Lec4-mwf

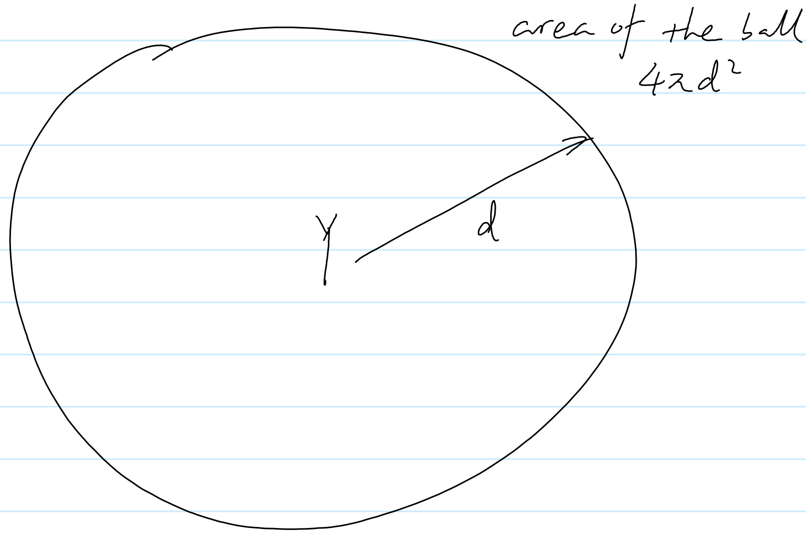
Monday, January 14, 2008 9:51 PM

- Lec4supp
- Fig 2.4-2.5 of schwartz.

Free-space model-15min

Saturday, January 12, 2008 10:53 AM

— No obstacles



First, assume that power emitted from the transmitting antenna isotropically. (omni-directional)

P_T = transmission power

From the conservation of power, the receiver power density at distance d is

$$\delta p_r(d) = \frac{P_T}{4\pi d^2}$$

If the receiving antenna has an effective area (or aperture) A_R, the received power is

$$P_r(d) = \frac{P_T}{4\pi d^2} \cdot A_R \cdot \eta_R$$

$\eta_R < 1$ is the efficiency parameter of the receiving attenna, due to

- transmission line attenuation
- filter loss
- antenna loss, etc.

Effective area of isotropic antenna is

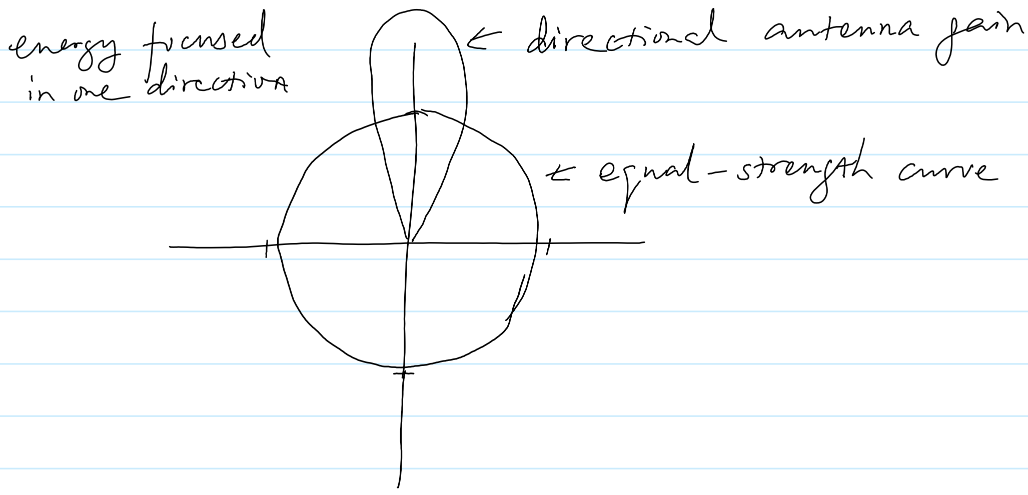
$$A_R = \frac{\lambda^2}{4\pi}, \quad P_R(d) = P_T\left(\frac{\lambda}{4\pi d}\right)^2$$

— Note that this decreases as $\lambda \downarrow$ (or $f \uparrow$)

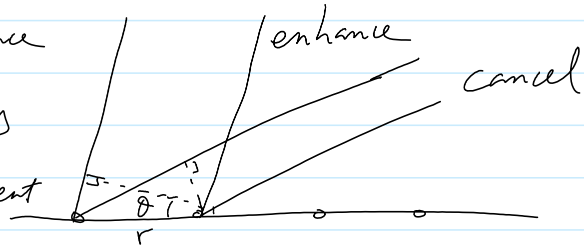
$\lambda = c/f$ c=3*10^8 m/s

Gain due to directional Antennas

Directional antennas can provide a gain factor over omnidirectional antennas



phase difference among the received signals of each antenna element



phase difference $= \dfrac{r \cdot \sin\theta}{\lambda}$

directional antenna built from

antenna arrays

Both transmitter & the receiver can have such type of gains.

