

# Lec9

Saturday, January 27, 2018 10:08 AM

Two-dimensional case:

⑥ What equi-lateral geometric shapes can tessellate an infinite 2-dim space?

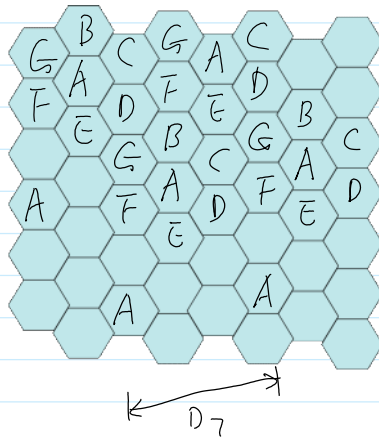
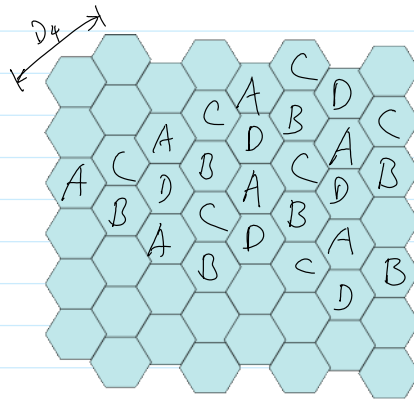
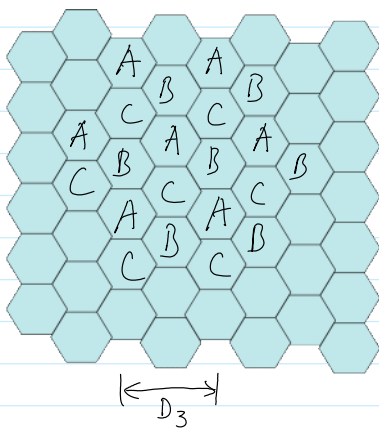
⑦ square, triangle, hexagon

↑ closest to circle.

Hexagon Cells

Show 3, 4, 7 - reuse patterns (p66-67)

cells of the same channel also forms a hexagon.

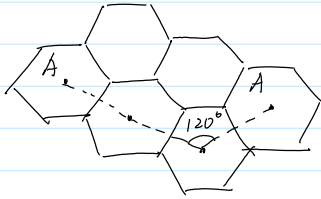


Reuse factor C  
 $\Rightarrow$  every cluster of C cells repeats itself

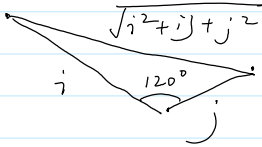
Reuse distance D

For uniform hexagon cells, the reuse factor C must satisfy

$$C = i^2 + ij + j^2, \quad i, j \text{ are positive integers.}$$



Two closest cell  
that use the  
same channel.



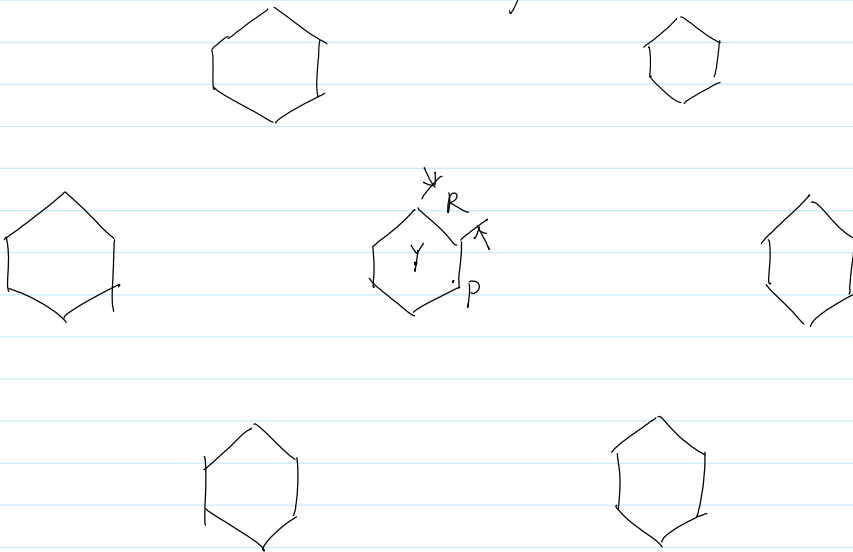
We can show that  
a cluster covers  
 $i^2 + ij + j^2$  times the area  
of a single cell  
 $\Rightarrow C = i^2 + ij + j^2$

