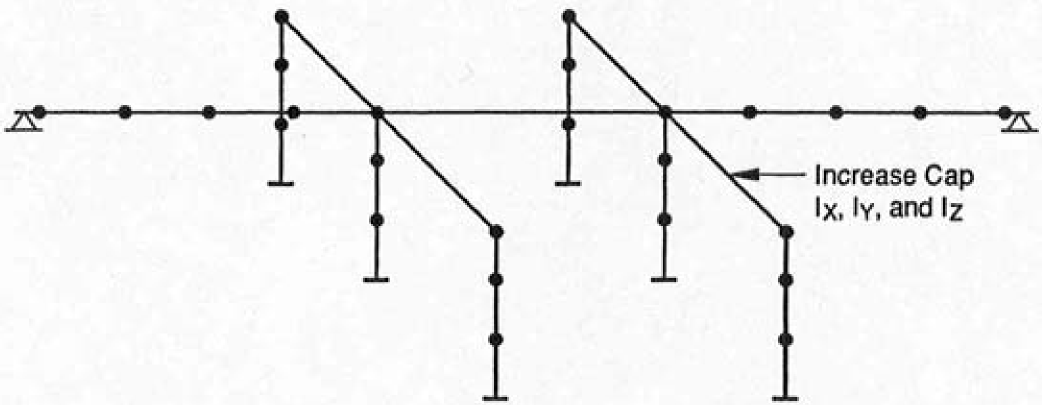
DYNAMIC MODEL ASSUMPTIONS AND ADJUSTMENTS

Following is a discussion and example of how to select and adjust stiffnesses at abutments to simulate realistic structure movements.

Also included is an example determination of abutment seat lengths for seismic movement in the longitudinal direction when restrainers are not used. The method is based on the 1983 AASHTO Seismic Guide Specifications.

Bents



The current BAG generated model for multi-column bents is a stick model which attempts to represent the horizontal stiffness of the actual structure. To simulate the very stiff deck unit all the cap moments of inertia are increased. The resulting column forces should be approximately the same because of the rigid cap.

Non-prismatic columns are preferably reinforced with uniform spirals assuming the flared portion as not effective for seismic loads. These columns should be modeled as prismatic members for seismic loads. If the flare is to be designed as a ductile component, the model should be non-prismatic.

Care should be taken to make sure that the total mass and stiffness of the structure is represented in the model. Large caps, flared columns, extra surfacing, and heavy utilities may add additional mass to the system and should be included in the analysis.

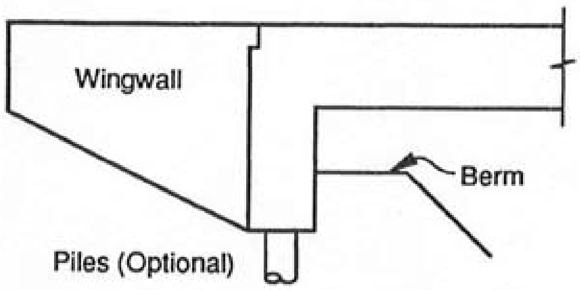
Columns in multi-column bents should be pinned at the footing to reduce the loads into the footings.

The bridge should be able to deform in a ductile manner beyond its elastic limit. The ductile action occurs whenever the elastic seismic moment is greater than the yield capacity of the column. Plastic hinges normally form in the column members. These components must therefore be capable of withstanding considerable plastic deformation. Precaution should be taken to force hinging to occur in the ductile column region rather than in the foundation unit. This means the foundation should be designed to match the probable yield moment capacity of the column. This reduces the likelihood of a brittle non-ductile failure in the foundation and controlled damage occurs within the generally visible column area.

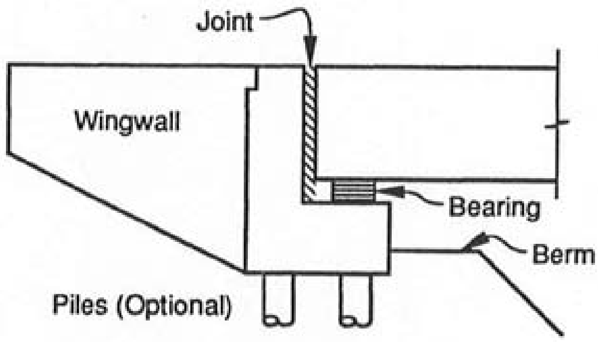
Abutments

Abutments can be divided into two general classifications for seismic analysis:

1) Monolithic
2) Seat type



Monolithic Abutments



Seat Type Abutments

Monolithic abutments tend to mobilize soils and absorb or dissipate seismic energy in both the longitudinal and transverse directions. When it is desirable to carry large forces into the soils at the abutment area, this abutment type is the best. Damage may be heavy but with adequate reinforcement and berm the abutment will perform satisfactorily, and the collapse potential will be very low. Maintenance problems generally preclude the use of monolithic abutments except for short structures (see Memo to Designers 5-2).

Seat type abutments are preferred as they permit the engineer more control over the degree of soil mobilization, even though the added joint introduces a potential collapse mechanism into the structure. Damage with this type of abutment is more controllable than with the monolithic abutment. Longitudinally the backwall gap should be minimized to permit mobilization of backfill soils. Transversely the superstructure may be held or released.

Abutment Stiffness

Abutments usually attract forces when the bridge is excited by seismic motions. After the 1971 San Fernando Earthquake, it was quite evident that most of the abutments had been subjected to large forces. For example, on many bridges abutment damage was the *only* damage reported, indicating that the abutment attracted a large portion of the seismic force. Therefore, the stiffness effects of the earth at the abutment must be considered if the dynamic model is to be realistic. This is especially true for seismic forces in the longitudinal direction.

The following preliminary stiffness coefficients are suggested for average abutment backfill conditions:

For Soils: K_s = Soil Stiffness per LF of wall/footing (k/in of soil deformation)

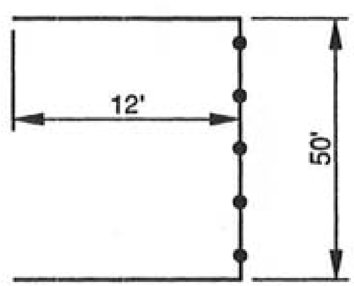
K_s = 200 k/in (based on material with V_s = 800 ft/sec[†] and 8'± effective height of wall)

For Piles: K_p = Stiffness per pile (k/in of pile deflection)[††]

K_p = 40 k/in for OSD standard 45 ton, 70 ton, and 16" CIDH piles.

Any abutment element (such as a wingwall) which is considered effective should be added into the computation of the abutment stiffness. The engineer must then evaluate that element to make sure it can carry the force reduced by the appropriate risk factor, Z.

Preliminary Abutment Stiffness Calculation Example:



Longitudinal

Soil: 200 k/in × 50 ft = 10,000 k/in
Piles: 40 k/in × 5 piles = 200 k/in

Assume soil acts ½ time; $K\ avg = \frac{10000}{2} + 200 = 5200\ k/in$

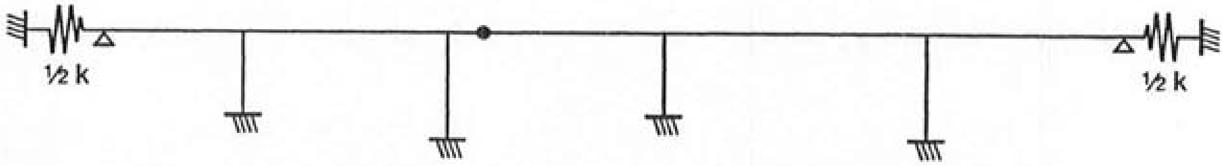
Transverse

Soil: Assume ⅔ Wingwall Acts: 200 k/in × 8 ft = 1600 k/in
Piles: 40 k/in × 5 piles = 200 k/in
Assume one wingwall is ⅓ effective, one fully effective.
K avg. = 1.333 × (1600) + 200 = 2330 k/in

[†]"Dynamic Analysis of Bridge Structures Subject to Earthquake Loads", by Agbabian Assoc.
[††]"Lateral Resistance and Deflection of Vertical Piles Interim Report No. 1", Wilfred S. Yee HPR-PR-1 (8), D-4-74, No. 6-71, Sept. 1971.

