sum_{i=1}^{k} \gamma_i L_i \tag{5.26}$$

in which R_i and L_i are nominal resistances and loads, generally close to the mean values of the resistances and loads; ϕ_i and γ_i are respectively the resistance and load factors depending on the target reliability and amount of uncertainty in each random variable. These factors are given by:

$$\eta_i = \frac{\mu_i}{X_{ni}}(1 + \alpha_i \beta \delta_i) \tag{5.27}$$

in which η_i is either the load or resistance factor; μ_i and X_{ni} are respectively the mean and nominal values; α_i is the sensitivity coefficient, being positive for load variables and negative for resistance variables; δ_i is the coefficient of variation of the variable X_i ; and β is the target safety index. β is given by

$$\beta = \Phi^{-1}(1 - P_f) \tag{5.28}$$

in which Φ^{-1} is the inverse standard normal cumulative distribution and P_f is the target limit-state probability. The above LRFD design format explicitly accounts for the uncertainty in both load and resistance. Note that γ_i is greater than 1 and ϕ_i is less than 1 and also higher reliability (larger β) and larger uncertainty in the random variables (δ_i) lead to larger γ_i and smaller ϕ_i and consequently larger required design resistance. It has been used in most recent code procedures, e.g., for buildings in US (ASCE-7 1998, AISC 2001) based on Ellingwood et al. (1982), and Galambos et al. (1982), offshore structures in US (API 1990), and bridges in US (AASHTO 1994, Kulicki et al. 1995) and Canada (CAN/CSA 2000).

The reliability checking in these procedures, however, has been mostly at the member rather than at the system levels. For most buildings under earthquake loads, the responses are dynamic and nonlinear and may have different hysteretic and degrading behaviors including those caused by brittle fractures at the joints so the problem is much more involved. Although it is possible to combine the FORM with a finite element analysis to solve this problem (e.g., Der Kiureghian 1996), it is generally difficult to express the limit states of interest directly in terms of the basic load and resistance random variables and consequently the modeling and computational problems become much more involved.

5.5.2 FEMA/SAC Procedure

In view of the damages suffered in recent earthquakes, in the SAC/FEMA Joint Venture for Steel Buildings, a reliability-based and performance-oriented design has been developed where all randomness and uncertainty in the load and resistance are explicitly considered and accounted for (FEMA 350, 2001). The critical issues related to such a statistical and reliability framework were reviewed in Wen and Foutch (1997). The theoretical basis for the development of the design procedure can be found in Cornell et al. (2002). The final design format still retains the basic LRFD flavor with additional quantitative treatment of the effect of uncertainty. The results are being adopted in the AISC Seismic Provisions (Malley 2002) and mostly likely will serve as a prototype for wider adoption in other codes and standards. The probability basis of this design procedure is briefly described in the following.

In the SAC/FEMA procedure, performance is checked at two levels, immediate occupancy (IO) and collapse prevention (CP) with associated target probability of 50% and 2% in 50 years respectively. Fig. 5.3 shows the performance checking of such a procedure. Assuming the probabilistic performance curves can be described by a distribution such as the lognormal, the two points on the curves allow one to describe and check the performance for a wide range of response. If the probability curve is higher than the target, stiffening, strengthening, or other mitigation measures are needed. Given the target probabilistic performance curve, the design is then to solve the inverse problem of finding the required structural capacity to meet the requirement. The simple closed form solution of the reliability analysis given in Section 5.4.2 allows one to solve the inverse problem. Referring first to Eq. 5.13, the inverse problem of determining the required structural median capacity, $\tilde{C}$, for a prescribed target mean limit state probability , $E[P_t] = P_0$, can be solved as follows:

$$\left\{\exp\left[-\frac{1}{2}\frac{k}{b}(\beta^2{}_{CR}+\beta^2{}_{CU})\right]\right\}\tilde{C} \geq \left\{\exp\left[-\frac{1}{2}\frac{k}{b}(\beta^2{}_{DR}+\beta^2{}_{DU})\right]\right\}\tilde{D}^{P_0} \quad (5.29)$$

Note that Eq. 5.29 is not a rearrangement of Eq. 5.13, therefore the exponents are different. It can be rewritten as

$$\phi\tilde{C} \geq \gamma\tilde{D}^{P_0} \quad (5.30)$$

ϕ is the capacity (resistance) factor and γ is the demand (load) factor. $\tilde{D}^{P_0}$ = median demand corresponding to $S_a{}^{P_0}$, a spectral acceleration of exceedance probability of P_0. From Eq. 3.15, one obtains

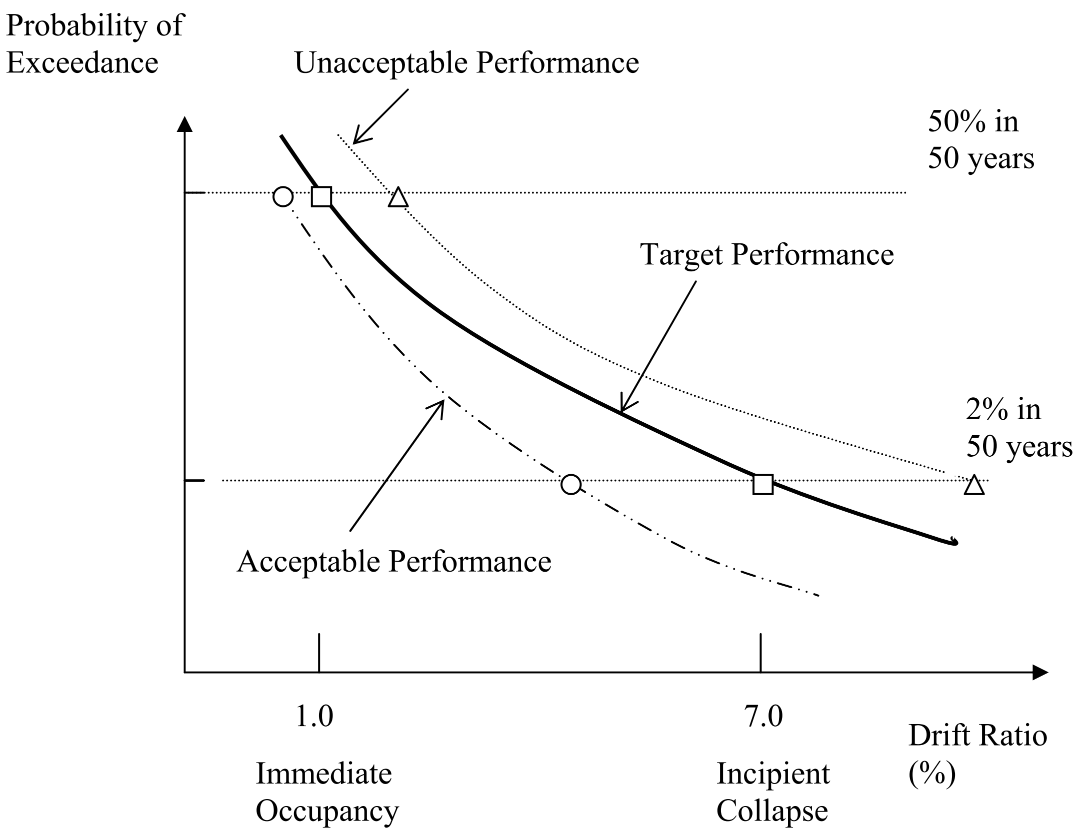

Figure 5.3. Bi-level acceptance criteria in terms of 50-year limit state probability in log scale.

$$\tilde{D}^{P_0} = a\,(S_a^{P_0})^b \qquad (5.31)$$

in which $S_a^{P_0}$ is solved from Eq. 3.17. Note that from Eq. 5.31, smaller P_0 (higher reliability) gives larger $S_a^{P_0}$, hence larger $\tilde{D}^{P_0}$ and from Eq. 5.30 larger randomness and uncertainty in the demand and capacity give larger γ and smaller ϕ leading to larger design capacity $\tilde{C}$.

Continuing with the example problem in the previous section of the 3-story steel building, if the target 50-year incipient collapse (defined according to an IDA) probability is 2% what should be the median drift capacity against collapse? Note that since the target probability is lower than the current value of 0.0374 (Eq. 5.20), structural drift capacity against incipient collapse needs to be enhanced. From Eqs. 5.29 and 5.30 the demand and capacity factors are calculated to be

$$\gamma = \exp\left[\frac{1}{2}\frac{3}{1.2}(0.30^2 + 0.25^2)\right] = 1.21 \tag{5.32}$$

$$\phi = \exp\left[-\frac{1}{2}\frac{3}{1.2}(0.35^2 + 0.28^2)\right] = 0.778 \tag{5.33}$$

and the median drift capacity corresponding to 2% in 50 years spectral acceleration exceedance probability is

$$\tilde{D}^{2\%} = a\,(S_a{}^{2\%})^b = 0.06\,(0.698)^{1.2} = 0.039 \tag{5.34}$$

The required design median drift capacity against incipient collapse is therefore according to Eq. 5.30

$$\tilde{C} = \frac{1.21}{0.802} \times 0.039 = 0.0588 \tag{5.35}$$

Compared with the current median capacity of 0.05, an increase of 18% is required. The above design satisfies the target *mean* limit state probability considering all the randomness and uncertainty. Alternatively, one can also use a confidence level approach to set the design requirements as follows.