$\Rightarrow$  The reuse-distance is related to the  
reuse-factor  $C$  by

$$D = \sqrt{C} \quad 2 \cdot \frac{\sqrt{3}}{2} R = \sqrt{3C} R.$$

SIR calculation:

- same assumption as linear case
- Six 1st-tier interferers.



received signal  $P_T / R^n$

1st-tier interferers

$$\approx \frac{P_I}{(D-R)^n} + \frac{P_I}{(D+R)^n} + 4 \cdot \frac{P_I}{D^n}$$

$$(D-R)^n \cdot (D+R)^{-n} \cdot D^n$$

$$\Rightarrow S2R \approx \frac{1}{\left(\frac{D}{R}-1\right)^{-n} + \left(\frac{D}{R}+1\right)^{-n} + 4\left(\frac{D}{R}\right)^{-n}}$$

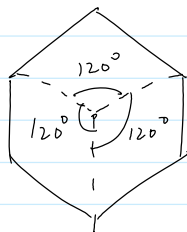
C	D/R = $\sqrt{3}C$	S2NR (dB)	
		n=3	n=4
3	3	5.3	9.3
7	4.58	11.6	18
12	6	14.8	22.2

Various other approximations are used in the literature. The relative difference of the approximations decreases as the reuse factor becomes large.

### Implications

AMPS: Threshold 18dB,  
Need 7-reuse at  $n=4$   
In practice, we  $C=7$  plus 120°  
sectoral antenna.

$\Rightarrow$  only two first-tier interferers  
per sector

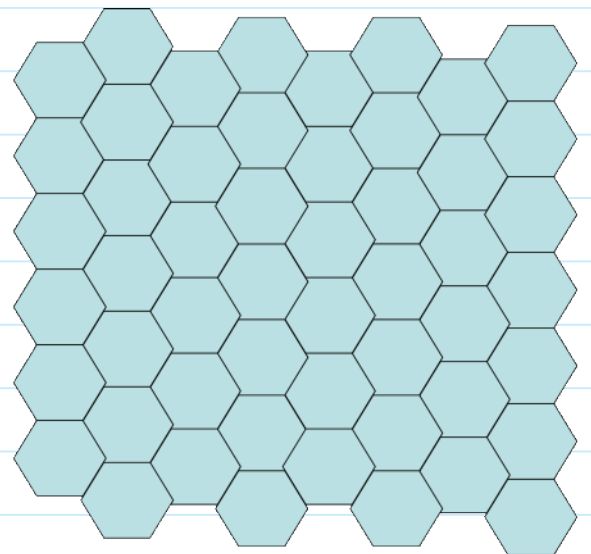
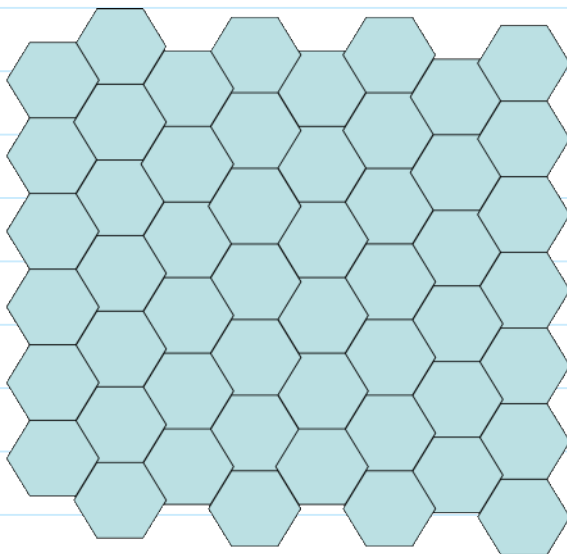
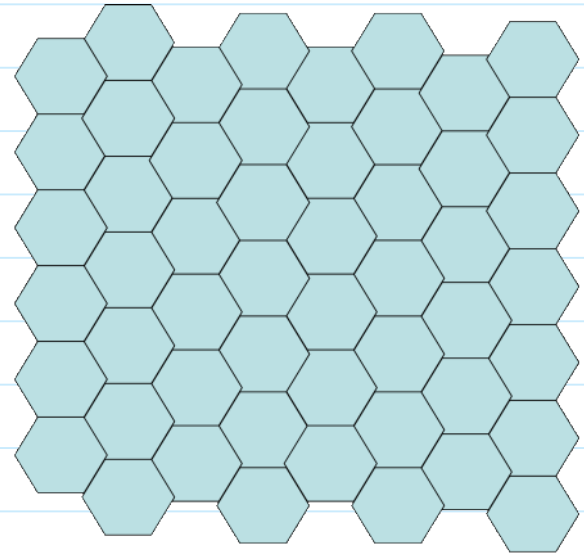
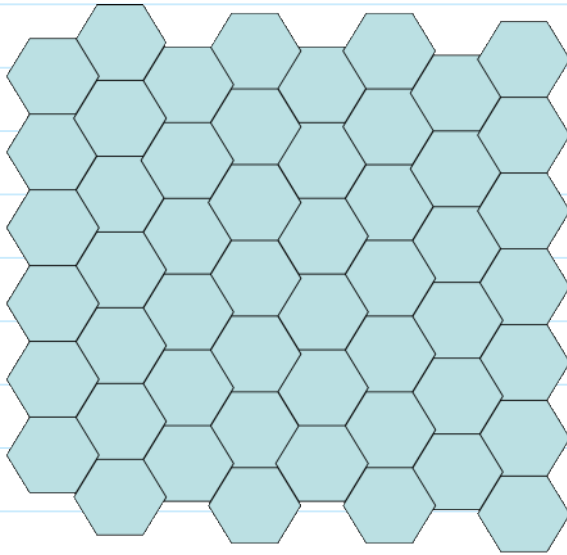