There are several judgmental assumptions which were made in the previous example:

1) In the longitudinal direction, when the structure is moving toward the soil, the full passive resistance of the soil is mobilized but when the structure moves away from the soil, no soil resistance is mobilized. The total structure stiffness would be too high if the full passive resistance were used at both abutments, so as an approximation one-half of the total stiffness is allocated to each abutment.



For unusual cases where abutments are unequal, two trials may be necessary. A full stiffness would be assigned to one abutment and zero stiffness assigned to the other with a second trial to examine the reverse condition.

In any case, it is important that the *total* stiffness of the system in the longitudinal direction is correct to compute the period accurately.

This reduction of stiffnesses at abutments then requires adjustment of the computed resultant forces. When half springs are used, the resulting forces from the analysis should be doubled at the abutment.

2) In the transverse direction, the flexible wingwalls are not usually considered fully effective and some judgment is used to calculate a realistic stiffness. Also the soil between the wingwalls is more effective than the exterior soil. Generally if the bridge is over 50' wide, it is difficult to mobilize the lateral forces created with wingwalls. In this case a release condition may be preferable.

   Variations in transverse stiffness for the longer and/or narrower bridges do not significantly affect the periods of vibration.

The preliminary abutment stiffness may require adjustment after analysis if the resulting forces or deformations are excessive.

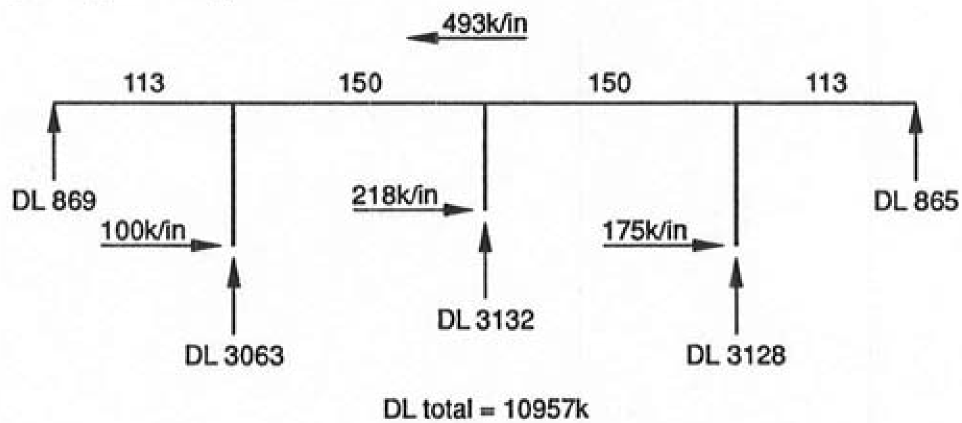
As a rule-of-thumb: Abutment effective soil stress in excess of 5 ksf (Maximum stress = 5/0.65 = 7.7 ksf) is considered excessive for effective soil resistance. (0.65 represents the ratio of effective stress to maximum stress under cyclic loading). Higher allowable soil stresses may be used with spread footings in good soil. Forty kips/pile is the assumed load a 45 or 70-ton pile will accept.

When the soil stresses are excessive, the abutment stiffnesses are too large. The abutment stiffnesses should be adjusted and subsequent analyses made until the engineer is satisfied with the soil stress level at the abutments. Excessive deformations at the abutment may cause problems. Abutments in San Fernando which moved up to 0.2 feet appeared to survive with little need for repair. If possible, this limit should be maintained. Excessive deformations may create stability and integrity problems both at the abutment and at the bents. Deformations greater than 0.2 feet in the abutment foundations should be evaluated for these effects. Abutments designed to 'fuse' or release before overloading foundation components can withstand very large deformations as long as adequate seat width is provided.

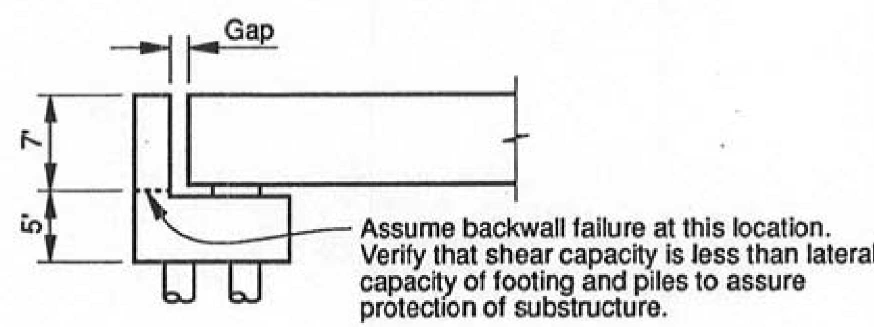
The following example illustrates the adjustment of the longitudinal abutment stiffness for a large seat-type abutment.

Longitudinal EQ Evaluation Example:

EQ Data (10-80'), A = 0.5g



Abutment 71' Wide on 30 Piles



Evaluate Fully Released Condition at Abutment

$$\text{Period} = 0.32\sqrt{\frac{10957}{493}} = 1.51 \text{ sec}$$
$$\text{ARS } (0.5 \text{ g}, 10\text{-}80') = 0.50 \text{ g}$$
$$\text{EQ Force} = 0.50(10957) = 5479 \text{ k}$$
$$\Delta = \frac{5479}{493} = 11.1''$$

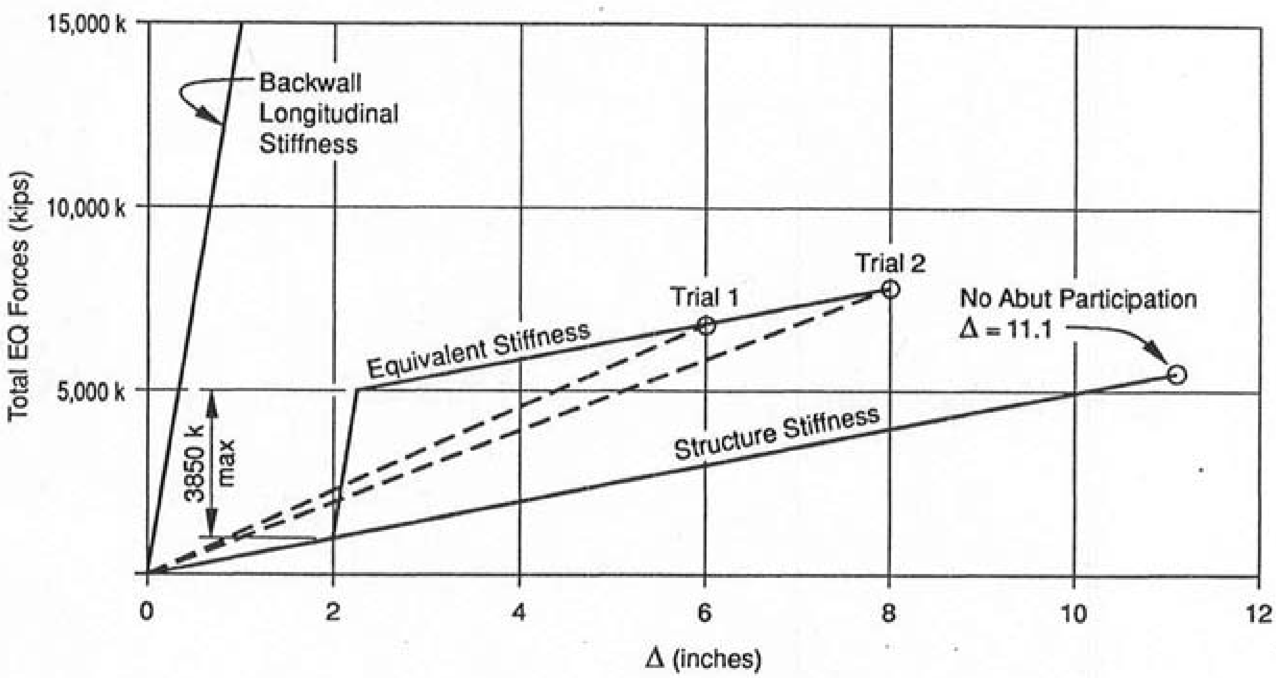
Evaluate Abutment Longitudinal Stiffness (Backwall Only)

(200 k/in/ft)
71(200) = 14,200 k/in
(Structure Stiffness = 493 k/inch)

Maximum Abutment Force (7.7 ksf on Soil), assume backwall breaks off.