At the transmitter side,

G_T = gain factor of the transmitting antenna

G_T is proportional to the effective radiating area A_T (the antenna size in wavelengths) of the transmitting antenna.

$$G_T = \frac{4\pi \eta_T A_T}{\lambda^2}$$

wavelength

Isotropic Antenna

$$\frac{4\pi}{\lambda^2} \cdot A_T = 1$$

η_T = effeciency factor for transmitting antenna. (similar to η_R)

The received power is then

$$P_r(d) = \frac{P_T}{4\pi d^2} G_T A_R \eta_R$$

The effective area A_R obeys a similar relationship.

If we define the attenna gain at the receiver end

$$G_R = \frac{4\pi \eta_R A_R}{\lambda^2}$$

Then

$$P_r(d) = P_T \, G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2$$

then

$$P_r(d) = P_T \, G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2$$

— Friis free-space equation.

Notes:

① The Friis equation is good only for the far field (or Fraunhoefer region)

Far-field distance

$$d_f = \frac{2D^2}{\lambda^2} \quad \text{— in \# of wavelengths}$$

where D is the longest physical linear dimension of the antenna.

② $\lambda = \frac{c}{f}$

③ $P_r(d)$ can be written as

$$P_r(d) = P_r(d_0)\left(\frac{d_0}{d}\right)^2$$

In dB (decibel), $10\log_{10} X$
the power decreases by 20dB as the distance is increased by 10 dB.

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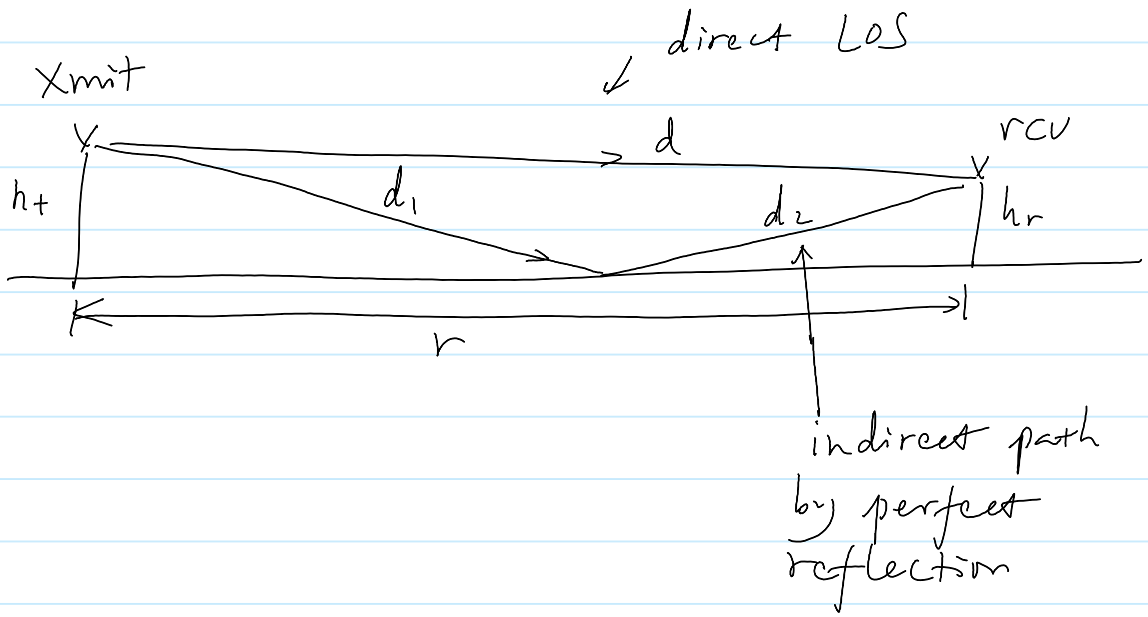
2-ray model - 10min

Saturday, January 12, 2008 11:21 AM

In the free-space model, only 1 path between sender and receiver. This is often not the case for terrestrial communication, where the ground becomes a natural reflector.