If a design criterion is that there must be a confidence level of at least q that the actual (but uncertain) limit state probability is less than the allowable value of P_0, then the formulation given in Eqs. 5.30 can be rearranged in terms of the factored capacity/demand ratio as follows

$$\lambda_{con} = \gamma \tilde{D}^{P_0} / \phi \tilde{C} \tag{5.36}$$

in which $\gamma, \phi, \tilde{D}^{P_0}$, and $\tilde{C}$ are defined in Eqs. 5.29 to 5.31, λ_{con} is the demand/capacity ratio depending on the confidence level given by

$$\lambda_{con} = \exp\left[-K_x \beta_{UT} + \frac{1}{2}\frac{k}{b}\beta^2{}_{UT}\right] \tag{5.37}$$

$$K_x = \Phi^{-1}(q)\text{, } q = \text{confidence level} \tag{5.38}$$

$$\beta_{UT} = \sqrt{\beta^2{}_{CU} + \beta^2{}_{DU}}\text{, total uncertainty in capacity and demand.} \tag{5.39}$$

For example, continuing with the example, if the target P_0 = 2% in 50 years and a confidence level of q = 85% is desired. From Eqs. 5.37 to 5.39, one obtains

$$K_x = \Phi^{-1}(0.85) = 1.04$$

$$\beta_{UT} = \sqrt{0.25^2 + 0.28^2} = 0.375$$

$$\lambda_{con} = \exp\left[-1.04 \times 0.375 + \frac{1}{2}\frac{3}{1.2}0.375^2\right] = 0.807 \qquad (5.40)$$

The required design median drift capacity is then determined from Eq. 5.36

$$\tilde{C} = \frac{1.21 \times 0.039}{0.807 \times 0.802} = 0.073 \qquad (5.41)$$

Compared with current design of 0.05, an increase of 46% is needed to satisfy this design criterion.

5.6 Target Reliability

A critical element in a reliability-based design procedure is the selection of the structural performance levels and the associated acceptable (target) reliability. Another related question, which has not been explicitly addressed in current codes, is the target reliability against different hazards. For example, when both wind and earthquake are important, should uniform reliability be required for design against each hazard? Determination of the target reliability levels for various limit states against one and more hazards requires broader social-economical considerations. The selection of the design hazard levels and the corresponding structural performance requirements has largely been based on professional experience and judgment. While collective professional wisdom may be the only recourse at present, it could lead to unsafe or wasteful designs. One good example of the problem of reliance on probability only in design decision is the use of Maximum Considered Earthquake (MCE), normally defined as the 2% in 50 years event, as the design earthquake. Due to the flatness of the tail of seismic hazard curve in certain locations in eastern United States, the spectral acceleration at this low probability level could come close or even exceed some in the most seismic region in western United States. For example, the MCE in terms of the spectral acceleration at central San Francisco is 1.5 g at 0.2 sec and 0.75g at 1 sec while at Memphis it is 1.25g at 0.2 sec and 0.4g at 1 sec according to the 1997 NEHRP hazard map. It is difficult to justify such large design ground motion (even after some reduction) when the projected per capita annual cost due to earthquakes at Memphis is only $15 compared with $200 at San Francisco (Searer 2002, FEMA 2001). In other words such a procedure could produce overly conservative and wasteful designs. A more rational procedure is to arrive at the target reliability by considering the long-term risk-benefit tradeoff.

5.6.1 Target Reliability Implied in Current Practice

One commonly used approach to the determination of target reliability levels is comparison of risks of consequence of structural limit states with other societal risks, such as those of accidents, deceases, and industrial hazards. Alternatively, one can compare the notional (calculated) probability of limit states with those implied in current designs and adjust accordingly. This approach has been used, for example, by Ellingwood et al. (1982) in calibrating the target reliability of structural members against practice acceptable at the time in developing the AISC LRFD design recommendations, which have been adopted in the ASCE-7 (1995). The need for a more rational approach to determining target reliability and acceptable risk has received serious attention by researchers and engineers recently (e.g. Ellingwood 1999). One of such approaches is to strike a balance between the initial cost and potential large losses over the structure's lifetime caused by the hazards. Since the lifecycle cost depends on the occurrence of limit states and hence the demand and system capacity, it is also highly uncertain and needs to be properly treated as such. An optimal design decision under uncertainty can be reached by minimization of the expected lifecycle cost. The recent progress in application of this approach to seismic design is briefly described in the following.

5.6.2 Target Reliability according to Minimum Lifecycle Cost Design Criteria

The design procedure based on optimization considering cost and benefit is generally referred to as level IV reliability-based design. For example, Rosenblueth (1976) had made strong and convincing arguments for the profession to move from a semi-probabilistic, second moment, or full distribution design format to one based on optimization since it is the only rational procedure to ensure long term benefit to the society. A rational approach is based on consideration of costs over the structure's lifetime including construction cost, maintenance cost, damage cost, cost of loss of revenue, cost of injury and death, and discounting of cost over time (e.g. Ang and Leon 1997,Wen and Kang , 2001). Since limit states that cause serious consequences and large costs normally have very small probabilities of occurrence, the design problem is to balance the initial cost of the structure against the expected cost of the consequence of failure.

Central issues in this approach are proper consideration of the uncertainty in demands and capacity and accurately accounting of various costs in order to arrive at the optimal solution. Following Wen and Kang (2001), over a time period (t) which may be the design life of a new structure or the remaining life of a retrofitted structure, the expected total cost due to all hazards can be expressed as a function of t and the design variable vector X,

$$E[C(t,X)] = C_0(X) + E[\sum_{i=1}^{N(t)} \sum_{j=1}^{k} C_j e^{-\lambda t_j} P_{ij}(X,t_i)] + \int_0^t C_m(X) e^{-\lambda\tau} d\tau \qquad (5.42)$$

in which E [.] = expected value; C_0 = the construction cost for new or retrofitted facility; X = design variable vector, e.g., design loads and resistance; i = severe loading occurrence number including joint occurrence of different hazards such as live, wind, and seismic loads; t_i = loading occurrence time; a random variable; N(t) = total number of severe loading occurrences in t, a random variable; C_j = cost in present dollar value of consequence of j-th limit state at t = t_i including costs of damage, repair, loss of service, and deaths and injuries; $e^{-\lambda t}$ = cost discount factor over time t, λ = discount rate per year; P_{ij} = probability of j-th limit states being exceeded given the i-th occurrence of one or multiple hazard; k = total number of limit states; and C_m=operation and maintenance cost per year. Implicit in the formulation is the assumption that the structure will be restored to its original condition after each hazard occurrence. The design decision problem is therefore to determine X such that E[C(t,X)] is minimized.

The method was applied to the design of a 3×5 bay, 9-story special moment resisting frame steel office building in Los Angeles, California, Seattle, Washington and Charleston, South Carolina. The building is designed for a wide range of base shear and meets the drift and other requirements of NEHRP 97. The system strength is measured by a system yield force coefficient (system yield force determined from a static pushover analysis using DRAIN2D-X divided by the system weight). Five limit states in terms of story-drift are used according to the performance levels of FEMA 273 (1996). The empirical seismic hazard procedure of FEMA 273 is used to calculate the ground excitation demand for a given probability level. The probability of drift ratio was determined from the uniform hazard response spectra and a method of equivalent nonlinear single degree of freedom system (SDOF) for the building following Collins et al. (1996). The drift ratio is then multiplied by a correction factor (see Section 5.4.3) to incorporate building capacity uncertainty and then converted to damage factor according to FEMA-227 (1992). The maintenance cost is not considered in this study. Initial costs are estimated according to Building Construction Cost Data (1996). The nonstructural items were not considered in the initial cost since they are not functions of the design intensity. The damage cost, loss of contents, relocation cost, economic loss (dollar/ft^2), cost of injury ($1000/person for minor and $10,000/person for serious injury) and cost of the human fatality ($1,740,000/person) are estimated based on FEMA-227. All costs are given in 1992 US dollars. A constant annual discount rate λ of 0.05 is assumed. Fig. 5.4 shows an example of various lifecycle (50-year) costs as functions of the structural strength measured by a system yield coefficient S_y (system yield force divided by weight) at Seattle. Note that the expected cost due to earthquake clearly dominates except for very small S_y at which the structure becomes more vulnerable to wind load. The optimal design strength is determined by minimizing the total lifecycle cost. At this optimal strength, the increase in initial cost is balanced by the decrease in overall expected failure cost.

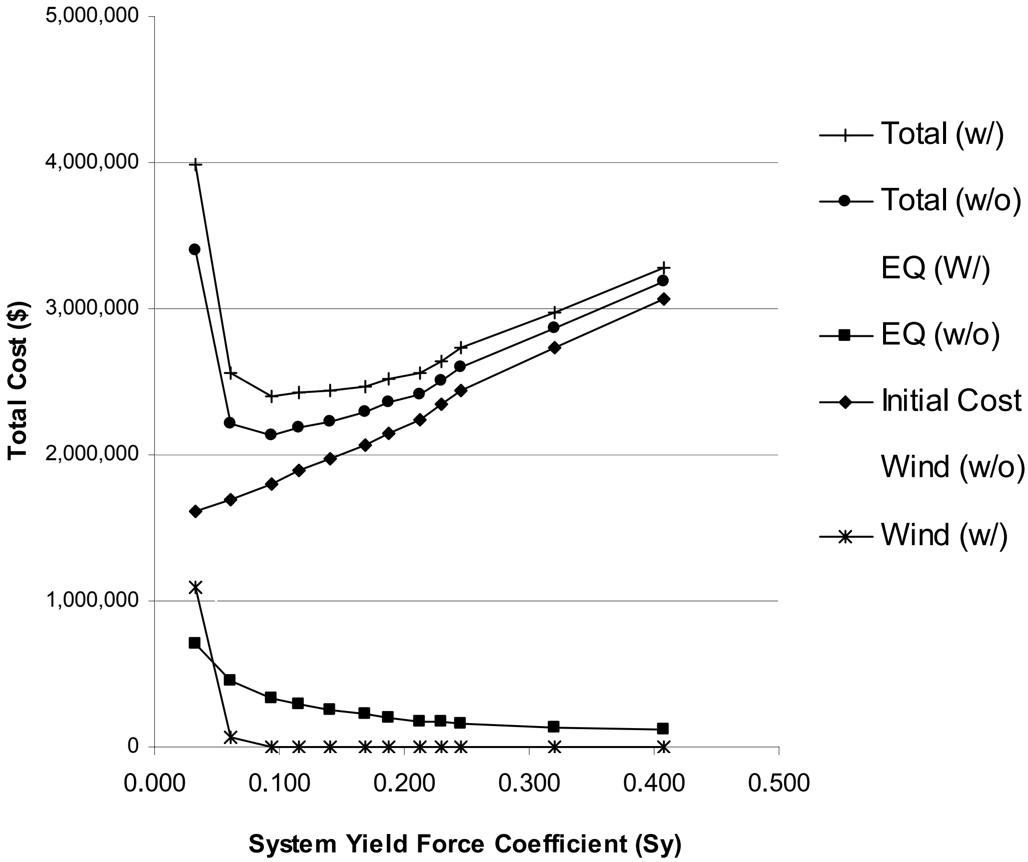

Figure 5.4. Lifecycle Costs of a 9-story Office Building as Function of System Yield Force Coefficient at Seattle (life = 50 year, discount rate = 5%, w/ and w/o indicate with and without consideration of cost due to deaths and injuries, Wen and Kang 2001)

