Lec7

Sunday, January 26, 2020 10:35 AM

OFDM (Orthogonal Frequency Division Multiplex)

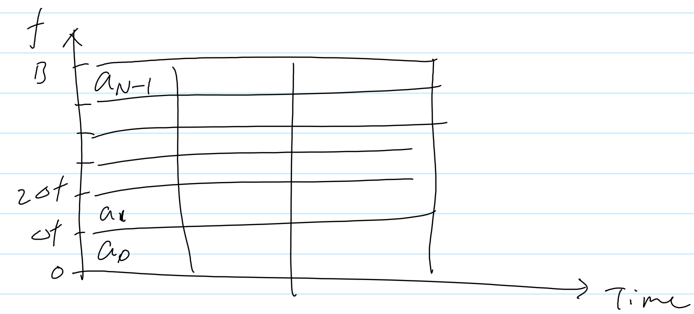
- used in both WiMAX & LTE
- also used in 802.11g & 802.11a

OFDM divides a large BW into a number of narrowband subcarriers, each of which carries information

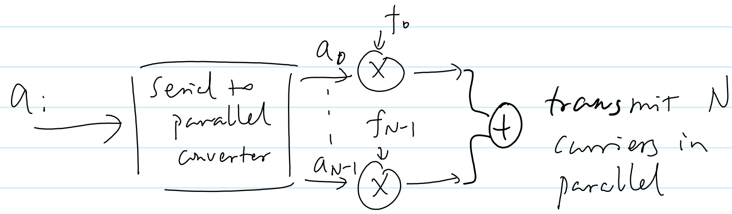
In OFDMA, we further divide time into slots. Each user can be allocated to any subset of the freq.-time slots.

- Although somewhat analogous to GSM's freq-time division. There are two main differences
  1. Each user can access the entire band
  2. No guard band across sub-carriers.

in Baseband



To see how this can be achieved with no guard band:



1. Let a_k be the symbol on sub-carrier k. The aggregate signal is then

$$s(t) = \sum_{k=0}^{N-1} a_k e^{j2\pi k \cdot \delta f \cdot t} \quad (\cdot e^{j\omega_c t})$$

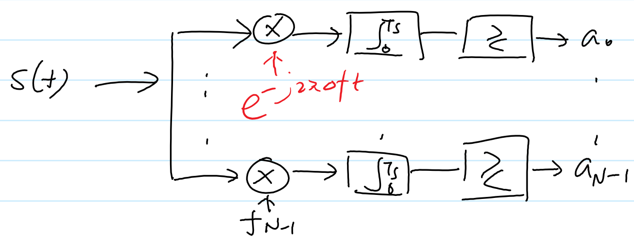
The aggregate signal is then

$$s(t) = \sum_{k=0}^{N-1} a_k e^{j2\pi k \cdot \Delta f \cdot t} \quad (\cdot e^{j\omega_c t})$$

- Seem similar to FDMA?

② We want the subcarriers to be orthogonal to each other (so that we can decode each a_k at receiver).

- In FDMA, filters are used to extract each subcarrier
  - need guard bands & filters
- OFDM uses a different approach, which allows much tighter packing of the sub-carriers



"matched filter"

- When stream k passes through the desired branch k:

$$\int_0^{T_s} e^{j2\pi k \Delta f t} \cdot e^{-j2\pi k \Delta f t} \, ds = T_s$$

- When stream k passes through another branch $m \neq k$, we want

$$\int_0^{T_s} e^{j2\pi k \cdot \Delta f t} \cdot e^{-j2\pi m \cdot \Delta f t} \, dt = 0 \quad \forall k \neq m$$

A sufficient condition is

$$\Delta f = 1/T_s$$

The total bandwidth is

$$\Delta f \cdot N = N/T_s$$

- No guard band is needed!

<u>Example:</u> LTE (long-term evolution)

- Total bw: up to 20MHz
- each sub-carrier: 15kHz $\quad 20M/15k \approx 1667$ subcarriers

  Symbol duration: $66.67\mu s = \frac{1}{15kHz}$

- At 20MHz bandwidth
  downlink: up to 150Mbps with 2x2 MIMO
  300Mbps with 4x4 MIMO

  uplink: up to 75Mbps

Benefit (i): Handle frequency-selectivity for wide-band signals

- Without OFDM

  $B = 20MHz \qquad T_s = \frac{1}{B} = 50ns \ll 2\lambda T_{av}$

- With OFDM

  $T_s = 66.67\mu s > 2\lambda T_{av}$

  - each sub-carrier experience flat fading.