$7 \times 71 \times 7.7 = 3827$ k   Backwall Only   say 3850 k maximum

Plot Total EQ Forces vs Deflection to Visualize Various Trials



Trial 1. Assume 2" backwall gap and 6" total movement. Construct force/Δ Line as follows:

(a) Follow structure stiffness for 2" (for gap).
(b) Using slope of the abutment stiffness continue for 3850 k maximum more.
(c) Continue after soil failure along structure stiffness line until 6" total is reached. Compute equivalent stiffness at 6800 k/6" or 1133 k/in.

$$\text{Avg Period} = 0.32\sqrt{\frac{10957}{1133}} = 1.00 \text{ sec; ARS} = 0.8 \text{ g}$$

$$\Delta = \frac{0.8(10957)}{1133} = 7.7\text{" Too high}$$

Try 8" total movement (2" gap + 6")

Trial 2

$$\text{Equiv Stiffness} = \frac{7900}{8} = 988 \text{ k/inch}$$

$$\text{Avg Period} = 0.32\sqrt{\frac{10957}{988}} = 1.07 \text{ sec; ARS} = 0.75 \text{ g}$$

$$\Delta = \frac{0.75(10957)}{988} = 8.3 \text{ Use } 8.5"$$

Determine EQ Forces for 8.5" Movement

Abutment	3850	3850 k	
Bent 2	8.5(100)	850 k	
Bent 3	8.5(218)	1853 k	Check ARS•DL
Bent 4	8.5(175)	1488 k	0.75(10957) = 8218 k
		8041 k	Within 5% ok

Generally one of three options may be selected by the engineer in the longitudinal design of the abutment:

1) Provide a very large gap in order to isolate the superstructure movements from the abutment.

2) Provide a gap for thermal considerations and permit the abutment backwall to fail thus protecting the abutment footing and piles.

3) Permit the total abutment to move, preferably keeping the total movement of abutment footing and piles under about 3 inches.

Usually the fully free condition (1) is more difficult to design for, however, with stiff columns (i.e., low deformations) a small gap (already required for temperature) may be adequate. Option (1) generally requires a larger joint.

Option (2) is generally preferred and assures that the foundation components will be protected.

Option (3) will allow more movement and damage to occur at the abutment and would require an evaluation of stability of the total bridge if movements exceed 3 inches. The use of Option (3) is quite valid for lower seismic areas and for bridges with adequate stability, which can survive the effects of abutment movements.

Abutment Keys

Abutment keys when required to transmit total seismic loads should be designed for the force required to mobilize the soils and piles below them. Refer to Memo to Designers 5-1 for examples of the design of keys for various abutment configurations.

Wingwalls

Wingwalls may or may not be utilized to mobilize soil resistance. It is sometimes satisfactory to assume the wingwalls are broken off. This condition occurs for monolithic abutments on very wide structures where large forces are drawn to the abutment. In this case, a second analysis with reduced stiffness in the transverse direction is required.

Thicker, tapered, or multiple wingwalls add stiffness and may be used to deal with moderate lateral forces, although this solution may not be the most economical one.

Seat Widths

Design displacements are as important as design forces because many of the loss-of-span type failures in past earthquakes have been attributed in part to relative displacement effects.

The support length at abutments and hinge seats should accommodate the structure's overall inelastic response displacements, independent movement of different parts, out of phase rotation of substructure components resulting from traveling surface wave motions, and rotational displacements due to skewed supports. The recent work of Elms et al (2), (3) can be used to give the order of magnitude of abutment movement and the recent work of Werner et al (4), (5) gives some indication of the effects of traveling waves on the responses of a limited number of bridges. However, much research remains to be done in both these areas.(1)

Best displacement estimates can be obtained if an inelastic time history method of analysis is performed. However, this is not recommended because of the complexities involved in performing the analysis.

The multi-mode dynamic analysis gives us our next best estimate of the displacements for a total structure if the flexibility of the foundation is included in the model. However, the current state-of-the art in this area gives the designer little help in evaluating the differential hinge and abutment displacements due to severe earthquake motions. For this reason it is necessary to specify minimum bearing support lengths.

For hinges, current policy requires 24-inch seats and restrainers. This solution accounts for the uncertainties of determining realistic displacements, although for very tall structures wider seats may be justified.

At abutments, it is recommended that a bearing seat length be provided equal to the displacements resulting from the elastic analysis, or the value, N, from the following modified expression (from the 1983 AASHTO Guide Specifications) whichever is greater:

$$N = (12 + 0.03L + 0.12H)\left(1 + \frac{S^2}{8000}\right)$$

N = Support width in inches normal to centerline of bearing, minimum = 2'-6".
L = Length in feet, of the bridge deck to the adjacent expansion joint, or to the end of the bridge deck. For single span bridges L equals the length of the bridge deck.
H = Average height in feet of columns supporting the bridge deck to the next expansion joint. H = 0 for single span bridges.
S = Skew of abutment in degrees.

Refer to Figure 4-1.

It must be recognized that displacements are very sensitive to the flexibility of the foundation. If the foundation is not included in the elastic analysis, consideration should be given to increasing the displacements for bridges founded on very soft soils when making the comparison above. This increase may be of the order of 50% or more but as with any generalization considerable judgment is required. A better method is to determine upper and lower bounds from an elastic analysis which incorporates foundation flexibility. Special care in regard to foundation flexibility is required for bridges with higher piers.

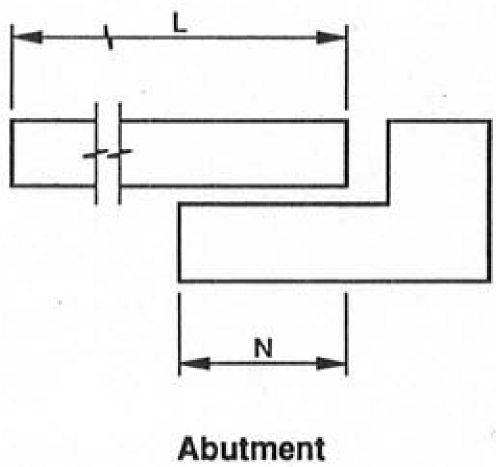


Figure 4-1 Dimensions for Minimum Support Length Requirements at Abutments

Seat Width References

1. Hall, W. J. and Newmark, N. M., "Seismic Design of Bridges – An Overview of Research Needs," Proceedings of a Workshop on Earthquake Resistance of Highway Bridges, Applied Technology Council, Berkeley, California, January 1979.

2. Richards, R. and Elms, D. G., "Seismic Behavior of Retaining Walls and Bridge Abutments," Report No. 77-10, University of Canterbury, Christchurch, New Zealand, June 1977.