In the 2-ray model, the transmitted EM wave reaches the receiver through 2 paths

- the direct line of sight path
- indirectly by perfect reflection from a flat ground surface



We will show that in the 2-ray model

$$P_r(d) \propto \frac{1}{d^4}$$

The transmitted signal is $E_T e^{j\omega_c t}$

Direct-path received signal

$$\frac{E_T}{d} e^{j\omega_c \left(t - \frac{d}{c}\right)}$$

Indirect-path received signal

$$-\frac{E_T}{d_1+d_2} e^{j\omega_c \left(t - \frac{d_1+d_2}{c}\right)}$$

perfect reflection leads to phase shift of π.

Total received signal

$$\frac{E_T}{d} e^{j\omega_c\left(t-\frac{d}{c}\right)} \cdot \left[1 - \frac{d}{d_1+d_2} e^{-j\omega_c\left(\frac{d_1+d_2-d}{c}\right)}\right]$$

Total received power

$$\underbrace{\frac{E_T^2}{d^2}}_{\substack{\text{equivalent} \\ \text{to free-space}}} \underbrace{\left| 1 - \frac{d}{d_1+d_2} e^{-j\omega_c\left(\frac{d_1+d_2-d}{c}\right)} \right|^2}_{\text{due to two rays}}$$

$P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2$

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The cancelation effect of 2-ray - 15min

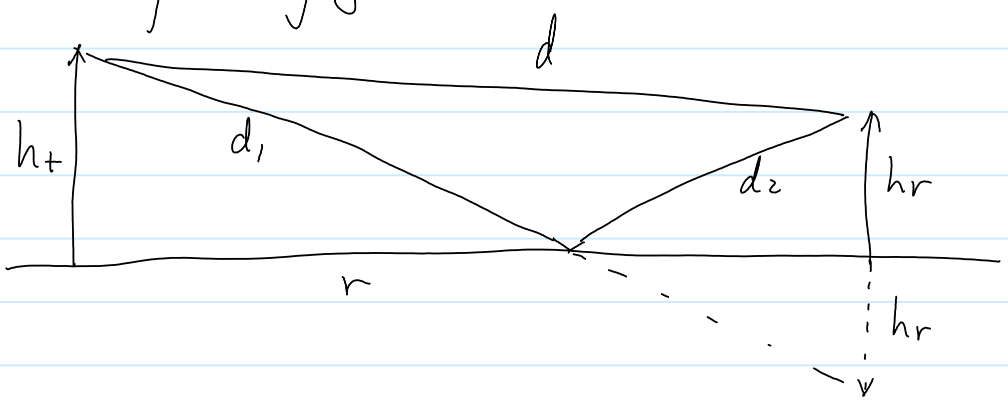
Saturday, January 12, 2008 11:31 AM

We will show that when $h_t, h_r << d$,

$$\left| 1 - \left(\frac{d}{d_1+d_2}\right) e^{-j\omega_c\left(\frac{d_1+d_2-d}{c}\right)} \right|^2$$

$$\approx \left(\frac{4\pi h_t h_r}{\lambda d}\right)^2$$

Recall from figure



$$\begin{cases} d = \sqrt{r^2 + (h_t - h_r)^2} \\ d_1 + d_2 = \sqrt{r^2 + (h_t + h_r)^2} \end{cases}$$