GSM: Threshold 7dB  
Need 3-reuse (at  $n=4$ )

Revisit the Assumptions.

# Cellular patterns in 2-d

Tuesday, February 19, 2008 4:26 PM

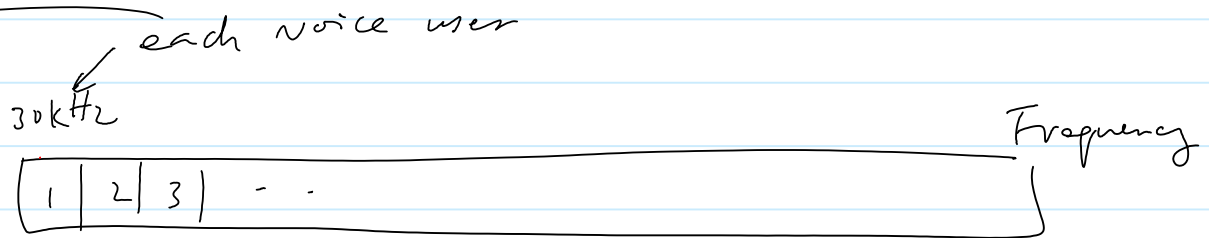


# TDMA vs FDMA - 10min

Sunday, January 06, 2008

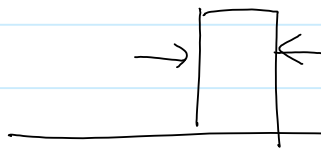
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FDMA

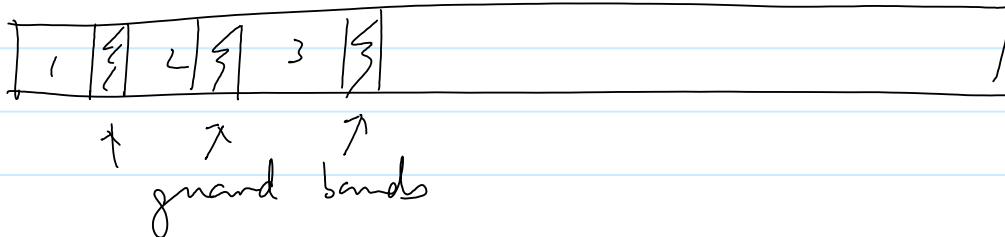
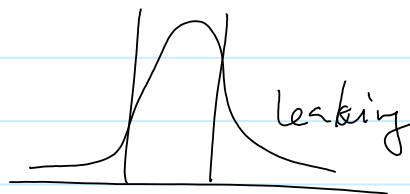