- However, the above orthogonality may be lost if another copy of the signal is delayed due to multi-path

$$\int_0^{T_s} e^{j2\pi k\delta f(t-\tau)} u(t-\tau) \cdot e^{-j2\pi m\delta f t} dt$$

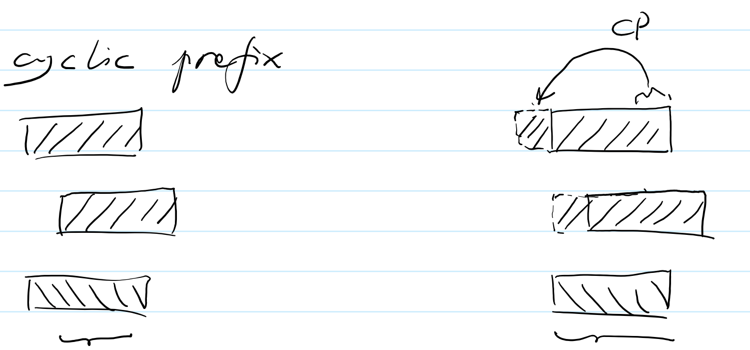
$$\neq 0$$

- OFDM uses a cyclic prefix

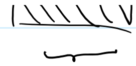
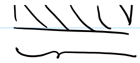
no delay

with delay

match filter



integral $\neq 0$ $\qquad$ integral $= 0$

match filter  

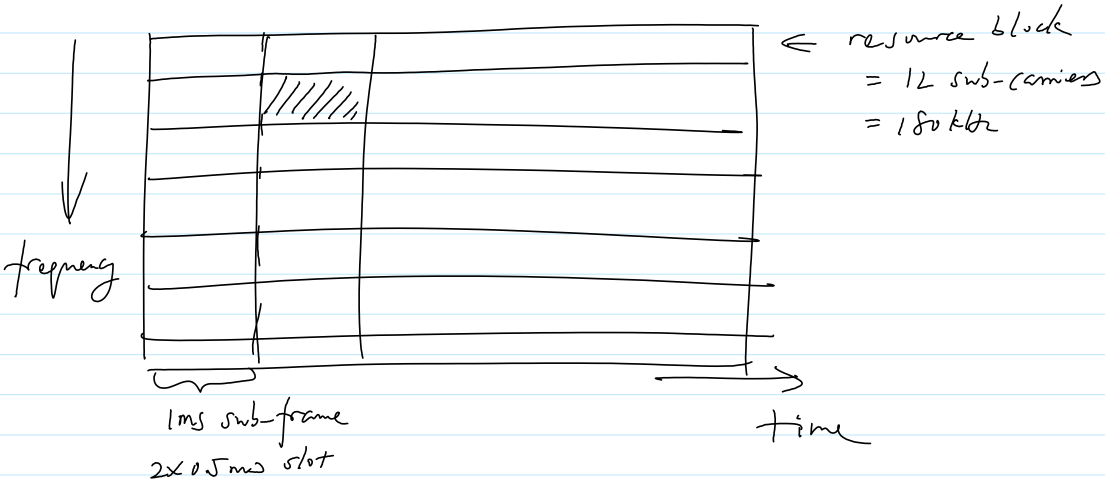
integral $\neq 0$ integral $= 0$

- CP needs to be longer than ~~the~~ delay spread

  normal CP: 4.7μs

  extended CP: 16.67μs

OFDMA (Orthogonal Frequency Division Multiple Access)



- BS can schedule users to a subset of frequency & time-slots
  - no dedicated channel

- Take advantage of good channel condition both in time & freq.
  - Opportunistic scheduling
  - Diversity

60

Benefit ②:

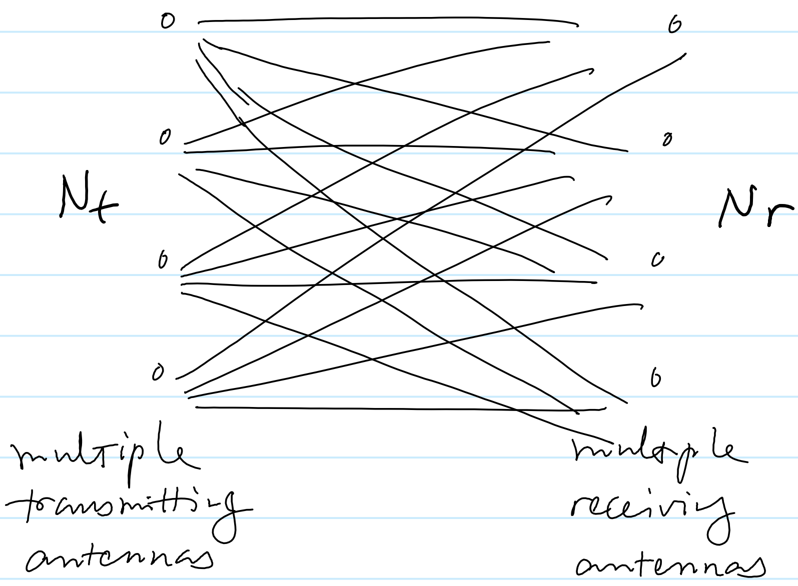
- High peak data rate, esp. when

Combined with MIMO

— Higher spectral efficiency

MIMO intro - 5min

Wednesday, January 16, 2008 4:11 PM



If the channel matrix is "right", the data rate can grow as $\min\{N_t, N_r\}$

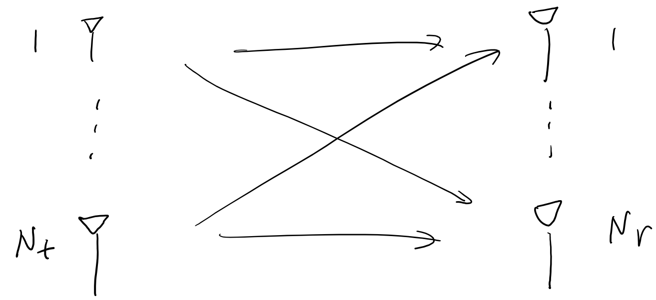
$\Rightarrow$ Increase data throughput

50

MIMO - 15min

Tuesday, January 28, 2020 4:19 PM

- Consider a system with N_t transmitting antennas and N_r receiving antennas.



- Let $\vec{s}$ be the vector of transmitting symbols, $\vec{s} \in R^{N_t}$
  $\vec{r}$ be the vector of receiving symbols, $\vec{r} \in R^{N_r}$
- The relationship between $\vec{r}$ and $\vec{s}$ is given by the channel model below:

$$\vec{r} = \sqrt{\rho} \cdot H \cdot \vec{s} + z$$

$$= \sqrt{\rho} \begin{array}{c} N_t \\ N_r \left[\begin{array}{ccc} & & \\ & & \\ & & \end{array} \right] \end{array} \begin{array}{c} 1 \\ \left[\begin{array}{c} \\ \\ \\ \end{array} \right] N_t \end{array} + \begin{array}{c} 1 \\ \left[\begin{array}{c} \\ \\ \\ \end{array} \right] N_r \end{array}$$

$[H_{ij}]_{N_r \times N_t}$

- Assume to be normalized such that $\|H\|_F^2 = N_r \cdot N_t$
- For example, this is true if $H_{ij} = 1$ in all elements
- "The gain between each transmitter - receiver pair is about the same"

transmitted symbol

- $s \in R^{N_t}$
- $E[\|\vec{s}\|^2] = 1$
- "Total transmission power should not grow with N_t"

noise

- $z \in R^{N_r}$
- i.i.d. zero mean with variance $\sigma^2 = 1$
- "More antennas pick up more noise"

- If the power is uniform on all antennas, then $E(s_i^2) = \frac{1}{N_t} \triangleq \sigma_s^2$

- Thus, ρ can be interpreted as the received SNR, comparable to a SISO system.

Eigen-Beamforming

- Assume that both the sender & the receiver knows the channel matrix H
- Using SVD (Singular Value Decomposition), H can be written as

$$H = U \quad \Lambda \quad V^T$$

$$= \begin{array}{c} N_r \\ N_r \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \end{array} \begin{bmatrix} \sqrt{\lambda_1} & & & \\ & \sqrt{\lambda_2} & & 0 \\ & & \ddots & \\ 0 & & & \sqrt{\lambda_N} \end{bmatrix} \begin{array}{c} N_t \\ \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} N_t \end{array}$$

- $N_r \times N_r$ unitary matrix

- $N_r \times N_t$ but diagonal
- $N = \min\{N_t, N_r\}$
- Each λ_i is also the eigenvalue of HH^T

- $N_t \times N_t$ unitary matrix

- By unitary matrix, we mean that

$$UU^T = U^T U = I_{N_r \times N_r} \qquad VV^T = V^T V = I_{N_t \times N_t}$$

$$UU^T = U^T U = I_{N_r \times N_r} \qquad VV^T = V^T V = I_{N_t \times N_t}$$

- Vx will rotate a vector x but won't change its length.

- In eigen-beamforming, the sender multiply the information symbols x by the matrix V, i.e.

$$\vec{s} = V \cdot x$$

$$= \begin{bmatrix} v_1 & \cdots & v_{N_t} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_{N_t} \end{bmatrix}$$

- The information x_1 is now sent over all antennas as $v_1 x_1$
- Like the antenna array example, this roughly forms a beam for certain direction.