3. Elms, D. G. and Martin, G. R., "Factors Involved in the Seismic Design of Bridge Abutments," Proceedings of a Workshop on the Earthquake Resistance of Highway Bridges, Applied Technology Council, Berkeley, California, January 1979.

4. Werner, S. D., Lee, L. C., Wong, L. H. and Trifunac, M. D., "An Evaluation of the Effects of Travelling Seismic Waves on the Three Dimensional Response of Structures," Agbabian and Associates, El Segundo, California, October 1977.

5. Werner, S. D., Lee, L. C., Wong, L. H. and Trifunac, M. D., "Effects of Traveling Waves on the Response of Bridges," Proceedings of a Workshop on the Earthquake Resistance of Highway Bridges, Applied Technology Council, Berkeley, California, January 1979.

EQUIVALENT STATIC ANALYSIS OF RESTRAINERS

General Procedure

1. Compute the maximum permissible restrainer deflection and limit deflection to the hinge seat width.

2. Compute the maximum longitudinal earthquake deflections on both sides of the superstructure joint under consideration. For curved bridges, compute the joint opening resulting from a lateral earthquake.

3. Compare the deflections from steps 1 and 2 (above) and determine the course of action.

4. Determine the number of restrainers required.

5. Check the deflections of the restrained system and revise the restrainer and/or column assumptions if required. Repeat steps 1 – 5 if necessary.

Assumptions

- A segment is defined as a portion of superstructure between expansion joints.

- Three separate analyses may be required to evaluate the restrainers at a particular joint, one each for the segment on either side of the joint and an evaluation of the joint opening from lateral earthquakes for curved bridges. The segments should be assumed to be moving longitudinally away from the joint. Usually the lighter segment will govern the restrainer design, but if one segment is heavier and significantly stiffener, it may require fewer restrainers. In either case the analysis which requires the *fewer* number of restrainers will govern.

- The mass to be used for computing the earthquake force shall be the mass of one segment adjacent to the joint under consideration.

- Assume one end of the restrainer is fixed and the other end is attached to the superstructure segment moving away from the joint.

- The longitudinal stiffness of the structure/restrainer system shall be computed by mobilizing the longitudinal stiffness of one adjacent segment in addition to the longitudinal stiffness of the segment under consideration. This adjacent segment can only be mobilized when the gap between the segment under consideration and the adjacent segment is closed. If this joint gap is equal to or greater than the estimated earthquake deflection, then the adjacent segment cannot be expected to be mobilized. If this gap represents a significant portion of the estimated earthquake movement, then a reduced stiffness should be assumed. The abutment may be included as part of the adjacent segment when gap considerations permit.

- Expansion joint gaps in recently constructed hinges with expanded polystyrene in the joint are not capable of transmitting any appreciable force until the joint is fully closed. Older hinges with 'expansion joint filler' in the joint may be considered closed after 50% of the gap is compressed if the material is still in the joint. Many of these older joints have been cleaned/rebuilt and the material removed. Do not assume there is material in the joint unless you know for sure it is there.

- Multiple simple-spans on bearings require an evaluation of the longitudinal adequacy of the bearings. If the bearings are not adequate to transfer the earthquake forces to the substructure then only the restrainers can be utilized to determine the longitudinal stiffness of the system. Adjacent segments should not be considered when computing the stiffness of multiple simple-span systems.

- For retrofit analysis, a determination must be made in regard to column adequacy. As a general rule, older columns with widely spaced ties, lap splices in main reinforcement and inadequate footings cannot be expected to develop large ductile forces. Whenever the applied earthquake moments exceed the nominal strength, these older columns should be assumed to have failed and a moment release introduced at that location. It is not too unreasonable to assume that 50 to 100% of the columns are damaged in this way, depending on how many columns are involved and how many inadequate details are involved. The presence of lap splices and the lack of top footing reinforcement generally increases the chances for damage at the bottom of existing columns.

Detailed Procedure

1. Compute the maximum permissible restrainer deflection and compare to the hinge seat width.

   1a. Maximum permissible restrainer deflection, D_r.

   $$D_r = D_y + D_g$$

   Where D_r = the maximum permissible restrainer deflection
   D_y = the restrainer deflection at yield
   D_g = the gap in the restrainer system

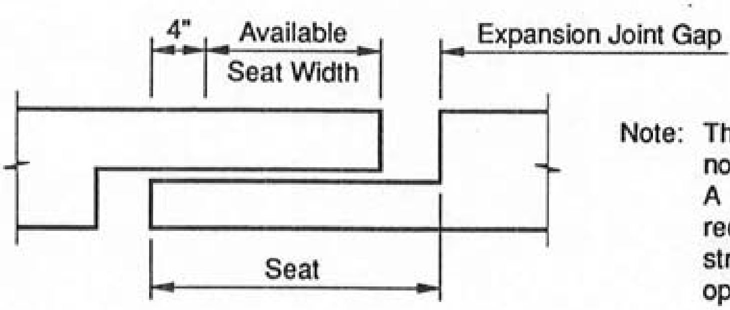
   Yield deflection, $D_y = F_y L/E$

   Where F_y = Yield stress in restrainer
   = 176.1 ksi for cables (39.1/.222)
   = 120 ksi for rods

   L = Restrainer length

   E = Initial modulus of elasticity of restrainer (before initial stretching)
   = 10,000 ksi for cables
   = 30,000 ksi for rods

1b. Compare the available hinge seat width with the maximum permissible restrainer deflection, D_r.



Note: The 4 inch dimension shown provides for a nominal 'resonable' allowable seat width. A larger or smaller dimension may be required. Expansion joint gap for new structures shall be the maximum estimated opening.

If the maximum permissible restrainer deflection (D_r from 1a.) is greater than the available seat width then the hinge could become unseated before the restrainer capacity is reached. In this case, either D_r must be reduced by, (a) shortening the restrainers, (b) decreasing the restrainer gap, or (c) reducing the stress in the restrainers or the seat width must be increased.

2. Compute the maximum longitudinal earthquake deflections on both sides of the superstructure joint under consideration.

2a. Compute the unrestrained system stiffness, (K_u) of the segment nearest to the joint under consideration. Assume the segment is moving away from the joint under consideration. Consider all columns or piers which can be mobilized. The next adjacent segment (including the abutment, if present) may also be added if they can be mobilized. The segments on either side of the joint should be evaluated separately.

DO NOT INCLUDE THE RESTRAINERS IN THIS CALCULATION EXCEPT FOR FULLY RELEASED SEGMENTS OR SIMPLE SPANS.

K_u = the unrestrained total system stiffness.

Where K_u = the equivalent stiffness of the total system considering the stiffness of all substructures mobilized and any gaps in the system.

Stiffnesses, (K) of various components...

Columns & Piers ..... $K = 12EI/L^3$ for Fixed-Fixed ends
$K = 3EI/L^3$ for Fixed-Pinned ends
$K = 0.0$ for Pinned-Pinned ends

Abutments .... $K = 200$ W

Piles .... $K = 40$ k/in/pile

Where E = Modulus of elasticity
I = Moment of inertia
L = Length
W = the normal bridge width

Note: The maximum force which can be transferred to the soil at the abutment is 7.7 A_n where 7.7 = maximum soil stress (ksf) and A_n = abutment area of soil mobilized (normal).

Note: On retrofit jobs, the capacity of the columns or piers should be evaluated. If failure is expected, a reduced stiffness should be used to model the "failed" condition. It is not too unreasonable to assume that 50% of the columns or piers will be damaged, as it is unlikely that all of the columns or piers will fail simultaneously. Simple spans on bearings will require a similar analysis. If failure of the bearings are expected from longitudinal forces, then the restraint offered by the substructure cannot be relied upon. In this case the stiffness of the system will come entirely from the restrainers.

2b. Compute the longitudinal earthquake deflection for both of the segments adjacent to the joint under consideration. Assume no restrainers in the system for this calculation. (Except for fully released segments such as simple spans.)