filter

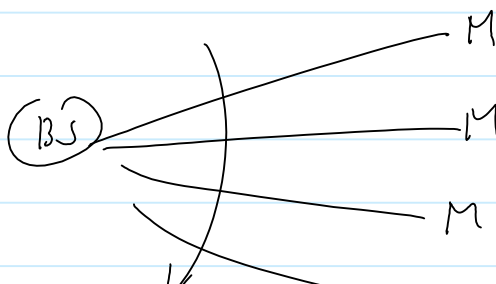
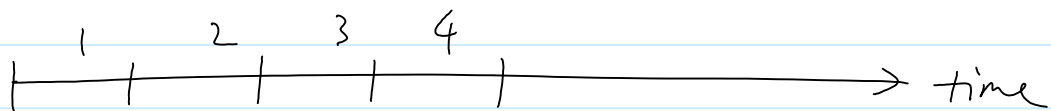
Ideal filter

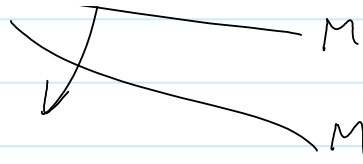


Reality



TDMA



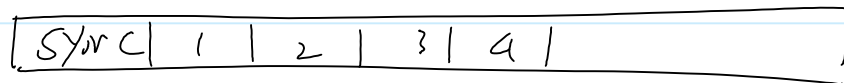


Both BS and M maintain accurate & identical clock.

Each user speaks with the BS at a specific time.

(Q) What is the advantage of TDMA?

- Time slots can be tightly packed, with no intervals in between.
- In reality, some time is used for synchronization sequence (guard time)



Sync. sequence is a code-sequence known to the receiver, so that the receiver knows when data time slots starts

- Base station only needs one radio  
⇒ the cost per user of BS equipment is lower

20min

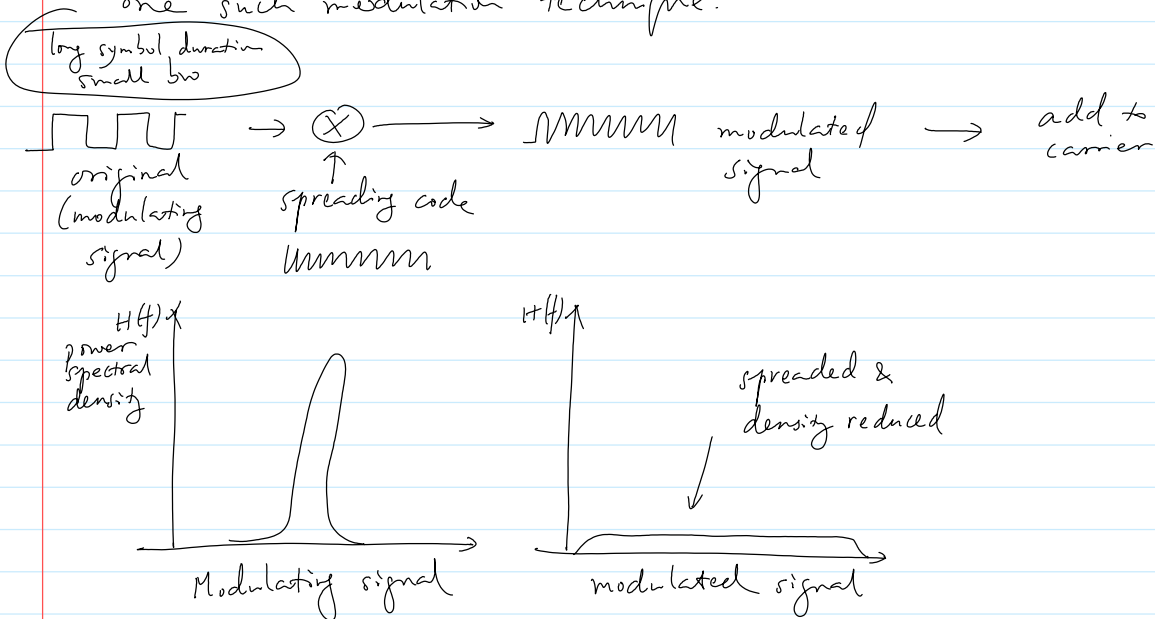


The 3rd Multi-access technology that appears in 2G is CDMA.