Comparison of the optimal strength against seismic loads based on life-cycle cost (LCC) with current design according to NEHRP 1997 in the first two rows of Table 5.1 indicates that current design requirements could be enhanced, or in other words, the target reliability index could be set higher, to ensure long-term benefit at all three locations. Comparison of the last three rows of the table shows that the design is often dominated by the hazard that has high intensity and large uncertainty, causing large consequences (earthquake at Los Angeles and Seattle and hurricane winds at Charleston). The design, however, is not *controlled* by the dominant hazard as is recommended in most current codes, since the lesser hazard still contributes to the lifecycle cost, e.g. earthquake at Charleston and winds at Seattle. The sensitivity of the optimal design to important design parameters such as structural life, discount rate and death and injury cost was investigated. The optimal design was found to be highly dependent on failure consequence and moderately sensitive to the structural life span and discount rate assumptions. It may or may not be sensitive to death and injury cost assumptions,

Table 5.1. Comparison of Design System Strength (Lateral System Yield Force Divided by Weight) under Winds, Earthquakes, and Both Hazards (Wen and Kang 2001)

Location / *Hazard (Design Basis)*	Los Angeles	Seattle	Charleston
Earthquake (NEHRP 1997)	0.140	0.100	0.075
Earthquake (LCC)	0.198	0.109	0.097
Wind (LCC)	0.073	0.073	0.121
Earthquake and Wind (LCC)	0.198	0.115	0.146

NEHRP=National Earthquake Hazard Reduction Program, LCC=Life Cycle Cost

dependent on location and hazard risk characteristics and percentage contribution of such costs to the overall lifecycle cost. The question of uniform reliability against different hazard was also examined. The implied reliabilities against wind and earthquake of the optimal design are vastly different indicating that, contrary to common belief, uniform reliability against different hazards is not required.

The results show that the minimum expected lifecycle cost approach properly takes the uncertainty in structural demand and capacity and the broader social-economical issues into consideration. It is a promising method for selecting appropriate target reliability for design.

5.7 List of Symbols and Notation

β : dispersion parameter in a lognormal distribution $= \sqrt{\ln(1+\delta^2)}$
β : reliability (or safety) index
C: structural member or system capacity against a prescribed limit state
$\tilde{C}$: median (50 percentile)value of C
D: displacement demand variable, e.g., interstory drift ratio, global drift ratio
$\tilde{D}$: median value of D
δ : coefficient of variation
$f_X(x)$: probability density function of random variable X
$f'(\theta)$: prior distribution of parameter θ
$f''(\theta)$: posterior distribution of parameter θ
Φ : cumulative distribution of a standard normal variate
Φ^{-1}: inverse function of Φ.
ϕ : resistance (or capacity) factor in LRFD design

g(**X**): performance function of basic random variables **X:** $X_1, X_2, \ldots X_n$ corresponding to a given limit state

γ: load or demand factor in LRFD design

$H_t(a)$: seismic hazard function; probability of spectral acceleration exceeding "a" in "t" years

Incremental Dynamic Analysis (IDA): dynamic analysis of structural against collapse with incremental increase of ground excitation intensity.

$L(\theta)$: sample likelihood function of parameter θ

λ : scale parameter in a lognormal distribution = $\ln\mu - 0.5\beta^2$

λ : discount rate

μ : mean value

P_q: q percentile value of P

ρ : reliability/redundancy factor in Uniform Building Code

σ : standard deviation

q: percentile value; confidence level

S: soil classification

S: sensitivity coefficient due to capacity uncertainty

S_a: spectral acceleration

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6. DAMAGE AND LOSS ASSESSMENT

6.1 Introduction – overview

In Chapters 3 – 5, we have examined sources of uncertainty associated with identification and specification of the seismic hazard and its demands on engineered facilities, and with response of systems to the spectrum of intensities of excitation. We have identified at least two (complementary) reliability-based tools for measuring the limit state probability, the probability that the system violates stipulated limits or conditions of behavior.

The first of these tools is the *fragility*, defined as the probability of entering a specified limit state (LS) conditioned on the occurrence of a specific hazard intensity, d, among the spectrum of (uncertain) hazards, D. This limit state probability, P[LS|D = d], can be determined from a model of the system, in which the uncertain variables are treated as random variables that are interrelated through fundamental principles of engineering science and mechanics. To take an example from structural engineering, if the limit state is defined by the event in which the maximum interstory drift in a frame, θ_{max}, exceeds a limiting drift, θ_{limit} [defined, e.g., so as to minimize damage to nonstructural building components], the limit state LS = $\{\theta_{max} > \theta_{limit}\}$. For fragility assessment purposes, θ_{max} is obtained from a nonlinear finite element model of the structural system. In this same context and on the basis of the USGS hazard mapping project, D often is defined by the spectral acceleration at the fundamental period of the structural system. Both θ_{max} and θ_{limit} are functions of random variables, characterized by both aleatory and epistemic uncertainties. The fragility modeling process allows the combined effect of the uncertain variables to be propagated through the model by numerical means (e.g. simulation) or (rarely) in closed form. Quantification of the uncertain nature of the limit state can be used as a basis for decision.

The second of these tools takes the additional step of summing (or integrating) [LS|D = d] over the spectrum of possible hazard intensities challenging the facility, leading to the (unconditional) *limit state probability*:

$$P[LS] = \sum_{d} P[LS|D = d]\; P[D = d] \tag{6.1}$$

[cf eqs (3.1) and (4.42)] in which the P[D = d] represents the hazard, which depicts the relative likelihood of occurrence of hazards of specific intensity. This latter probability often is expressed in the form of a complementary cumulative probability distribution function, $H_D(d) = P[D > d]$, denoted the hazard curve, as discussed in Chapter 2.

Risk analyses performed in support of consequence-based engineering require an assessment of the consequences of specific challenges to a system in addition to their probability of occurrence. These consequences are expressed most commonly in terms of damage and economic loss. Accordingly, the limit states identified by the analyst must be “mapped” to specific damage states and, in turn, to losses. This mapping process is one of the most significant research challenges to consequence-based engineering at the present time, and requires a consideration of socio-economic as well as technical factors. Assuming that this mapping can be completed successfully (as discussed in the sequel), expected damages and losses due to earthquakes during a service period, T, due to the occurrence of a LS can be computed by multiplying the limit state probability in eq (6.1) by the cost of occurrence of the LS, discounted to present worth, or

$$E[Loss] = \sum_{t} P[LS_t]\, C[LS_t] \tag{6.2}$$

where the summation is taken over the service period. The expected loss is a common method for characterizing facility risk, and is a rational basis for decision for a “risk-neutral” investor. The principle of designing to achieve minimum total expected lifetime cost has been recognized in the structural engineering area for decades (e.g., Rosenblueth, 1976; Frangopol, 1985). These early studies, with their focus on structural design, had a relatively narrow focus. More recent studies (Kanda and Shah, 1997; Ang and De Leon, 1997; Wen and Kang 2001) have applied the concepts of life-cycle cost minimization to earthquake hazard mitigation, but again, with a focus on identifying the optimal structural design strategy or optimal target reliability for design purposes. In these studies, the loss model C[LS] was assumed to be deterministic. As will be seen subsequently, this is an oversimplification.

An estimate of expected facility loss requires consideration of all significant limit states, or the summation (integration) of a large number of terms such as those in eq (6.2). This general formulation can be applied at the individual component or sub-structure level, at the building or facility level, at the building category level (for an inventory of similar buildings), or at the regional level by expanding the dimensions of the summation.

If the probability distributions for all uncertain parameters in eq (6.1) are known, the analysis models are accurate, and the costs in eq (6.2) are available, the result is a point estimate of the limit state probability or risk. Unfortunately, this is seldom the case. Limitations in knowledge and ability to model the hazard and its demand on the system, the response of the system to specific demands, the ensuing state of the system and damage, and relation of costs to limit or damage states give rise to epistemic uncertainties (see Chapter 2) at all stages of the analysis. Losses stemming from the occurrence of specific events are hard to judge; this is particularly so for so-called indirect losses, as described subsequently. In the presence of these epistemic uncertainties, P[LS] and E[Loss] are, themselves, functions of uncertain (random) variables, which can be characterized by probability distributions that are analogous to the sampling distributions in classical statistical inference. A sampling distribution has a mean (or median) and variance (or standard error), which enables the decision-maker to attach a confidence level to specific statements regarding limit state probability or loss. Recent studies in performance-based engineering indicate that many stakeholders would prefer performance statements to be accompanied by a statement of confidence that measures the credibility of the supporting analysis.

Damage and loss assessment methodologies require an integrated approach for dealing with seismic hazard, structural response and fragility assessment, and the mapping that must occur between limit states and damage and cost, developed within an underlying framework for reliability analysis and uncertainty modeling. It is assumed that this framework is, or will become, available. In the remainder of this Chapter, we summarize existing approaches to damage assessment and loss estimation, with a view toward identifying significant research issues requiring resolution to support consequence-based engineering and decision-making.

6.2 Performance levels and objectives

The performance of buildings, bridges and other infrastructure during Loma Prieta and Northridge Earthquakes and the substantial economic losses suffered as a result has led to a fundamental rethinking of the objectives of engineering design and construction. Safety of facility occupants and the public at large is (and always will be) a paramount objective in civil engineering. The current regulatory process for civil infrastructure (building codes, standards, local building departments, state/federal highway agencies, and other regulatory authorities) functions reasonably well with regard to assurance of general public safety. The loss of life and injury from recent earthquake disasters in the United States has been modest in comparison to such losses in other parts of the world. On the other hand, the economic losses from direct facility damage, loss of business opportunity and community disruption have been substantial and, in the view of many stakeholders, unacceptable. It is these losses that are providing the major impetus for the development of the new paradigm of performance-based engineering (PBE). In PBE, a facility is designed so as to achieve a level of performance (above and beyond the life safety requirement) tailored to the expectations of the specific owner/occupant. There is considerable momentum in the professional community toward PBE as a long-term goal in earthquake engineering and hazard mitigation (indeed, PBE is the major focus of the research programs in the PEER Center). While PBE currently is in a state of flux, as would be typical of a new approach, all proposals to date have features in common. Several of these features are relevant for development of damage and loss assessment methodologies and for consequence-based engineering, as they relate specifically to social expectations regarding performance of infrastructure facilities. The brief review below is aimed at identifying these features.