Compute the longitudinal deflection, $D_l = ARS(W)/K_u$ (in)

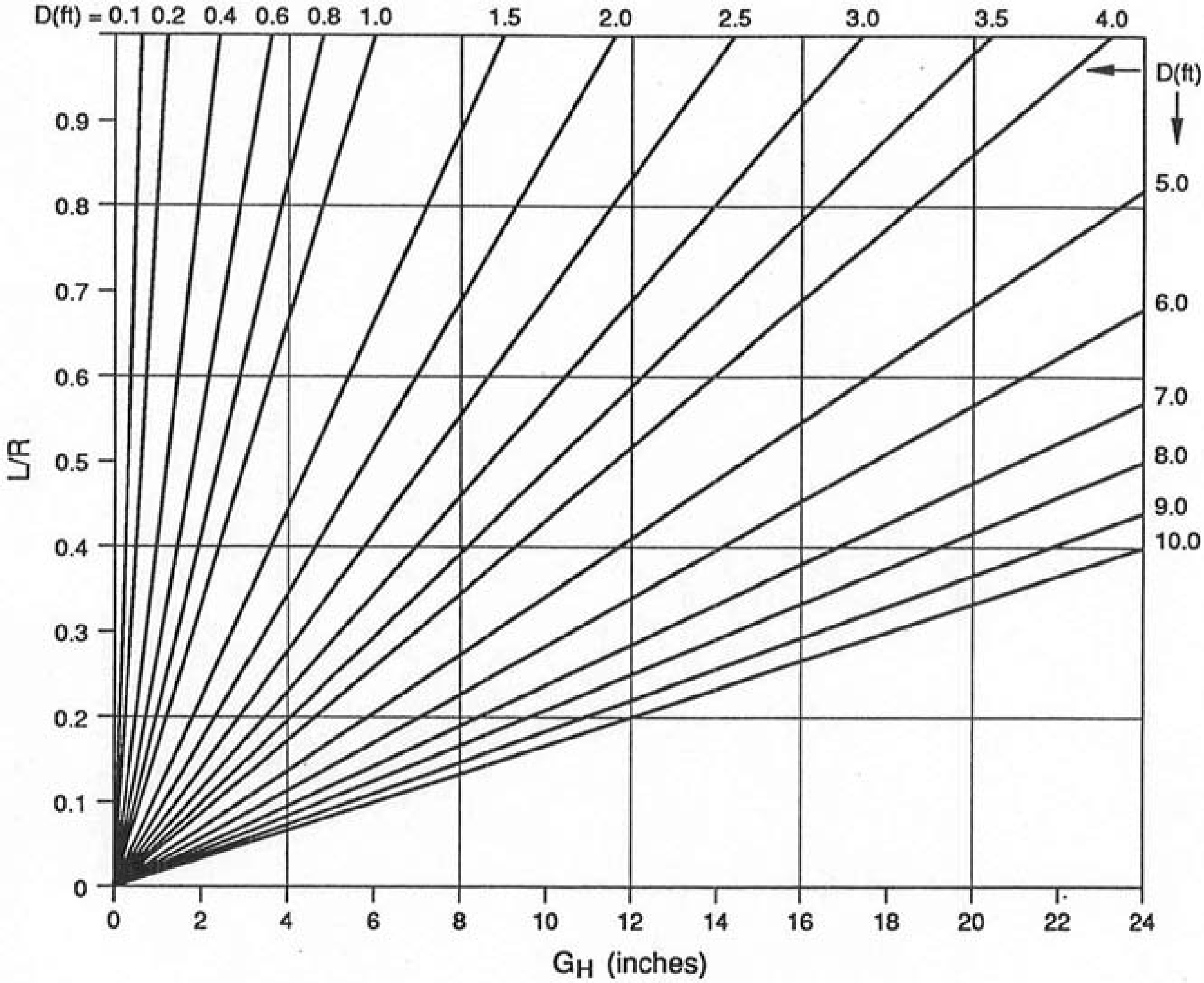
Where D_l = the longitudinal earthquake deflection of the unrestrained system;
ARS = the acceleration in g. for a given period of vibration, T (sec). Where T = 0.32 × the square root of (W/K_u) (Ref. *Bridge Design Specifications*, Figures 3.21.4.3 A – D and Section 3.21.6.1);
W = Weight of the segment (k);
K_u = the unrestrained system stiffness (k/in), from 2a.

For curved segments, compute the effect of transverse earthquake deflections on joint openings by:

$$G_h = D_t \sin\left[\frac{28.648L}{R}\right]$$

Where G_h = half gap at ends of curved segment (ft)
D_t = transverse earthquake deflection of segment (ft)
L = Length of segment (ft)
R = Radius of segment (ft)

A plot of the half-gap G_h for various L/R ratios and transverse deflections is shown on the next page.



Half Gap (G_H) for Various Lateral Deflections (D) and L/R Ratios

For curved segments the total gap opening at a joint from a transverse earthquake is obtained by adding the half openings from the ends of the two adjacent segments:

$G_t = G_{h1} + G_{h2}$

Where G_t = Total gap opening due to a transverse earthquake.
G_{h1}, G_{h2} = Half gap openings at joint.

2c. Compute the maximum joint opening by combining the effects of the longitudinal and transverse loads (*Bridge Design Specifications*, 3.21.1.1).

Where D_{eq} = the earthquake deflection of the unrestrained system
= the maximum of $D_\ell + 0.3(G_t)$ or $0.3(D_\ell) + G_t$

3. Compare the deflections from steps 1 and 2 and determine the course of action.

Compare the smaller of the two earthquake deflections from step 2c with the maximum permissible restrainer deflection from step 1a. If D_{eq} is *less* than D_r, then only a minimum number of restrainers will be required. Provide at least 2 separate cable restrainer units (or equivalent) across the joint. Locate these units as close as practicable to the outside edges of the bridge. If D_{eq} is *greater* than D_r by a significant amount, the analysis will show that a large number of restrainers will be required. This is because the analysis will determine the number of cables required to modify the earthquake deflection, D_{eq} to equal the restrainer capacity, D_r.

4. Determine the number of restrainers required.

$N_r = K_u(D_{eq}\text{-}D_r)/(F_yA_r)$

Where N_r = the number of restrainers required
K_u = the unrestrained system stiffness from 2a
D_{eq} = the deflection due to earthquake forces from 2c (the minimum of the two values from each side of the joint should be used)
D_r = the maximum restrainer deflection from 1a
F_y = Yield stress – 176.1 ksi for cables, 120 ksi for rods
A_r = Area of one restrainer
¾" cables = 0.222 sq in
1" rods = 0.85 sq in
1¼" rods = 1.25 sq in
1½" rods = 1.58 sq in

5. Check the deflection of the restrained system and revise the restrainer and/or column assumptions if required. Repeat steps 1 – 5 if necessary.

5a. Determine the deflection of the restrained system, the maximum of:

$D_t = ARS(W)/K_t + 0.3\ (G_t)$ or $D_t = 0.3\ [ARS(W)/K_t] + G_t$

Where D_t = the deflection of the restrained system;
ARS = the acceleration in g. for a given period of vibration, T (sec). Where T = 0.32 × the square root of (W/K_u) (Ref. *Bridge Design Specifications*, Figures 3.21.4.3 A – D and Section 3.21.6.1);

W = Weight of the segment (k);
K_t = the total restrained system stiffness (k/in) = $K_u + K_r$
K_u = the unrestrained system stiffness (k/in)
K_r = $F_y(N_r)A_r/D_r$
F_y = Yield stress in restrainer – 176.1 ksi for cables (39.1/0.222), 120 ksi for rods
N_r = the number of restrainers
A_r = Area of one restrainer
¾" cables = 0.222 sq in
1" rods = 0.85 sq in
1¼" rods = 1.25 sq in
1½" rods = 1.58 sq in
D_r = the maximum restrainer deflection from 1a

5b. Adjustment procedure.