CDMA is based on spread-spectrum technologies.

Spread-spectrum modulations refer to any modulation techniques in which the BW of modulated signal is much larger than the BW of the modulating signal.

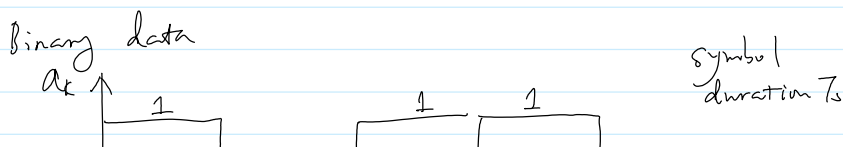
DS-SS (Direct Sequence - Spread Spectrum) is one such modulation technique.

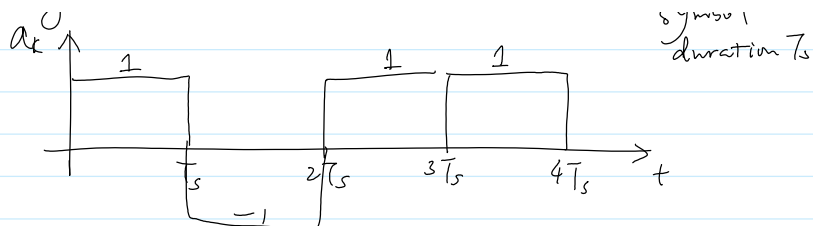


With good spreading codes (Pseudo-random sequence), the modulated signal looks like a random background noise

- special significance in military apps
- Communication is stealthy.
- We will focus more on the capacity aspect.

To be more precise:





With normal BPSK

$$a_k \rightarrow \begin{array}{c} \textcircled{\times} \\ \uparrow \\ \cos \omega_0 t \end{array} \rightarrow s_d(t)$$

$$s_d(t) = \sqrt{2P} \cos(\omega_0 t) \cdot a_k(t)$$

$\uparrow$  total power  $P$                        $\uparrow$  phase modulation  $\phi, \pi$

- At the receiver

$$s_d(t) \rightarrow \begin{array}{c} \textcircled{\times} \\ \uparrow \\ \cos \omega_0(t) \end{array} \rightarrow r_d(t) \rightarrow \boxed{\int_0^{T_s}} \quad \text{[Note: The integral symbol is partially obscured by a box in the original image.]}$$

$$r_d(t) = \sqrt{2P} \cos^2(\omega_0 t) \cdot a_k(t)$$

$$\int_0^{T_s} r_d(t) dt = a_k(t) \cdot \underbrace{\int_0^{T_s} \sqrt{2P} \cos^2(\omega_0 t) dt}_{\text{constant} > 0}$$

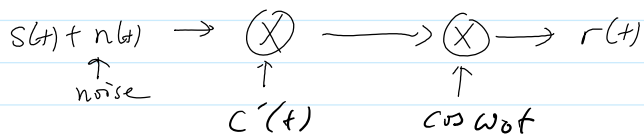
Spreading sequence



$$a_k \rightarrow \begin{array}{c} \textcircled{\times} \\ \uparrow \\ \cos \omega_0 t \end{array} \rightarrow \begin{array}{c} \textcircled{\times} \\ \uparrow \\ c(t) \end{array} \rightarrow s(t)$$

$$s(t) = \sqrt{2P} \cos(\omega_0 t) \cdot a_k(t) \cdot c(t)$$

At the receiver



$$r(t) = \sqrt{2P} c(t) c'(t) \cdot a_k(t) \cdot \cos^2(\omega_0 t) + n'(t)$$

① If  $c(t) = c'(t)$  (i.e., the <sup>same</sup> spreading code and perfect synchronization achieved),

$$\text{Then } r(t) = \sqrt{2P} \cdot a_k(t) \cos^2(\omega_0 t) + n'(t)$$

- No spreading effect at the receiver!
- $r(t)$  is then fed to the detector to retrieve  $a_k(t)$ .

