

School of Civil Engineering

CE 614 – Statistical and Econometric Methods

Assignment #2A

(Count Data – Poisson Regression – Likelihood Function Calculation)

You are given data from six household and want to develop a model to study the number of cars owned by the household as a function of income (the only explanatory variable provided). The data are:

Household number	Number of cars owned	Annual household income in thousands of dollars
1	0	20
2	5	250
3	2	60
4	3	80
5	1	40
6	1	35

The model is estimated with only one explanatory variable (annual income, with no constant) and maximum likelihood results give $\beta = 0.0067$ (see estimation results on the next page). The likelihood function for a Poisson regression is: $L(\beta) = \prod_i P(y_i)$ and with $\lambda_i = \text{EXP}(\beta X_i)$ applying the Poisson equation (Eq. 11.1 in the text) gives Eq. 11.3 (page 284 in the text);

$$P(y_i) = \frac{\text{EXP}[-\text{EXP}(\beta X_i)] [\text{EXP}(\beta X_i)]^{y_i}}{y_i!}; \quad (11.1)$$

$$\text{so, } L(\beta) = \prod_i \frac{\text{EXP}[-\text{EXP}(\beta X_i)] [\text{EXP}(\beta X_i)]^{y_i}}{y_i!} \quad (11.3)$$

Assignment:

With the data provided, manually calculate the Likelihood function using Eq. 11.3. Then compute the log-likelihood and compare it to the Limdep output provided on the following page.

Variables available for your specification are (file TOBIT-LL-13.TXT)

Variable Number	Explanation
x1	Number of cars owned by the household
x2	Annual household income in thousands of dollars

Limdep output with TOBIT-LL-13.TXT data:

```
--> read;nvar=2;nobs=6;file=D:\old_drive_d\new_laptop\CE697N-disk\TOBIT-LL-13.TXT$
--> poisson;lhs=x1;rhs=x2$
```

```
+-----+
| Poisson Regression
| Maximum Likelihood Estimates
| Model estimated: Aug 12, 2013 at 11:10:49AM.
| Dependent variable           X1
| Weighting variable           None
| Number of observations        6
| Iterations completed         5
| Log likelihood function      -8.240650
| Number of parameters         1
| Info. Criterion: AIC =       3.08022
|   Finite Sample: AIC =       3.24688
| Info. Criterion: BIC =       3.04551
| Info. Criterion:HQIC =       2.94128
+-----+
```

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+-----+
| Poisson Regression
| Chi- squared =      2.43645  RsqP=   .6954
| G - squared =      2.83718  RsqD=   .6784
| Overdispersion tests: g=mu(i) : -1.537
| Overdispersion tests: g=mu(i)^2: -1.695
+-----+
```

```
+-----+-----+-----+-----+-----+
| Variable | Coefficient | Standard Error | b/St.Er. | P[|Z|>z] | Mean of X |
+-----+-----+-----+-----+-----+
| X2       | .00671260  | .00167798      | 4.000    | .0001    | 80.8333333
```