$$\Rightarrow \quad d_1 + d_2 = \sqrt{d^2 + 4h_t h_r}$$

$$= d \cdot \sqrt{1 + \frac{4h_t h_r}{d^2}}$$

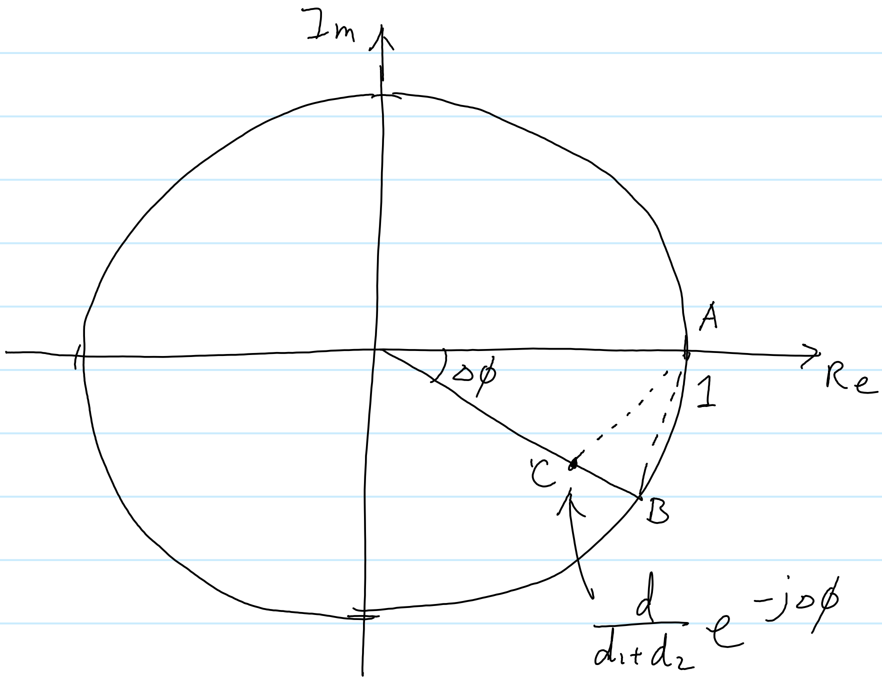
If (A) $4h_t h_r << d^2$,

then

$$d_1 + d_2 \approx d + \frac{2h_t h_r}{d}$$

Next, let

$$\Delta\phi = \omega_c \frac{d_1 + d_2 - d}{c}$$



We know

$$|BC| = 1 - \frac{d}{d_1+d_2}$$

$$\doteq 1 - \frac{d}{d + \frac{2h_t h_r}{d}} \simeq \frac{2h_t h_r}{d^2}$$

$$|AB| = 2\sin\frac{\Delta\phi}{2}$$

If (B) $\Delta\phi \ll 1$

$$\Leftrightarrow \frac{4\pi}{\lambda}\frac{h_t h_r}{d} \ll 1$$

then

then

$$|AB| \simeq \Delta\phi \simeq \frac{4\pi h_t h_r}{\lambda d}$$

When $d \gg \lambda$, $|AB| \gg |BC|$

Hence,

$$\left| 1 - \frac{d}{d_1 + d_2} e^{-j\Delta\phi} \right| = |AC|$$

$$\simeq |AB| \simeq \Delta\phi = \frac{4\pi h_t h_r}{\lambda d}$$

$$\therefore \quad P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2 \left(\frac{4\pi h_t h_r}{\lambda d}\right)^2$$

$$= P_T G_T G_R \frac{(h_t h_r)^2}{d^4}$$

Key point: It is the phase difference that determines the combined effect of 2 rays

— We will see this again in multipath fading

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The cancelation effect of 2-ray - supp

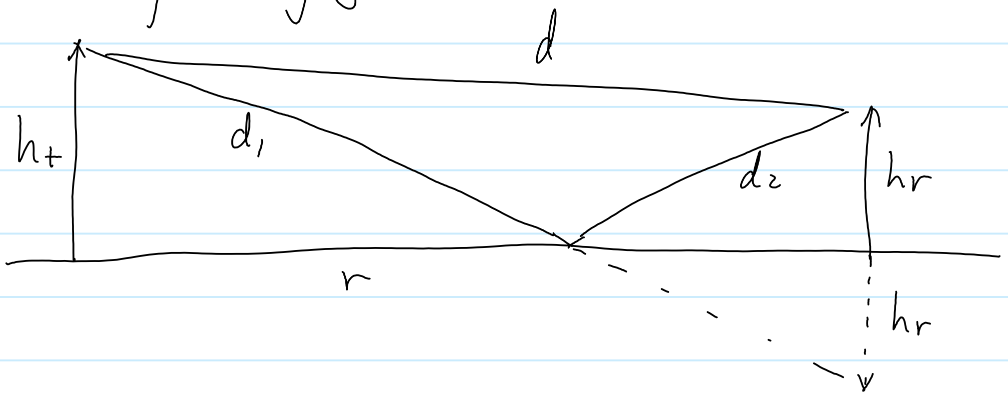
Saturday, January 12, 2008 11:31 AM

We will show that when $h_t, h_r \ll d$,

$$\left| 1 - \left(\frac{d}{d_1+d_2}\right) e^{-j\omega_c\left(\frac{d_1+d_2-d}{c}\right)} \right|^2$$