What is the bandwidth of the signal

- Modulating signal  $a_k(t)$ :

$$\text{Symbol rate } T_s, \text{ bw } B_s = \frac{1}{T_s}$$

- same without spreading  $s_d(t)$

- With spreading  $s(t)$

$$\text{Chip rate } T_c \ll T_s, \text{ bw } B_c = \frac{1}{T_c} \gg B_s$$

At the same total power  $P$ , the PSD of the modulated signal  $\ll$  PSD of the modulating signal (by  $1/w$ )

- At the receiver, if  $c(t) = c'(t)$ , the bw of  $r(t)$  is still  $B_s$

② Consider instead the transmission of another user

$$\tilde{s}(t) = \sqrt{2P} \cos(\omega_0 t) \cdot \tilde{a}_k(t) \cdot \tilde{c}(t)$$

At the receiver

$$\tilde{r}(t) = \sqrt{2P} \tilde{c}(t) c(t) \cdot \tilde{a}_k(t) \cos^2(\omega_0 t)$$

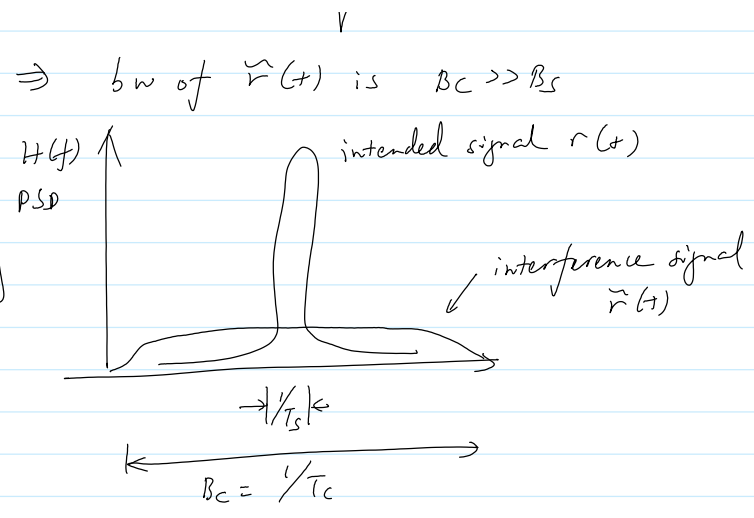
Since  $\tilde{c}(t) \neq c(t)$ ,  $\tilde{r}(t)$  looks like another pseudo-random sequence

If  $\tilde{c}(t)$  &  $c(t)$  are perfectly

$$\Rightarrow \text{bw of } \tilde{r}(t) \text{ is } B_c \gg B_s$$

1 1 -1 -1

If  $C(t)$  are perfectly orthogonal  $\Rightarrow$  no interference!  
 However, # of orthogonal codes is small



1 1 -1 -1  
 1 -1 -1 1

When we put a band-pass filter with bw  $B_S = 1/T_s$ , the power of the interfering signal is reduced to

$$P \frac{\frac{1}{T_s}}{\frac{1}{T_c}} = P/W$$

where  $W = \frac{T_s}{T_c}$  is processing gain

If  $W$  is large, the receiver will be able to decode even when two CDMA users transmit simultaneously.

$\Rightarrow$  Each "code" serves as a channel: A user can successfully communicate provided that the same code is not used by other users in the same cell.

(Q) If the # of codes is infinite, can CDMA support infinite # of channels then?

(A) No.

The signal-to-noise-interference ratio at the receiver is then (assuming received signal strength is the same)

$$\frac{P}{\frac{P}{W}(N-1) + \eta}$$

$\uparrow$                        $\uparrow$   
 # of users          background noise

The # of users  $N$  that can be supported

depends on the required SNR level.