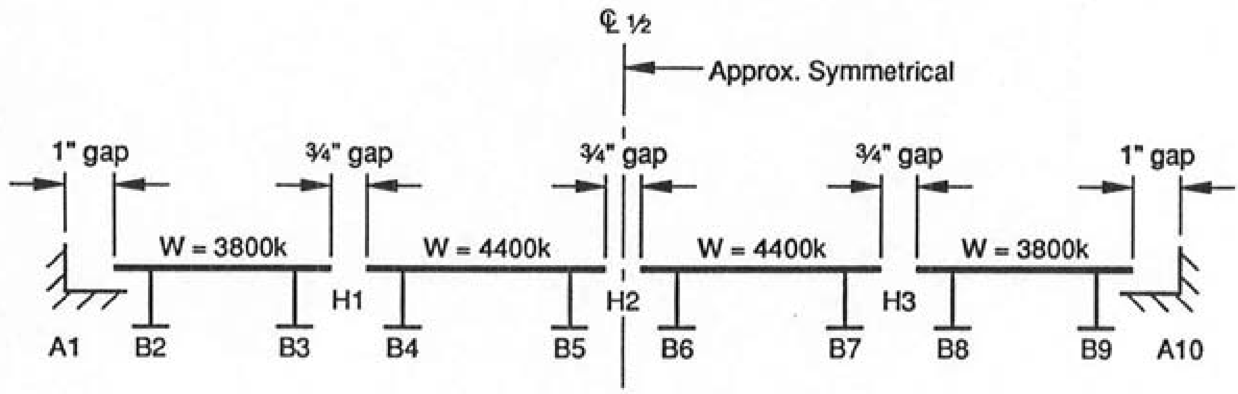
If the deflection of the restrained system, (D_t) is not equal to the permissible restrainer deflection, (D_r), then the adjustment procedure must be used. Usually this adjustment is accomplished by changing the number of restrainers, but revision of gaps can sometimes be used for minor adjustments. Column or pier capacity under the restrained system deflection, (D_t) should be verified to assure that the initial assumptions are still valid. If not the model must be adjusted and steps 1 – 5 repeated.

If D_r is *greater* than D_t, the number of restrainers may be reduced. After reduction, the new restrainer configuration should be checked to assure that D_r is not less than D_t.

If D_r is *less* than D_t, the number of restrainers should be increased. Steps 1 – 5 should be repeated until D_r is equal to or greater than D_t.

Example Equivalent Static Restrainer Analysis

Example 1 – 3 Hinged Retrofit



Seismic Data:	A = 0.6 g, 10-80' alluvium
Hinge Data:	Seat width = 6", diaphragms = 2'-6" thick
	¾" gap in hinge – no material in hinge
	Hinge has steel angles – allow 3" minimum seat
Abutment Data:	40' wide × 10' high
	1" gap – no material in joint
Columns:	All columns 24' long, longitudinal I = 32 ft^4. Assume column bottoms to be poorly detailed with inadequate lap splices and lightly reinforced footings. Assume the bottom of all columns have failed (i.e., pinned) and that 50% of all column tops failed.
Restrainers:	Minimum length = 2'-6" thru diaphragm
	+ 2'-0" thru bolsters
	4'-6" minimum

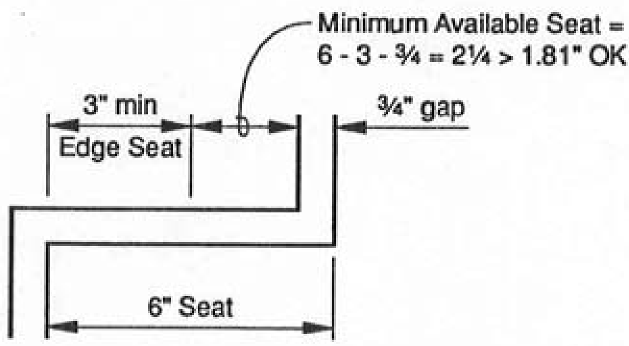
Try 5' cables – Restrainer C-1 – ¾"ø cables. Leave ¾" gap for thermal.

Example 1

Step 1a — Compute Maximum Restrainer Deflection, D_r

$$D_r = D_y + D_g = \frac{176.1(5)12}{10{,}000} + 0.75$$
$$= 1.06" + 0.75" = 1.81"\ D_r \text{ for 5' cables with } \tfrac{3}{4}" \text{ gap}$$

Step 1b — Check seat width



Seat will allow up to 2¼" movement or 2.25 - 0.75 = 1.5" cable movement (with ¾" gap)

$$\text{Maximum cable} = \frac{1.5(10000)}{176.1(12)} = 7.1' \text{ say } 7' \text{ with } \tfrac{3}{4}" \text{ gap}$$

Retainer Summary

Length	D_y	D_g	D_r
5'	1.06"	0.75"	1.81"
7'	1.5"	0.75"	2.25"

Step 2 — Compute Unrestrained Longitudinal EQ Deflection

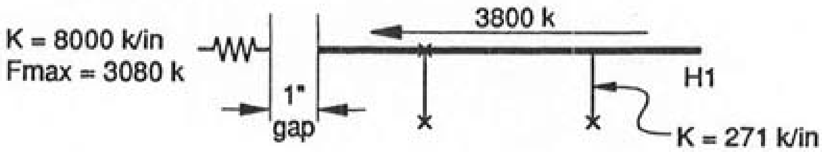
Stiffness of 1 superstructure unit = $3EI / L^3$

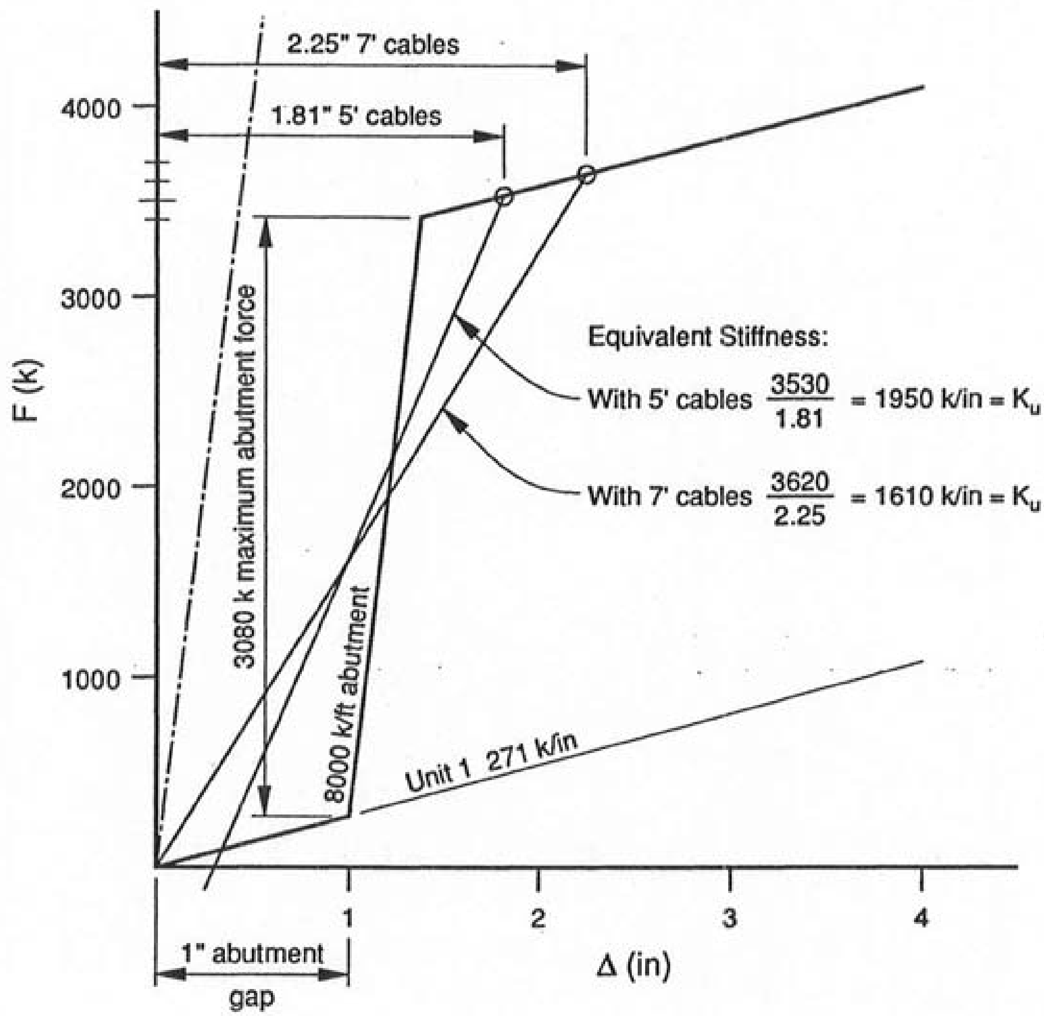
$$= \frac{3(468000)32}{24^3 \times 12} = 271 \text{ k/in/unit}$$