A performance level is a statement of facility behavior or condition that can be related to user or occupant needs and serve as a vehicle for communication between owner, architect, engineer, and other stakeholders. When coupled with a stipulated hazard, it becomes a performance objective. In current codes, standards and regulatory documents, the (unwritten) performance objective is "life safety under a design-basis event," or something similar to it. Since the regulatory process in the United States focuses on life safety by statute, other performance objectives (e.g., serviceable performance under "ordinary" conditions) have received little formal attention, and have been left to the stakeholders to resolve individually. However, as noted above, this situation is beginning to change. The following paragraphs briefly review some highlights of recent progress in this regard.

FEMA 273/274/356. Perhaps the most widely publicized effort to establish performance-based engineering procedures was developed by the Applied Technology Council (ATC) under the auspices of the Building Seismic Safety Council (BSSC) and American Society of Civil Engineers (ASCE), with the support of the Federal Emergency Management Agency (FEMA). The work product was published as *NEHRP Guidelines (and commentary for seismic rehabilitation of buildings (FEMA 273/274)*, and has since been revised as a *Prestandard and Commentary (FEMA 356)*. While the effort was directed toward rehabilitation of existing buildings, it identified four specific performance levels (Operational, Immediate Occupancy, Life Safety and Collapse Prevention), and contains a set of tables that relate these levels to specific transient and permanent deformations for a broad spectrum of structural and nonstructural components. The relation between performance level and deformation is based on engineering judgment. Moreover, the structural and nonstructural performance levels are identified separately, with the implication that they can be considered separately in performance assessment of a specific facility. This implication is questionable.

SEAOC Vision 2000. The Vision 2000 project [SEAOC 1995] identified five performance levels: Fully Operational, Operational, Life Safety, Near Collapse and Collapse. The *SEAOC Blue Book* also recognizes five levels, but identifies them with the non-descriptive designators SP1, SP2, SP3, SP4 and SP5, accompanying each with a textual description of the damage to be expected. Consistent with the scope of the *Blue Book*, the focus is on structural systems. The non-descriptive nature of the designators is a barrier to communication with stakeholders outside the structural engineering community.

SAC (FEMA 350). The SAC project focused on the performance of steel moment frame structural systems. Other structural systems and the performance of nonstructural components were not considered. The multiple performance levels in the above two studies were reduced to two: Immediate Occupancy and Collapse Prevention. It was noted that it was very difficult to associate a specific structural response computed by finite element analysis (e.g., interstory drift, connection rotation) to performance levels (damage states) associated with life safety.

ICC Performance Code (2000). The ICC Performance Code stipulates four performance levels: Mild Impact, Moderate Impact, High Impact and Severe Impact. These four levels of impact are accompanied by textual descriptions, e.g., "Moderate Impact" is tantamount to repairable structural damage, moderate contents and nonstructural damage, short delay in re-occupancy. Facility Performance Groups are associated with a performance level and hazard intensity (e.g., "very large" events shall have no more than a "moderate impact" on hospitals), creating a *de facto* performance objective for each facility category and removing this decision from the hands of the engineer, architect or owner.

NFPA 5000 Code, Performance-based Option: Chapter 5 contains performance criteria for safety from fire, structural failure, as well as other desired attributes. Section 5.2.2 dealing with safety from structural failure identifies three levels: Serviceability, Immediate Occupancy, and Collapse Prevention. Each is accompanied by a textual description of what is involved at each level, e.g., Immediate Occupancy permits repairable cracking or permanent deformation of the structural and nonstructural elements, but the structure must not be unsafe for continued occupancy. Specific design scenarios (including load combinations) are stipulated in Section 5.5.3 for each performance level, removing this decision from the engineer, architect or owner.
The above brief review indicates that the coupling between structural performance levels and more general socioeconomic performance objectives is still relatively weak. Such performance objectives would include (but not be limited to):

- Loss of life
- Injury
- Direct economic losses from building damage
- Losses from infrastructure disruption

The latter two appear to be of special concern to many stakeholders in the building process, perhaps because current codes and standards appear to have met their life safety goals during and following recent earthquake disasters but have not prevented severe economic losses (both in terms of loss of facility and loss of opportunity, to be discussed subsequently). It is clear that the need to manage and control such losses in the future will drive many of the criteria for decision-making in consequence-based engineering. Measuring performance quantitatively is difficult; sophisticated reliability methods will be required.

6.3 Damage assessment

Performance-based engineering requires a relation between structural behavior and damage states. There are several significant research issues:

- The mapping between structural response and a damage state (drift levels giving rise to cracking, spalling, partition damage, cladding damage, etc)

- A mapping between structural response and a heuristic performance state (e.g., an association between interstory drift and life safety).

- Modeling the uncertainties in the analysis of structural behavior and system capacity that is necessary to identify the limit state and the relation between that limit state and specific levels of damage.

Such relations are an essential part of the database required to transform estimates of engineered facility response to damage levels and losses. Proposals to date have focused on structural deformations as the basis for measuring damage and loss. The implications of this approach and limits of its applicability require further examination.

6.4 Loss assessment methodologies

6.4.1 Review and appraisal of previous research

Loss assessment requires a methodology for predicting economic and social consequences of different natural hazards of varying intensity that can be used as a decision tool. These methodologies range from the qualitative to the highly quantitative. Their development and improvement has been motivated by the series of natural hazards (earthquake, wind and flood) that have occurred during the past three decades. In their development and implementation, loss assessment methodologies can be specific for one facility, generic for general categories of facilities (e.g., pre-stressed concrete bridges; light-frame wood construction), or regional in nature (residential construction in Southern Illinois). Moreover, they can address facilities holistically (a building or bridge system) or subsystems within the facility (moment-resisting frame; bridge girders). There obviously are gradations within these spectra of alternatives, and the selection of an appropriate decision tool depends on the availability of the supporting databases and the nature of the decision at hand.

Several loss assessment methods are reviewed briefly in chronological order to identify common features, facilitate understanding of the problem and identify some of the research issues that remain to be addressed.

<u>Seismic Design Decision Analysis</u>. This early study (Whitman, et al 1975) developed a procedure for examining tradeoffs between future losses against additional the cost of providing seismic resistance through alternate design strategies. The focus was on structural systems. A damage probability matrix,

$$P_{ij} = \Pr[\, D_i | MMI_j] \qquad (6.3)$$

was developed that expressed probabilities of being in specific discrete damage states, D_i, conditioned on the occurrence of earthquakes of specific Modified Mercalli Intensities, MMI_j, for a specific building category (reinforced concrete frames in apartment buildings between 5 and 20 stories in height in Boston, MA) and design strategy (conformance to requirements for Uniform Building Code seismic zones 0,1,2 and 3). Damage states were expressed qualitatively (e.g., light, moderate, heavy), and were related to cost, expressed as a damage ratio equal to the ratio of repair cost to replacement cost. With the hazard expressed in terms of probability of occurrence of earthquakes in terms of MMI, overall mean damage ratios then could be expressed in terms of alternative strategy. All sources of uncertainty (other than that in the seismic hazard) were encapsulated in the development of the damage probability matrices; these matrices were determined from a review of building performance during San Fernando earthquake of 1971 as well as several other large earthquakes that had occurred in the preceding two decades, supplemented with a heavy dose of professional judgment. Despite its simplicity, this early model has all the essential ingredients of the later more sophisticated models discussed below, in that it integrates information on earthquake magnitude and occurrence, structural response, damage and cost. The notion of a damage probability matrix (DPM) is one that recurs repeatedly in later studies.

ATC-13 (1985). This methodology was intended for estimating community/social losses, and addresses damage and losses associated with 40 different general categories of buildings rather than for individual buildings. Damage factors, expressed as the ratio of repair cost to replacement cost, were determined by expert opinion for each type of building construction as a function of Modified Mercalli Intensity. Damage probability matrices similar to those in the SDDA were determined for seven damage states, ranging from "no damage" to "destroyed." Losses considered included building repair and contents replacement, loss of use, injuries and fatalities. The influence of the damage states identified in this study can be seen in more recent proposals for estimating losses in specific building categories.

HAZUS (NIBS, 1997). This loss estimation methodology for buildings was developed by the National Institute of Building Sciences (1997) in a format to facilitate its use with a standard geographic information system. As with ATC-13, HAZUS is a category-based rather than building-specific evaluation procedure. It is intended for use as a tool for estimating future social and economic losses to a community, and for emergency preparedness and disaster recovery planning activities. HAZUS requires a portfolio of fragilities for generic structural and nonstructural components, related to four damage states: slight, moderate, extensive and complete – which are related to a damage factor, as defined in ATC-13 (repair cost/replacement cost). (Uncertainties in cost of specific damage states are not considered.) The fragilities are assumed to be described by lognormal distributions; where possible, the distribution parameters are defined from laboratory testing, but in most cases they are determined simply by judgment. Damage to building contents is treated separately from structural or nonstructural damage. Overall damage is obtained by summing damage to structural components, nonstructural components and contents.

Damage probability matrices from fragility assessment. Advances in structural reliability analysis supported by finite element platforms during the 1990's has made it possible to systematize and quantify the approach for establishing relations between earthquake intensity and motion characteristics to structural response and damage, making it possible to get away from the empiricism of earlier approaches. Singhal and Kiremidjian (1996) constructed fragilities and damage probability matrices using a Monte Carlo simulation approach involving nonlinear finite element dynamic analysis of building response to an ensemble of artificial non-stationary ground motions. Ground motion intensities for fragility assessment were characterized by spectral acceleration, S_a. As in the earlier SDDA study, the DPM expresses conditional probability of being in specific damage states vs. MMI, requiring a probabilistic relationship between S_a and

MMI. Uncertainties in structural capacity were included. The notions of "fragility" and "damage" were used interchangeably; dynamic structural response measured by the Park/Ang damage index was simply mapped to five damage states – none, minor, moderate, severe, and collapse. Cost impacts were not considered. The implementation of this method was category-based rather than building-specific. Three generic reinforced concrete frames were considered, representing the following categories – low-rise buildings, mid-rise buildings, and buildings over 8 stories – a building classification similar to that in ATC-13.