⇒ The # of available channels in CDMA systems is limited by interference, not by reuse pattern

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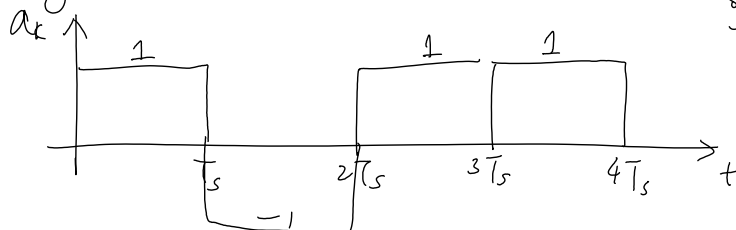
For example, in IS-95

- Each freq channel is 1.25 MHz
- Chip sequence at 1.2288 Mcps
- data sequence at 19.2 kbps

$$\text{Processing gain} = \frac{1228.8}{19.2} = 64$$

# Sender Side:

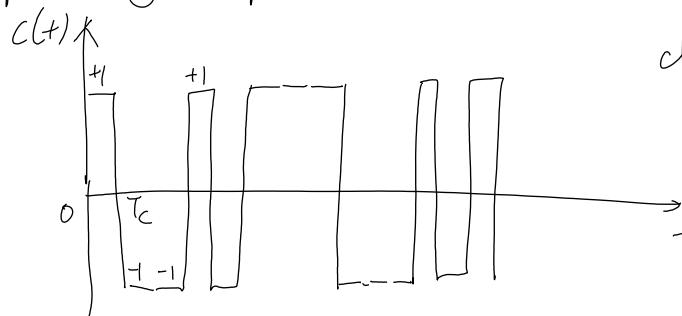
Binary data  $a_k$



symbol duration  $T_s$

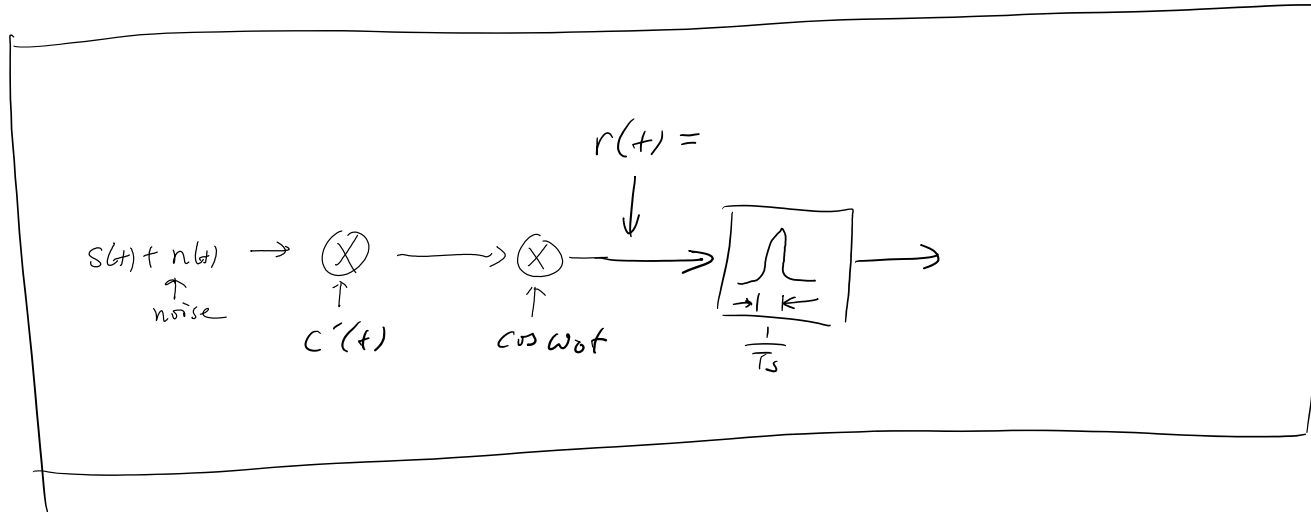
$$a_k \rightarrow \begin{matrix} \text{X} \\ \uparrow \\ \cos \omega_0 t \end{matrix} \rightarrow \begin{matrix} \text{X} \\ \uparrow \\ c(t) \end{matrix} \rightarrow s(t) =$$