Abutment stiffness = 200(40) = 8000 k/in
Maximum abutment force = 7.7(400) = 3080 k

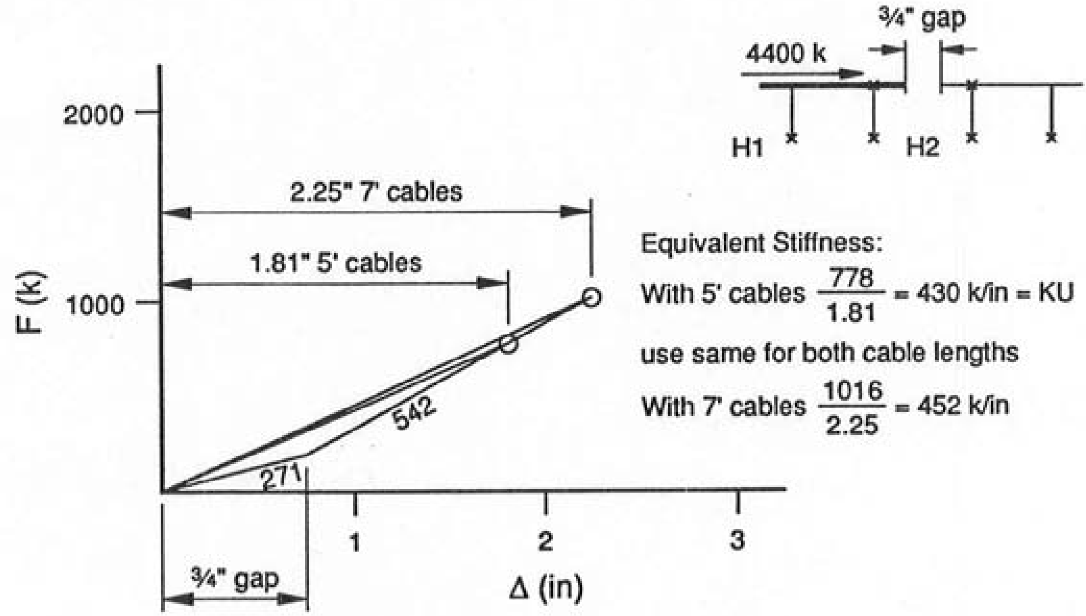
Step 2a — Evalute System Stiffness K_u

Unit 1 moving away from Hinge 1 — Mobilize Abutment

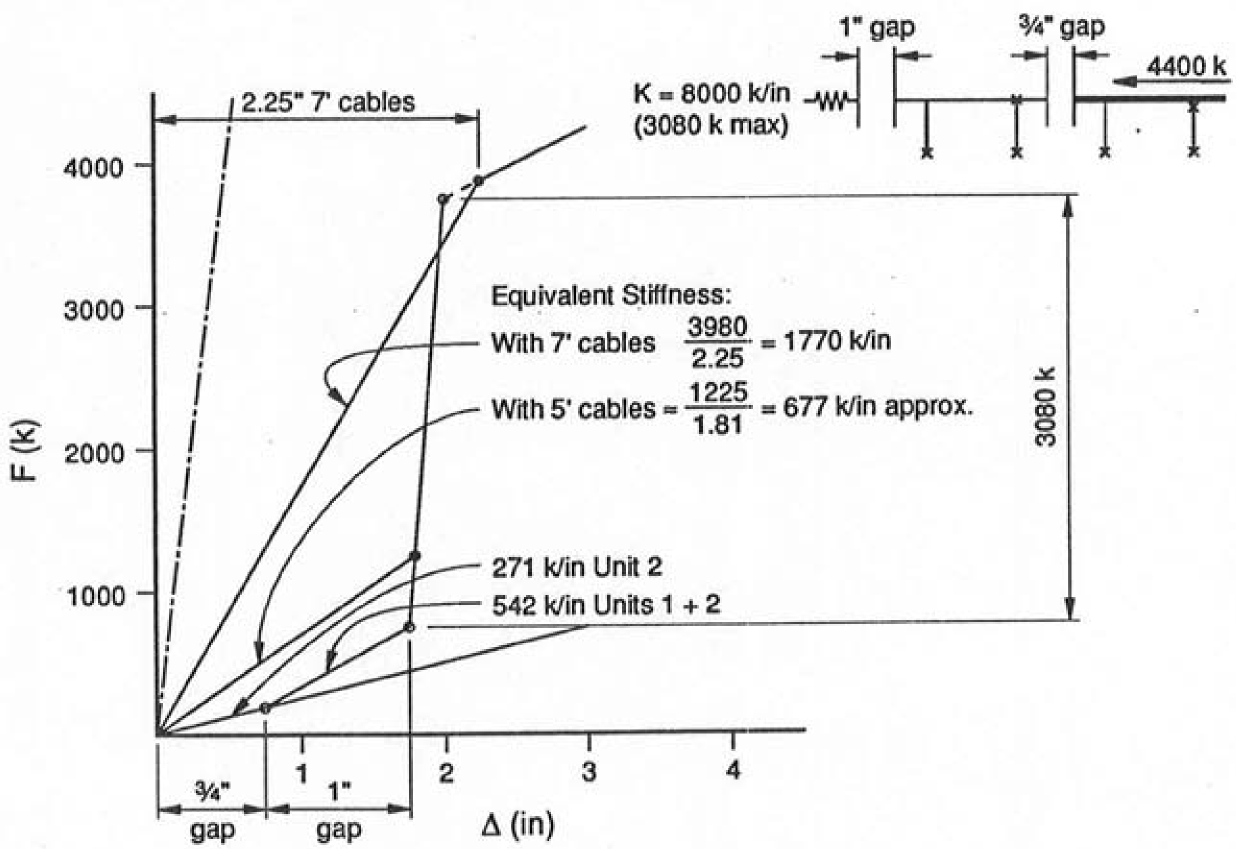




Unit 2 moving away from Hinge 1 — Mobilize Unit 3



Unit 2 (or 3) moving away from Hinge 2 — Mobilize Unit 1 + Abutment



Step 2b — Compute Maximum Unrestrained Seismic Deflections

Unit 1 moving away from Hinge 1

$W = 3800$ k

$K_u = 1950$ k/in with 5' cables

$= 1610$ k/in with 7' cables

$$T = 0.32\sqrt{\frac{W}{K_u}} = 0.32\sqrt{\frac{3800}{1950}} = 0.45 \text{ sec with 5' cables}$$

$$= 0.32\sqrt{\frac{3800}{1610}} = 0.49 \text{ sec with 7' cables}$$

ARS (from curves) = 1.7 g with 5' cables

= 1.65 g with 7' cables

$$D_{eq} = \frac{ARS(W)}{K_u} = \frac{1.7(3800)}{1950} = 3.31" \text{ (5' cables)}$$

$$= \frac{1.65(3800)}{1610} = 3.89" \text{ (7' cables)}$$

Unit 2 moving away from Hinge 1

$W = 4400$ k

$K_u = 430$ k/in (for both 5' and 7' cables)

$$T = 0.32\sqrt{\frac{W}{K_u}} = 0.32\sqrt{\frac{4400}{430}} = 1.02 \text{ sec; ARS} = 0.93 \text{ g}$$

$$D_{eq} = \frac{ARS(W)}{K_u} = \frac{0.93(4400)}{430} = 9.5"$$ Larger than Unit 1 moving away from H1 *Does Not Govern*