Later studies have built on this earlier work, providing methods for updating fragility and damage probabilities based on data collected from damage surveys following earthquakes. Singhal and Kiremidjian (1998) developed a Bayesian method for updating analytical fragility estimates for low-rise reinforced concrete frames in their 1996 study with observed building damage data from the Northridge Earthquake. This updating focused on the estimate of the median fragility; the logarithmic standard deviation was assumed constant. Confidence intervals on the fragilities for minor, moderate, severe damage and collapse were determined from the confidence interval on the median, defined by its posterior density. To develop a database of damage from the damage surveys following the Northridge earthquake, which were developed, in part, from local property tax values, it was necessary to establish a relation between damage state (e.g., moderate), damage index (e.g., Park/Ang DI = 0.2 - 0.5), and damage factor (10% - 60%, expressed as ratio of damage cost to replacement cost). ATC-13 results relating DF to damage state were used for this purpose. The relation between DF and DS was assumed to be piece-wise linear, an assumption also tested by Ang and DeLeon (1997). The confidence intervals on the estimate of the median broaden for the more severe damage states.

Other recent studies (Gardoni, et al, 2002; Shinozuka, et al, 2000) have shown how to incorporate both empirical evidence based on observational data and analytical predictions in the development and updating of structural fragility parameter estimates for bridge columns, have incorporated both aleatory and epistemic uncertainties in the updating process, and have considered model error. Extending the application from individual bridges to categories of bridges, Shinozuka, et al, (2000) constructed fragilities for categories of bridges as a weighted average of fragilities of individual bridges.

<u>ATC-</u>38. This study surveyed over 500 buildings located near strong-motion recording sites following the Northridge Earthquake, in an effort to correlate damage levels with strong motion parameters. Survey data included structural and nonstructural design and damage measures. No data were provided on repair costs, however.

<u>Assembly-based vulnerability</u>. In a recent study, the earthquake hazard, structural response, damage to individual assemblies, and costs of damage (repair and replacement) have been integrated together in a probabilistic framework to develop functions (denoted "vulnerability" functions) that describe the relation between earthquake intensity (measured by spectral acceleration) and cost (Porter, Kiremidjian and McGrue, 2001). The costs are measured by a damage factor, defined as in ATC-13. In contrast to the

studies reviewed above, however, this loss assessment methodology is based on engineering analysis, is highly quantitative and data-intensive, and is focused on the performance of individual buildings rather than generic building categories. The use of expert opinion is minimized. The fragilities are incorporated in such a way as be compatible with, and allow the use of, the fragilities from the HAZUS report, if required. Uncertainties at all stages of the analysis (including damage states and associated costs) are propagated through the model by Monte Carlo simulation. Uncertainties in the seismic hazard are not considered, the structural models appear relatively simple, and the distinction between aleatory and epistemic uncertainties is blurred. Nonetheless, the treatment of uncertainty at all stages of the analysis is among the most comprehensive of all methodologies developed to date, and the inclusion of uncertainty in the relation between damage and cost is noteworthy. This methodology has been tested for 19 wood frame buildings as part of the CUREE-Caltech Wood Frame Project to estimate losses in wood-framed residential construction (Porter, et al, 2002). The results can be used to determine optimal strategies for purchasing or insuring properties or for rehabilitation and retrofit, given the occurrence of specific earthquake threats.
While the studies above were conducted over a period of nearly three decades and had different goals and objectives, there are common features that emerge from them:

> The mapping of estimated engineering response to "damage state" is a recurring theme; furthermore, damage states tend to be treated as discrete rather than continuous. There are two studies that have related damage state to damage factors (cost ratios, as defined below) – SDDA (1975) and ATC-13 (1985) – and other studies have adopted the taxonomy of damage states in these studies, with minor variants. The arguments in support of this classification are more persuasive if considered in the overall context of risk communication or the need to interface with a broad group of stakeholders than they are on purely scientific or engineering grounds. It is easier to relate "severe" damage than "maximum drift between a and b" to a monetary unit (or range). However, there is some question as to the applicability of this classification scheme if damage predictions for individual facilities were to be aggregated by a decision-maker in making a damage prediction on a regional basis. It would seem more logical to aggregate facility damage over a region using quantitative metrics and convert the aggregated estimate of earthquake impact at the end decision point.
>
> The component or assembly-based methods tend to be more sophisticated than the category-based methods, and the epistemic uncertainties associated with their use are smaller. Moreover, it is evident that logical approaches can be developed to combining component-based fragilities, damages or losses in forecasting the likely damage to a community or region. In contrast, category-based methods, while applicable to regional loss estimation, would be more difficult to de-aggregate to assess potential losses to an individual facility. Accordingly, it is suggested that this approach be de-emphasized in further loss estimation development.

Damage probability matrices are an essential ingredient of expected loss modeling, and it appears that there are significant uncertainties associated with their construction. In at least one study, the DPMs were hind-cast from economic losses. The databases available to support this process are limited, and the uncertainties are large.

Most recent studies have asserted that the fragility can be modeled by a two-parameter lognormal distribution. This hypothesis has actually been tested in several studies (e.g., Shinozuka, et al, 2000). The epistemic uncertainty in the fragility model can be vested in the median, as the uncertainty in the logarithmic standard deviation has a second-order impact on the fragility (Ellingwood, 2001). This approach to modeling epistemic uncertainty has been common practice in seismic risk assessment in the nuclear industry for some time.

Those studies that have considered the economic impact of damage invariably have done so in terms of the ratio of cost of repair to replacement cost (denoted the cost ratio in the sequel). There are large uncertainties in the relation between cost ratio and damage, which only one of the studies (Porter, et al, 2001) seems to have incorporated. It appears that more refined models relating economic losses to damage states will have to be developed if consequence-based engineering is to reach its potential.

To develop loss models that can be applied on individual, community and regional bases, one needs to establish a perspective from which costs are measured, how loss of fixed assets is to be treated relative to loss of wealth or income streams, and what role is to be given to losses that can be identified but are difficult or impossible to measure. It must be borne in mind that some losses appear immediately while others appear only years later. Furthermore, the same cost elements have different consequences to different segments of society.

6.4.2 Loss accounting

Individual or local losses are borne by individuals and organizations situated in the immediate vicinity of facility damage or failure. They include direct losses of property and income, as well as income derived from the use of the facility. Emergency response costs should be included in this accounting. Regional losses, such as those in contiguous urban areas, include disruption to business and economic activity in the affected area, as well as costs of disaster recovery operations. National losses impact the entire United States, and represent a social cost to the nation. The feature that distinguishes national losses from regional and local losses is that economic impacts are greatly reduced. In a market economy, adverse regional and local impacts often create incentives for compensating behavior elsewhere, and are balanced by increased employment and business activity elsewhere in the national economy.

Earthquake loss assessments must be conducted on a carefully balanced combination of local and regional accounting if the losses are to be compared equitably with other costs associated with regulation and natural hazard risk mitigation. Basing policies on a purely national perspective may place an undue burden on local communities; on the other hand, basing policies on purely local considerations is inefficient, as the benefit to the nation as a whole is overestimated.

6.4.3 Loss categories

In a general sense, studies conducted in the United States and abroad to measure losses associated with various natural hazards tend to categorize damage and loss by immediacy of impact (direct, indirect) and whether they can be monetized (tangible vs. intangible) (Ellingwood, et al 1993). Tangible losses can be direct (e.g., emergency response and cleanup; property and other fixed capital assets) or indirect (e.g, loss of production; opportunity costs). Similarly, intangible losses can be direct (e.g, number of fatalities) or indirect (disruption to lifestyles). Some examples are given below to convey a sense of the complexity required of the loss estimation methodology in order for it to be useful as a tool for public decision-making.

Tangible losses

Fixed capital assets include real property as well as manufacturing equipment, vehicles and other property. Their ownership may rest in either the public or private sector. They represent an investment in a durable entity that is expected to provide a stream of services with some monetary value over time. Such assets normally are valued on the basis of market price, replacement cost, depreciated cost, or restoration cost (Ellingwood, et al, 1993). From a social perspective, damage or loss of capital assets should be measured as a loss of services that they otherwise would provide. This measurement requires that a discount rate be selected. It is well-known that individuals, businesses, government agencies discount future costs and risks differently, with individuals (risk-averters) selecting a higher rate than public agencies (which tend to be more risk-neutral).

Emergency response and mitigation costs include medical assistance, evacuation, temporary housing costs, as well as governmental grants and subsidies.

Repair costs must include the cost of demolition/removal of debris, re-design and retrofit, material costs, including transportation and rental of equipment, and labor costs. Direct profit and overhead are typically between 10 and 20% of total direct costs. The cost of replacement should be evaluated under the assumption that the function of the new facility will be essentially the same as the old; if the function is to be expended or modified, the incremental cost should be taken out of the evaluation.

Loss of income streams should include lost salary and wage payments to workers displaced by the earthquake, and should be calculated net of losses that would be included in any measure of losses from capital assets. The lost income should be computed as the difference between the stream of income that would have occurred if the earthquake had not occurred and the income expected, given the occurrence of the earthquake in order to avoid double-counting losses.

Indirect costs should include disrupted activities (business and living expenses) and loss of opportunity.

Intangible losses

Intangible losses include those that either are not supported by available data on which monetary impact can be computed – environmental impacts - as well as losses that are truly intangible – disruption to daily lifestyles or peace of mind, stress-induced illness or other psychological factors, destruction of objects with sentimental value. Morbidity and mortality are included in this category because of the controversy that surrounds placement of value on human life and the distaste that this engenders. Direct losses that result from trauma sustained during or immediately following the earthquake and result in attributable injury, disability or death can be assessed in some instances. Indirect losses are those that show up in the population some time following the event. The fact that these losses are difficult or impossible to monetize does not mean that their impact should be ignored. They may, in fact, be a significant fraction of total losses from large earthquakes. Unfortunately, there is little research to support such loss estimates.
Uncertainties in the above costs are substantial, and most studies have not attempted to assign uncertainties to the cost estimates. Decision tools should include this source of uncertainty. In situations where only a range of cost values can be determined (perhaps by expert opinion) and other information is not available, the uniform distribution as the distribution of maximum entropy can be assumed.