Spreading sequence  $c(t)$



chip-duration  
 $T_c \ll T_s$

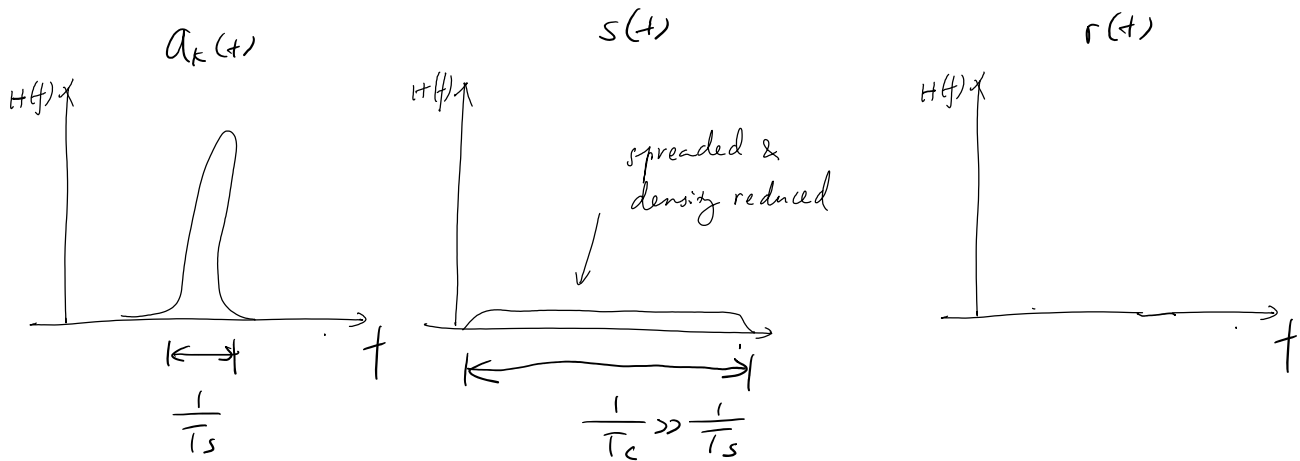
## Receiver Side



- ① For the intended signal, take  $c'(t) = c(t)$ .  
 - the same spreading code and perfect synchronization

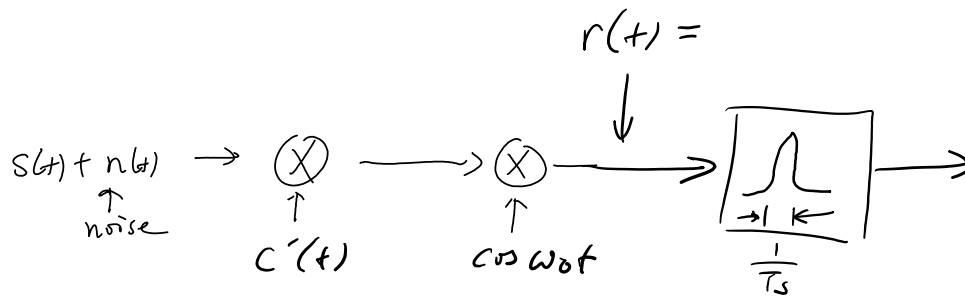
Then  $r(t) =$

What are the bandwidths of the signals?



# CDMA - handout3

Thursday, February 6, 2020 9:02 AM



⑦ Consider instead the transmission of another user

$$\tilde{s}(t) = \sqrt{2P} \cos(\omega_0 t) \cdot \tilde{a}_k(t) \cdot \tilde{c}(t)$$

At the receiver

$$\tilde{r}(t) =$$

Since  $\tilde{c}(t) \neq c(t)$ ,  $\tilde{r}(t)$  looks like another pseudo-random sequence

$$\Rightarrow \text{bw of } \tilde{r}(t) \text{ is } B_C \gg B_S$$

— When we put a band-pass filter with bw  $B_S = 1/T_s$ , the power of the interfering signal is reduced to

$$P \frac{1}{T_c} = P \beta$$



$$P \frac{\frac{1}{T_s}}{\frac{1}{T_c}} = P/w$$

where  $w = \frac{T_s}{T_c}$  is processing gain

— Another possibility is to use orthogonal codes such that

$$\int_0^{T_s} \tilde{c}(t)c(t)dt = 0$$

- No interference from  $\tilde{S}(t)$  at all!
- However, # of orthogonal codes is usually limited.
- Further, orthogonality may be lost if the synchronization is off.