$\Rightarrow$ "beamforming"

- Note that the total xmit power doesn't change since $E\{\|Vx\|_2^2\} = E[x^T V^T V x] = E[x^T x] = E[\|x\|_2^2]$

- At the receiver end, multiply $\vec{r}$ by U^T.

$$y = U^T r = \sqrt{\rho}\, \underbrace{U^T U}_{I} \Lambda \underbrace{V^T V}_{I} x + U^T z$$

$$= \sqrt{\rho}\, \Lambda x + \tilde{z}$$

$\tilde{z} = U^T z$
- each element still has the variance $\sigma^2 = 1$

$$= \sqrt{\rho} \begin{bmatrix} \sqrt{\lambda_1} & & 0 \\ & \ddots & \\ 0 & & \sqrt{\lambda_N} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_N \\ \vdots \\ x_{N_t} \end{bmatrix} + \tilde{z}$$

- Thus, we get $N = \min\{N_t, N_r\}$ equations

$$y_i = \sqrt{\rho}\sqrt{\lambda_i}\, x_i + \tilde{z}_i \quad , \quad i = 1, \cdots, N$$

- In other words, through eigen-beamforming, the channel can be viewed as equivalent to N seperate channels, each of which has an SNR of

$$\frac{\rho\, E[x_i^2]\, \lambda_i}{\sigma^2} = \rho\, E[x_i^2] \cdot \lambda_i$$

  - The total rate is

$$\sum_{i=1}^{N} B \log_2 \left(1 + \rho\, E[x_i^2] \cdot \lambda_i\right)$$

<u>The eigenvalues</u>

- How much total data rate can we get depends on the values of $\lambda_1, \cdots, \lambda_N$

- Consider $N_t = N_r = N$. The following is true

$$\sum_{i=1}^{N} \lambda_i = \|H\|_F^2$$

  - since $\|H\|_F^2 = N^2$ by our assumption

$$\Rightarrow \sum_{i=1}^{N} \lambda_i = N^2$$

- Consider two possibility

① If $H_{ij} = 1$ for all i, j.

$$H = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}}_{\text{all xmit-receive}} = \begin{bmatrix} \frac{1}{\sqrt{N}} & | & | & | \\ \vdots & | & | & | \\ \frac{1}{\sqrt{N}} & | & | & | \end{bmatrix} \underbrace{\begin{bmatrix} N^2 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix}}_{\text{only 1 non-zero}} \begin{bmatrix} \frac{1}{\sqrt{N}} & \cdots & \frac{1}{\sqrt{N}} \\ \hline & & \\ \hline & & \\ \hline & & \end{bmatrix}$$

all xmit-receive pairs are highly correlated

only 1 non-zero eigenvalue

- only one effective channel

$$\text{total rate} = B \log_2 (1 + \rho N^2 \cdot E[X_i^2])$$

- Even though the total rate still increase with N, the growth is very slow

② If H is such that $\lambda_1 = \cdots = \lambda_N = N$,

- N effective channels

$$\text{total rate} = N \cdot B \log_2 (1 + \rho \cdot N \cdot E[X_i^2])$$

- This growth is more desirable as it is linear in N.
- This is called the "spatial multiplexing" gain.

When will all λ_i's be approximately equal to N?

- One such case is when each element of H is i.i.d. zero-mean Gaussian with variance 1.

$$HH^T = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1N} \\ h_{21} & h_{22} & \cdots & h_{2N} \\ & - & - & \end{bmatrix} \begin{bmatrix} h_{11} & h_{21} & & \\ \vdots & \vdots & \vdots & \vdots \\ h_{1N} & h_{2N} & & \end{bmatrix}$$

$\Rightarrow$ The $(1,1)$-element of HH^T is

$$\sum_{i=1}^{N} h_{1i}^2 \approx N \quad \text{when } N \text{ is large}$$

The $(1,2)$-element of HH^T is

$$\sum_{i=1}^{N} h_{1i} \cdot h_{2i} \ll N \quad \text{when } N \text{ is large}$$

$\Rightarrow HH^T$ is roughly a diagonal matrix

$$\begin{bmatrix} N & & 0 \\ & N & \\ 0 & & \ddots \\ & & & N \end{bmatrix}$$

- Therefore, to get the spatial multiplexing gain, it is desirable that the channel of each transmit-receive pair is independent of others
  - sufficient "spatial diversity"
  - More suitable when there are many multipaths
    → "rich scattering" environment.
- Unfortunately, this also means that there is more overhead to estimate the CSI of each Xmit-receive pairs
  - overhead grows at N^2 (will see it again when we discuss LTE).

- Finally, in addition to the eigen-beamforming formulation above, there are other expressions of MIMO capacity that assume CSI only at the receiver, which may also produce spatial multiplexing gains.
- See reference on the course website.