6.4.4 Development of vulnerability functions

Vulnerability (or a *vulnerability function*) is defined in this study as the probability of incurring losses equal to (or greater than) a specified monetary unit, conditioned on the occurrence of an earthquake with a specified intensity (measured by spectral displacement, velocity or acceleration). For purposes of further discussion, we distinguish vulnerability from fragility. As discussed in previous chapters, the customary definition of fragility is the conditional probability of occurrence of a limit state, denoted LS, as a function of a specific intensity of earthquake excitation. That intensity ultimately must be expressed in units that are dimensionally consistent with the specification of the hazard because a fully coupled risk assessment begins with the convolution of the fragility and the hazard. The LS can be any undesirable event. (In the context of structural engineering, for example, the LS customarily is expressed in terms of strength or deformation units: moment from applied forces exceeding plastic moment capacity, lateral deformation within a story from applied forces exceeding 5% of story height, and so on.) In contrast, the vulnerability is a conditional probability expressed in terms of a decision variable, i.e., the probability of damage or economic loss exceeding a stipulated level, given the occurrence of a specific intensity of excitation. The vulnerability of a system can be determined by aggregating the vulnerabilities of a set of elements, components and sub-systems comprising that system, provided that component fragilities and cost functions that map specific structural limit states to states of damage can be constructed.

The development of vulnerability functions requires characterization of ground motion, determination of structural response, identification of degrees of structural damage, and cost of specific damage levels. A basic approach to vulnerability assessment is outlined in the following paragraphs. This provides an opportunity to discuss some specific research issues that must be addressed in developing loss estimation methodologies for civil infrastructure.

First, we must relate component or system response to earthquake demand. It is assumed that this relationship can be established through a relatively simple equation of the form developed in the SAC Project (Cornell, et al, 2002)

$$\theta = a\, S_a^{\,b}\, \varepsilon_1 \tag{6.4}$$

in which θ = structural response quantity computed from the numerical model of the system (e.g., an inter-story drift in a frame or a floor acceleration), S_a = spectral acceleration (other ground motion parameters might be considered, where appropriate), a and b are constants determined from the model, and ε_1 is a zero-median error term that describes the uncertainty in the relationship. Research conducted in the SAC Project indicates that for steel moment frames, $b \approx 1$; however, this simplification remains to be tested for other civil infrastructure. Relations such as that in eq. (6.4) generally require the use of advanced computational platforms; eq (6.4) is convenient for discussing fundamental concepts and for proof-of-concept studies involving relatively simple systems. While in structural engineering applications this relationship has been assumed to be univariate, for damage and loss assessment purposes the relationship may involve a vector of variable.

Second, facility damage must be related to the response, θ. This relationship might be expressed as,

$$D = c(\theta - \theta_{th})^{d}\, \varepsilon_2 \tag{6.5}$$

in which "damage", D, is a measure of the impact of response θ on the performance of the component or system, θ_{th} is a threshold below which damage can be assumed to be zero, and ε_2 is a random variable with median zero, describing the uncertainty in the damage metric. As noted in the review of previous studies, this relation usually has been assumed to be discrete, i.e., D = {none, minor, moderate, severe}; whether this assumption is important because of the manner in which damage data are reported requires further investigation.

Finally, the cost of damage (expressed in relative or absolute terms) is expressed as,

$$C = A\, D^{B}\, \varepsilon_3 \tag{6.6}$$

in which A and B are constants of the model and ε_3 is a random variable describing the uncertainty in the estimate of cost associated with specific damage states. Generally, the constant B is less than unity, as a component is likely to be replaced rather than repaired after sustaining a moderate level of damage, especially if it has been in service for some time (historical facilities would be an exception to this general rule).

Eq (6.4) can be determined with the assistance of advanced computational platforms. However, Eqs. (6.5) and (6.6) are mainly empirical at the current state-of-the-art and rely heavily on subjective judgment and experience. All three equations are dependent on category of buildings, bridges or other infrastructure facilities. Construction type, occupancy, size, when designed and constructed (by which code, if any), impact of aging and maintenance on performance and integrity all play an important role in these models. Note that all three stages of the analysis contain uncertainties, as embodied in the parameters ε_i, i = 1,2,3. While it is widely recognized that the uncertainties in ground motion and its impact on a system are substantial, the uncertainties in damage and cost are also significant. As a simple indication, the logarithmic standard deviation in ε_1 for an earthquake of given return period typically is approximately 0.30 (Cornell, et al, 2002). [The logarithmic standard deviation of a random variable is approximately equal to its coefficient of variation, a common dimensionless measure of uncertainty in that variable.] The variability in ε_2 appears to be greater than 0.30 for many structural and nonstructural items within a building, while the variability in ε_3 is on the order of 0.50 or more (Porter, et al, 2001). Neglecting these uncertainties can distort the risk analysis and ensuing decision process; their influence on the decision process remains to be addressed.

6.5 Proposed test-bed for uncertainty analysis and loss estimation methodology

The feasibility of the methodology should be tested on a subset of facilities where it is believed sufficient data on damage and failure costs exist. The project team has identified the following test cases (Table 6.1) for consideration in years 1 and 2. The criteria for selection include perceived relative importance in the typical building inventory in urban areas in the Eastern United States, availability of data, and the opportunity to leverage research ongoing independently on other projects. The scope of assessment can be expanded in the out-years. Fragility functions can be used with an appropriate cost model to demonstrate the economic benefits from design, QA/QC during construction, code enforcement, in-service maintenance and rehabilitation and retrofit following the occurrence of an earthquake.

Table 6.1 Facilities for analysis of uncertainty and loss estimation

Construction technology Building occupancy	Steel	Reinforced concrete	Masonry	Wood
Residential				X
Office	**X**	X		
Commercial		**X**	**X**	
Hospitals	X	X		
Schools			X	
Industrial	X			
Emergency facilities			X	

Three of these cases, indicated in bold-face type, are being analyzed concurrently in MAE Center Project DS-4.

6.6 Research issues

The review of damage and loss estimation methodologies in previous sections of this chapter has highlighted a number of research issues that must be addressed during the next four years to allow consequence-based engineering to achieve its full potential as a decision tool. Translating limit states and their (uncertain) probabilities of occurrence into states of damage and ensuing costs and losses requires an interface between engineering, economics and social policy. The following discussion highlights some of the issues.

6.6.1 Earthquake hazard

Early loss estimation studies specified earthquake hazard in terms of Modified Mercalli Intensity. Most recent studies (at least in the structural engineering area) have adopted spectral acceleration, velocity or displacement as being more descriptive of the impact of strong ground motion on the built environment. However, while spectral acceleration or velocity may be the parameter of choice in performing engineering analysis, it may not be the best parameter for communicating hazard intensity to a non-technical stakeholder group. MMI is relatively straightforward in this regard. Loss estimation methodologies must be developed so that the hazards, risks and losses can be communicated in alternate, albeit equivalent, vehicles.

Comprehensive loss estimation methodologies should be supported by a fully coupled probabilistic seismic risk assessment to quantify the cost-effectiveness of various design, mitigation and retrofit strategies. Measures of uncertainty necessary for such a PSA are not available in all cases. For example, the USGS seismic hazard curves represent an estimate of the median hazard at a specific site for a given spectral acceleration. The epistemic uncertainties associated with this hazard are large, particularly in the Eastern US, where not all potential sources of large earthquakes have been identified. In connection with probabilistic risk assessment of nuclear power plants, a number of site-specific seismic hazard analyses have been conducted, which involve construction of seismic hazard curves from alternate plausible seismo-tectonic sources proximate to the site. The scatter evident in these seismic hazard curves is consistent with a model of uncertainty in the median described by a logarithmic standard deviation, β_U, on the order of 50 to 100 percent. The impact of such a large uncertainty on the decision process can only be speculated at present, but it certainly would be very large.

6.6.2 Facility response and damage prediction

Damage and loss estimation require a framework for estimating the probability of specific damage states occurring due to earthquake ground motion. Two ingredients are required: a seismic fragility, which describes the probability of specific responses, and a relation between those response and "damage," which is likely to be defined qualitatively rather than quantitatively.

Fragility models of components and systems can be developed on several levels. The first is empirical, involving statistical analysis of post-disaster inspection data. This approach offers a broad picture of general categories of components and systems (e.g., reinforced concrete bridge columns). The uncertainties associated with this approach and their impact on decision-making is likely to be large. The second is theoretical, involving finite element (or other numerical) analysis of components and systems, using a postulated model based on the physics of the problem (e.g., the use of finite element analysis and random sampling to determine the probability that steel frames reach specified performance levels). This approach is component or system-specific; on the other hand, the verification and validation of the postulated model can become an issue, particularly when the system is challenged by rare events. A third is based on judgment and expert opinion, which has been used successfully in the nuclear industry when test data are limited or unavailable, and design-basis conditions have not been experienced.

There is a compelling need for relatively simple fragility analysis methods that can be used for rapid post-disaster condition assessment. One approach that has been used successfully with mechanical, electrical and structural components is to scale upward from the design calculations, using whatever limited data are available supplemented by judgment to determine the scaling factors and their uncertainties. In this approach, one might model the capacity as,

$$R = R_n F_s F_u F_{an} \tag{6.7}$$

in which R_n = capacity determined from design calculations, F_s = strength factor, describing the reserve strength, F_u = factor that describes the ability of the system to withstand demands well into the inelastic range through yielding, energy dissipation, or similar mechanisms, and F_{an} = analysis factor that describes the (epistemic) uncertainty in the analysis procedure used to determine capacity. Assuming that the scaling factors, F_i, are statistically independent, the median capacity and the uncertainty (defined by the logarithmic standard deviation) become,

$$m_R = R_n m_{Fs} m_{Fu} m_{Fan} \tag{6.8}$$

$$\beta_R = \sqrt{\beta_{Fs}^2 + \beta_{Fu}^2 + \beta_{Fan}^2} \tag{6.9}$$

The median "anchors" the fragility curve, which generally is assumed to be described by a lognormal distribution, while β_R determines its slope. Like those from the theoretical approach, the fragilities determined from this approach can be made to be facility-specific and are likely to be sharper than those from the empirical approach. However, the design documentation must be available in order to determine R_n, which serves as the starting point for the assessment. Such documentation may not be the case when older facilities are assessed.