Unit 2 moving away from Hinge 2

$W = 4400$ k

$K_u = 677$ k/in with 5' cables

$= 1770$ k/in with 7' cables

$$T = 0.32\sqrt{\frac{W}{K_u}} = 0.32\sqrt{\frac{4400}{677}} = 0.82 \text{ sec; ARS} = 1.20 \text{ g (with 5' cables)}$$

$$= 0.32\sqrt{\frac{4400}{1770}} = 0.50 \text{ sec; ARS} = 1.7 \text{ g (with 7' cables)}$$

$$D_{eq} = \frac{ARS(W)}{K_u} = \frac{1.20(4400)}{677} = 7.80" \text{ (5' cables)}$$

$$= \frac{1.7(4400)}{1770} = 4.23" \text{ (7' cables)}$$

Step 3 — Compare Deflections

Hinge	Length	D_{eq}	D_r	D_{eq} - D_r
1	5	3.31	1.81	1.5
1	7	3.89	2.25	1.64
2	5	7.80	1.81	5.99
2	7	4.23	2.25	1.98

Step 4 — Determine Number of Restrainers

Hinge	L	D_{eq} - D_r	K_u	$F_y(AR)$	$N_r = \dfrac{K_u(D_{eg} - D_r)}{F_y(AR)}$	No. of 10 Cable Units
1	5'	1.5	1950	39.1	75	8 ←
1	7'	1.64	1610	39.1	68	7
2	5'	5.99	677	39.1	104	10
2	7'	1.98	1770	39.1	90	9 ←

Try 8–10 cables units per hinge 5' long at H1 and H3
9–10 cables units per hinge 7' long at H2

Step 5 — Check Deflections

Compute K_r for restrainers

$10 \times 8 = 80$ cables with 5'
$10 \times 9 = 90$ cables with 7'

$$K_r = \frac{F_y N_r A_r}{D_r} = \frac{39.1(90)}{2.25} = 1564 \text{ k/in} \text{ (7' cables)}$$

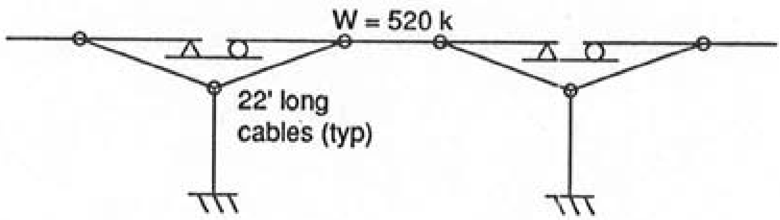
$$\frac{39.1(80)}{1.81} = 1728 \text{ k/in} \text{ (5' cables)}$$

Hinge	W	K_u	K_r	$K_t = K_u + K_r$	$T = 0.32\sqrt{\dfrac{W}{K_t}}$	ARS
1	3800	1950	1728	3678	0.32	1.82 g
2	4400	1770	1564	3334	0.37	1.78 g

$$D_t = \frac{ARS(W)}{K_t} = \frac{1.82(3800)}{3678} = 1.88" > 1.81" \, D_r \quad \text{OK}$$

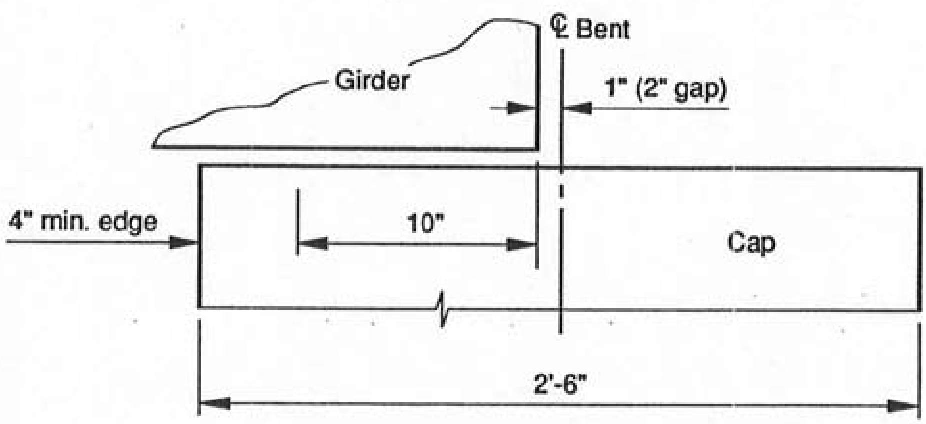
$$= \frac{1.78(4400)}{3334} = 2.35" > 2.25" \, D_r \quad \text{OK}$$

Example 2 — Multiple Simple Spans, Retrofit



Seismic Data:	A = 0.7 g, 10' – 80' alluvium
Bearings:	Assume fixed bearings no good in longitudinal direction. (Note: If bearings were okay they could be used to add to stiffness of system in longitudinal direction.) Assume keys will be checked/strengthened in transverse direction.
Restrainers:	22' long with ½" gap.

Available Seat Width:



Available seat = (1'-3") – 4" – 1" = 10"

Maximum restrainer deflection (D_y)

$$D_y = \frac{F_y L}{E} = \frac{176.1(22)12}{10000} = 4.65''$$

$$\text{Add Gap } (D_g) \quad \underline{0.50}$$

$$D_r = 5.15 < 10'' \quad \text{OK}$$

Total stiffness is from restrainers – try 20 cables

$$K_t = \frac{F_y N_r (A_r)}{D_r} = \frac{176.1(20)0.222}{5.15} = 151.8 \text{ k/in}$$

$$T = 0.32\sqrt{\frac{520}{151.8}} = 0.59 \text{ sec; ARS} = 1.62 \text{ g}$$

$$D_t = \frac{ARS(W)}{K_t} = \frac{1.62(520)}{151.8} = 5.55" > 5.15" \ (8\%)$$

Too large, add more cables (or lengthen)

Try 24 cables

$$K_t = \frac{176.1(24)0.222}{5.15} = 182.2 \text{ k/in}$$

$$T = 0.32\sqrt{\frac{520}{182.2}} = 0.54 \text{ sec; ARS} = 1.7 \text{ g}$$

$$D_t = \frac{1.7(520)}{182.2} = 4.85" < 5.15" \text{ OK (24 cables 22" long)}$$

Note: Because D_t is less these cables could be shortened — try 20'

20' Long Cables Total 24

$D_y = 20/22(4.65) = 4.23$
Add Gap <u>0.50</u>
$D_r =$ 4.73

$$K_t = \frac{176.1(24)0.222}{4.73} = 198.4 \text{ k/in}$$

$$T = 0.32\sqrt{\frac{520}{198.4}} = 0.52 \text{ sec; ARS} = 1.78 \text{ g}$$

$$D_t = \frac{1.78(520)}{198.4} = 4.67" < 4.73" \text{ OK} \quad \text{Use 20' long cables total 24}$$