In a general sense, performance is measured by damage and loss, requiring the response parameters to be “mapped” to heuristic performance goals. One might, for example, perform a finite element analysis of a building frame system using a suite of earthquake ground motions that yield a median spectral acceleration (at the fundamental period of the building) with a probability of 2% of being exceeded in 50 years. The building response to this suite of accelerograms is, of course, random. Suppose that the maximum interstory drift is selected as the parameter to measure system performance. The median and uncertainty in this response parameter for this suite of accelerograms for a steel moment frame might be 0.05 and 0.30. Three questions arise immediately: (1) What is the relation between the maximum interstory drift and damage, as a percentage of replacement cost? (2) Can damage to nonstructural components be inferred from the response of the structural frame? and (3) more fundamentally, if the structural performance metric is chosen to be some other response parameter, will the damage measure/replacement cost be the same? The uncertainties in such relations are very large, as Scawthorn, et al (1981) found when comparing plots of motion intensity vs damage to mid-rise steel and reinforced concrete buildings in the Sendai area subjected to the Miyagi-ken-oki earthquake of 1978. Damage accumulation is strongly dependent on frequency content and duration of strong motion. It now is widely recognized that peak ground acceleration is not a good indicator of damage potential. Vulnerability analysis must take the nonstationarity and duration of ground motion into account.

Current procedures such as those contained in FEMA 273 (1997) focus on force (or acceleration) and deformation for both structural and nonstructural components, and component detailing. The relation between these parameters and the performance of individual components and systems remains to be established. Other parameters – velocity, frequency content, duration of cycling – may also be important for damage prediction, but have received less attention than acceleration or deformation. The FEMA 273 guidelines on interstory drift (and those in the SEAOC vision 2000 document) continue to be controversial. Moreover, they have not been mapped into cost of damage.

6.6.3 Loss estimation

Relations such as eqs (6.4) through (6.6), whether explicit or implicit, can be created by statistical/empirical evidence (past damage) coupled with statistical procedures such as regression analysis, by expert opinion (Delphi), by engineering principles and analysis, or an appropriate combination of these techniques. They may represent individual buildings or broad categories of buildings; in the latter case, there is an additional source of uncertainty, i.e., variation among different buildings within a specific building type (light-frame construction). Constructing such functions will be a major research challenge in the development of consequence-based engineering. In the structural risk area, it has been found that one can go from building-specific to building-category vulnerabilities by direct summation. However, it is difficult to go in the opposite direction because details of specific construction necessarily are lost. Thus, it will be assumed that a facility-specific rather than facility-category approach should be taken.

The vulnerability of distributed systems can be determined using the behavior of individual components, subsystems and elements as building blocks and obtaining an estimate of the aggregated system using a summation process. The question must be answered as to how damage/costs of individual components contribute to damage/costs of the system as a whole. It seems obvious that the damage to the system can be obtained from the damage to its constituent parts by summation, but this assumption needs to be tested, as the constituents of a distributed system may interact in complex manner. Moreover, there is the issue of scale – if a complex distributed system is to be evaluated from the behavior of its constituents, there must be a portfolio of "standard" component and element fragility functions that can be integrated into system risk models, as well as standard "repair/replacement" cost models for repeated use in loss estimation. Associated with the use of such standard models is additional epistemic uncertainty, arising from the lack of granularity in the modeling of specific components.

Most previous studies have measured vulnerability as the cost to repair damage (or economic loss), expressed as a fraction of replacement cost. This normalized cost is plotted vs. severity (PGA, spectral acceleration, MMI, etc.) to yield a "vulnerability curve. One problem with this approach is that the normalized loss ratio (damage/replacement) depends on the nature of the building occupancy as well as the type of structural system. The ratio is different for residential (occupant-owned), multi-family residential (rental), small commercial, large commercial, schools, hospitals, fire stations, etc. The cost (or loss) function must be evaluated differently for residential buildings than for businesses (Kanda and Shah, 1997).

Extensive databases are required to support loss estimation. It would be helpful if these databases would express loss statistics in a consistent format. For example, databases developed by the insurance industry naturally quantify monetary damage in a manner that is useful for premium rate-setting: damage as ratio of market value (excluding value of land) vs replacement cost. Claims adjustment practices factor into these databases. Considering building facilities, the cost of damage from an earthquake often depends more on damage to nonstructural rather than structural components. For example, damage to fire sprinklers that causes water damage to contents and interior finish materials is major source of claims, even in buildings where little structural damage has occurred (Hamburger, 1996).

There are a number of miscellaneous costs that should be included in the modeling process. Among these is the impact of construction/installation quality on damageability of new and existing construction. The quality of construction and maintenance in service has a large impact on seismic vulnerability. Thus, in a minimum cost environment, the cost of frequent inspections during construction can easily be amortized over the life of the facility. Discount rates must be established to compute the present value of maintenance, repair or other mitigation strategies. Furthermore, indirect costs can be difficult to estimate, and yet may play a major role in loss estimation. The cost of "occupancy interruption," or "lost business opportunity" are important, and research must be conducted to determine how to model them.

6.6.4 Decision analysis and risk communication

Losses include human casualties, repair cost, loss of contents cost, loss of use cost, indirect costs. Perhaps the foremost issue is how exactly to measure such losses. Risk communication issues arise in the strategies selected for presenting information to the stakeholders - structural engineers, building owners or tenants, occupants, building regulators, lenders and insurers, urban planners, and the public. Tradeoffs between potential losses and the cost of reducing expected losses must be communicated in understandable terms. There also is the issue of communicating uncertainties in loss. This can be accomplished through a confidence statement, e.g., we are 90% confident that given the occurrence of a specific design/assessment scenario, the loss to a facility will be between 20% and 50% of replacement value. The ability to make such a statement requires a facility-specific type of fragility and vulnerability analysis, and an expanded database relating loss (expressed as a percentage of replacement cost) to facility response. Such databases are not available at present for civil engineering infrastructure.

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7. RESEARCH ISSUES

7.1 Introduction

The review of techniques for modeling, analysis and management of uncertainty in the previous sections of this report has highlighted a number of research issues that must be addressed during the next several years to allow consequence-based engineering to achieve its full potential as a decision tool. These issues are collected in several general categories: ground motion modeling, structural behavior and facility response, performance measurement, damage prediction and loss estimation, and risk communication and decision-making.

7.2. Ground motion modeling

The U.S. Geological Survey website provides an estimate of spectral acceleration at different return periods for rock for any specified latitude and longitude in the 48 contiguous states. The seismic hazard methodology from which these estimates are obtained now is relatively mature, and an estimate of median seismic hazard (spectral acceleration, S_a, vs annual probability that it is exceeded) now can be obtained for any building site. To assess the performance of a building or other structure at that site, a suite of ground motion records also is required. Such a suite often is constructed by scaling an ensemble of real accelerograms so that the resulting spectral accelerations match S_a at the fundamental period of the building or over a band of periods. Since the nonstationary nature of the ground motion in both frequency and time domains must be captured to achieve a reasonable simulation of facility performance in the nonlinear range, most recent studies have used scaled accelerograms that capture this non-stationarity naturally, rather than simulated records that must be enveloped and filtered to achieve the desired characteristics. A standard method for constructing an appropriate ensemble of ground motion records for different site conditions must be developed in order for independent investigators to achieve consistent results. Moreover, the importance of other parameters – velocity, frequency content, duration of strong ground motion – has not been examined thoroughly.

Regional damage and loss assessment using quantitative seismic risk analysis tools requires that the variation in ground motion intensity over an urban, county or state area be determined. The density of ground motion recording stations may not be sufficient to model this variation at the scale required in loss assessment, and the U.S. network provides such information only for rock. Techniques must be developed for adjusting suites of ground motion so as to take local site conditions into account.

Comprehensive loss estimation methodologies should be supported by a fully coupled probabilistic seismic risk. The USGS seismic hazard curves represent an estimate of the median hazard at a specific site. The epistemic uncertainties associated with this hazard are large, particularly in the Eastern US, where not all potential sources of large earthquakes have been identified. A number of site-specific seismic hazard analyses have been conducted in connection with probabilistic risk assessment of nuclear power plants; these involve construction of hazard curves from alternate plausible seismo-tectonic sources in the vicinity of the site. The scatter evident in these seismic hazard curves is consistent with a model of uncertainty in the median hazard described by a logarithmic standard deviation, β_U, on the order of 50 to 100 %. The impact of such a large uncertainty on consequence-based engineering is a cause for speculation at present, but it certainly would be very large. Clearly, a procedure for including the impact of epistemic uncertainty in seismic hazard on damage and loss estimation is required.

7.3 Facility response

Computational platforms for assessing response of individual facilities and complex systems of facilities to earthquake ground motion have increased in their levels of sophistication during the past decade. Notwithstanding these improvements, the epistemic uncertainties associated with the use of these platforms by different analysts, who may be equally well qualified but are working independently with competing platforms, are substantial. Two major sources of such uncertainties arise from (1) differences (often subtle) in the capabilities and limitations of the platforms, and (2) differences in the way that analysts may choose to model the systems of interest – granularity, simplifying assumptions, and similar factors. When a group of experts is brought together in the course of an analysis so that they can interact with one another periodically, it often is found that that their results, obtained independently, ultimately converge to a common solution. But that is not representative of how a facility risk analysis is likely to be performed as part of consequence-based engineering. Instead, the fundamental attributes of the methods used to analyze the system must be agreed upon and identified prior to the analysis to obtain comparable results. This is a research issue. Variations in response obtained using any (permitted) analysis having those attributes are part of the epistemic uncertainty in the analysis, and must be displayed along with other uncertainties in the risk analysis.

Fragility modeling is an essential ingredient of damage and loss assessment. As noted in previous sections, approaches to fragility modeling can be empirical (regression analysis of post-disaster data), theoretical (based on computational platforms), or judgmental (based on consensus estimation surveys, expert opinion). Appropriate fragility approaches must be tailored to specific damage assessment and loss estimation contexts and the epistemic uncertainties associated with the use of these different approaches must be identified. For example, one might envision fragility models tailored to several distinct purposes:

Rapid Post-Disaster Assessment: Requires a coarse model, tailored to groups of facilities or building inventories (perhaps compatible with HAZUS models), with high epistemic uncertainty;

Pre-disaster planning, urban planning, allocation of public resources, emergency facilities: Requires a model [perhaps developed by scaling upward from code provisions, as in Eqns 6.7 – 6.9] that is applicable on a wider scale, with epistemic uncertainties that reflect the level of the model. [Perhaps buildings of a particular occupancy designed by a specific code (e.g., *Uniform Building Code* 1994; *BSSC/NEHRP Recommended Provisions* 1997) would be treated as a group; the idea is akin to the "superstation" concept used to artificially increase the wind speed database used to develop design wind speed contours in *ASCE Standard 7-02*].

Damage Synthesis: Requires a model with a relatively low epistemic uncertainty tailored to an individual facility.

Comprehensive post-disaster assessment, insurance claims: Requires a relatively fine model of an individual facility, including structural members and connections, and interface of structural and nonstructural components (e.g., hung ceilings, parapets) to evaluate suitability for occupancy or determine need for rehabilitation, repair, or demolition.

7.4 Measurement of performance, damage prediction and loss estimation

Performance metrics for consequence-based engineering must be established. At the most fundamental level, performance metrics should include: lives lost, injuries, facility damage as a percentage of replacement cost, and losses due to economic disruptions, business downtime and lost opportunities (both direct and indirect losses). One of the most difficult aspects of performance-based engineering is the mapping of engineering calculations and predicted behavior to heuristically stated performance objectives. A foremost research issue is how exactly to measure such losses and assign uncertainties to them for decision purposes. It also would be helpful if these databases would express loss statistics in a consistent format.

Quantitative evaluation tools are available to the engineer to support analysis, design and condition assessment (e.g., finite element software; traffic flow models, etc). Unfortunately, public decision-making often is made in an environment where the social/political/economic goals are not expressed quantitatively. The uncertainties in the relations between these quantitative models and the social/economic goals is upon which decision is based has not been established in most situations. Even as deceptively simple a question as what is the relation between interstory drift and damage, expressed as a percentage of replacement cost, cannot be answered easily at the current state of the art. Moreover, techniques for modeling uncertainty in the performance of social and organizational systems in communities that are at seismic risk require additional research and development.

Relations such as eqs (6.4) through (6.6) must be developed from statistical/empirical evidence (past damage) coupled with statistical procedures such as regression analysis, by expert opinion, by engineering principles and analysis, or an appropriate combination of these techniques. The standard errors must be determined for uncertainty modeling.

Constructing such functions will be a major research challenge in the development of consequence-based engineering.

Most previous studies have measured vulnerability as the cost to repair damage (or economic loss), expressed as a fraction of replacement cost. The normalized loss ratio (damage/replacement cost) depends on the nature of the building occupancy as well as the type of structural system, and must be determined for general categories of building occupancy (e.g., general offices, single-family and multi-family residential, commercial, and industrial). Such databases on economic loss tend to be biased, in the sense that more severely damaged buildings tend to be inspected more carefully, skewing the damage statistics toward the more heavily damaged facilities. Conversely, lightly damaged buildings may not be inspected or evaluated. Thus, vulnerability curves based on such surveys would tend to overestimate the damage level to a particular structure, but would underestimate the total number of buildings damaged. Identifying the biases in current loss assessment procedures is a research issue.

Miscellaneous costs may have a large impact on loss assessment. The uncertainties associated with such costs is very high. For example, the quality of construction and maintenance in service has a large impact on seismic vulnerability, and costs associated with incremental increases in construction/installation quality on damageability of new and existing construction must be determined. Indirect costs may play a major role in loss estimation. The cost of "occupancy interruption," or "lost business opportunity" are important are important to the decision process, and research must be conducted to determine how to model them and their uncertainties.

Cost/benefit analysis and loss estimation requires that all losses be monetized in some way. Because expressing morbidity/mortality in terms of monetary losses appears to be politically unacceptable, it appears likely that a dual set of performance metrics will have to be developed – one involving monetized losses and the second involving social costs. This may require two risk metrics in consequence-based engineering rather than one.

7.5 Risk communication and decision analysis

Risk communication issues arise in the strategies selected for presenting information on expected facility performance to the stakeholders - structural engineers, building owners or tenants, occupants, building regulators, lenders and insurers, urban planners, and the public. There are tradeoffs between potential losses and the cost of reducing expected losses. Consequence-based engineering will fail to achieve its objectives if performance and risk issues cannot be explained to stakeholder groups and decision makers in understandable terms.

A first issue arises in attempting to convey notions of risk from rare events. Most stakeholders are not trained in, or comfortable with, the tools of quantitative risk analysis, especially when the decision process involves very rare events. Take, as a specific example, the manner in which a design-basis earthquake event is communicated. The U.S. Geological Survey (and the *BSSC/NEHRP Recommended Provisions* and *ASCE*

Standard 7-02) stipulate this "maximum considered" event as one with a 2% probability of being exceeded in 50 years (or, equivalently, a 2,475-year mean recurrence interval event). Most non-technical stakeholders have difficulty in understanding either of these measures. In particular, they interpret the return period specification as meaning that the earthquake will occur once every 2,475 years; they find it confusing when informed that this is not the case, and this makes the entire risk analysis less credible. Stating that the design-basis earthquake is one with a magnitude 7.0 occurring 20 km from the facility site is more understandable. Unfortunately, such a statement fails to convey any sense of uncertainty in the hazard or performance, and furthermore requires that the earthquake hazard be de-aggregated, a process which carries uncertainties of its own.

A second issue arises in stating confidence in the risk or loss assessment. Suppose, consistent with current LRFD practice, that a structural system is designed for the 10%/50 year (or 475-yr) event. The median probability of failure threatening life safety, obtained by convolving the fragility with the median seismic hazard curve, might be on the order of 10^{-4}/yr. How is that communicated to the building owner? By saying that "the probability of failing to meet the performance objective is 10^{-4}/yr"? Or that "we are 90% confident that the performance objective of life safety in 50 years will be achieved"? The answer to this question has an impact on the manner in which epistemic uncertainties must be propagated through the analysis.

Third, there is the issue of communicating uncertainties in loss. This can be accomplished through a statement measuring uncertainty in the loss (say 40%), a statement of probability [e.g., What is the probability that the underwriter will have to pay on a claim in excess of the standard deductible (producer risk)? What is the probability of financial ruin in the absence of insurance (consumer risk)?], or a statement of confidence (e.g., we are 90% confident that the loss will be between 20% and 50% of replacement value). How risk aversion should be included in the policy for individual facilities or for aggregations of facilities is an open question.

Fourth, a statement of risk that is not accompanied by a time frame is difficult to benchmark to other risks. A reference period on which the losses are based must be determined for consistency in risk communication. Should the losses be annual losses? Or should losses be based on a (approximately) 50-year service period for a facility? The insurance industry appears to prefer to work with annualized losses. On the other hand, annualized earthquake losses tend to be relatively small, might appear to diminish the importance of earthquakes in comparison to other natural hazards in public decision-making, and would give a false impression of the enormity of impact of a design-basis earthquake were it to occur in an urban area.

Finally, it is apparent that different risk communication strategies should be adopted for different groups. Public decision-making is likely to favor multiple decision-makers rather than the individual decision-maker. This being the case, there is a very real chance that what is communicated to individual groups will not be consistent across group lines, or will not appear to be consistent. Any real or apparent inconsistency will cast doubt on the entire process. Accordingly, a protocol must be developed to ensure that the fundamental issues related to risk and uncertainty are not distorted by the risk communication process.

7.6 Supporting databases

- Consequence-based engineering is data-intensive, and extensive databases will be required for successful implementation. While data needs have been discussed in the previous sections, here we summarize for quick reference some of the more important data needs. The accumulation of data necessary to define various uncertainties impacting the decision will be a continuing process. A set of protocols must be established for identifying significant (and insignificant) sources of uncertainty, and for collecting and modeling those that are identified as being significant. Development of such databases should have a high priority: Relations between system response, damage, and economic loss;
- Spatial distribution of damage;
- Relation between damage and direct economic loss;
- Relation between loss of function, recovery time and economic loss.
- Identification of stakeholder preference for consequence measurement;
- Identification of stakeholder preference for risk communication.

7.7 Other research issues

Other research issues may be important to the success of consequence-based engineering. These do not fall into any of the major categories identified above, but are listed below for completeness:

- There is a need for real-time analysis and updating of fragilities and vulnerabilities in the aftermath of an earthquake to make risk-informed decisions regarding re-occupancy/rehabilitation or repair/demolish. This requires:
    - "Dynamic" uncertainty analysis;
    - Quick methods for loss assessment, and validate component and whole-building fragilities using post-earthquake damage data.

- The effectiveness of local code enforcement in earthquake risk mitigation should be assessed;
- The effectiveness of inspection as a tool for code enforcement should be determined;

- Some (perhaps many) uncertainties will have to be postulated based on expert pinion to exercise the methodology and to guide the acquisition of additional data. How should that expert opinion be solicited and analyzed?
- A broader question is what is the role of performance-based engineering in the overall context of consequence-based engineering, and how do the uncertainties collected in the PBE activity interface with those in the current study?
    - Are they the same?
    - Are they presented in the appropriate (compatible) form?

- Do we need to jointly decide with PEER researchers on “domains of attraction” in uncertainty analysis and coordinate efforts, or do we proceed along independent lines of inquiry?

Finally, a practical issue relates to value engineering, or persuading the decision-maker that it is in his/her interest to invest in quantitative risk assessment and loss estimation in the hope of achieving enhanced facility performance levels at reasonable cost. The test-bed problems to be considered in the out-years should be structured so as to provide a convincing demonstration of the value added to the decision process.