$$\simeq \left(\frac{4\pi h_t h_r}{\lambda d}\right)^2$$

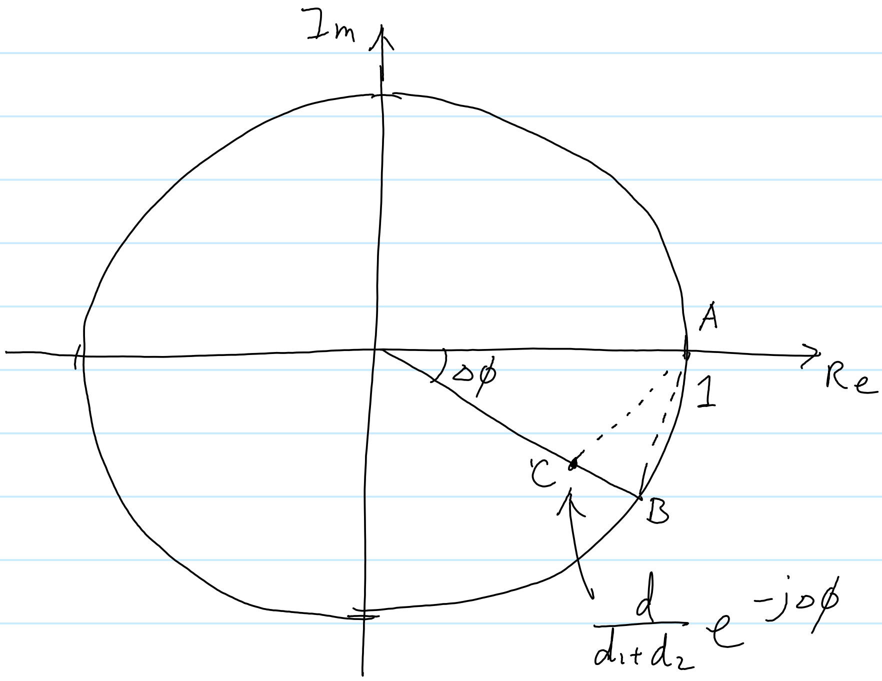
Recall from figure



$$\begin{cases} d = \sqrt{r^2 + (h_t - h_r)^2} \\ d_1 + d_2 = \sqrt{r^2 + (h_t + h_r)^2} \end{cases}$$

Next, let

$$\Delta\phi = \omega_c \frac{d_1 + d_2 - d}{c}$$



When $d \gg \lambda$, $\quad |AB| \gg |BC|$

Hence,

$$\left| 1 - \frac{d}{d_1 + d_2} e^{-j\Delta\phi} \right| = |AC|$$

$$\simeq |AB| \simeq \Delta\phi = \frac{4\pi h_t h_r}{\lambda d}$$

$$\therefore \quad P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi d} \right)^2 \left(\frac{4\pi h_t h_r}{\lambda d} \right)^2$$

60 $$= P_T G_T G_R \frac{(h_t h_r)^2}{d^4}$$

Key point: It is the phase difference that determines the combined effect of 2 rays

- We will see this again in multipath fading

Validity of Assumptions - 5min

Sunday, January 13, 2008 11:57 AM

Assumption (A)

$$4 h_t h_r << d^2$$

Example: $h_t = 50m$, $h_r = 2m$

$\Rightarrow d >> 20\ m$

Assumption (B)

$$\frac{4\pi h_t h_r}{d\lambda} << 1$$

Example: $f = 800\ MHz$, $\lambda = \frac{3}{8} m$

$\Rightarrow d >> 3.3\ km$

$f = 30\ GHz$, $\lambda = 0.01\ m$

$d >> 126\ km$

Assumption B is usually more stringent than assumption A.

(4)

More problematic for high frequencies. Instead of seeing smooth $1/d^n$ behavior, more likely to see small-scale multi-path fading.

Shadow fading -10min

Sunday, January 13, 2008 2:13 PM

The actual power received, measured over relatively long distances of many wavelengths, is found to vary randomly about the area-mean-power $P_T G_T G_R g(d)$.

Shadow fading is in general caused by variations in signal power due to signal attenuated by terrain obstructions, such as hills, buildings, or even leaves

The measured signal power thus may differ substantially at different location, even though they may be at the same distance from the xmitter

A good approximation for shadow fading is to assume that power measured in dB follows a Gaussian distribution centered about its average value.

$$10^{x/10} \quad \text{— log-normal}$$

6 – 10 dB

where $\quad x \sim N(0, \sigma^2)$

$$\Leftrightarrow \quad f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

Rational: A form of central limit theorem.

There are often many obstacles between the Xmitter and receiver.

The existance of an obstacle (e.g. a building) with width d_1 attennates the signal by

$$10^{-\alpha d_1}$$

where α is some constant, $d_1 = 0$ if there does not exist such an obstacle

With n potential obstacles, the attenuation is

$$10^{-\alpha \underbrace{\sum_{s=1}^{n} d_s}_{\text{becomes Gaussian when } n \text{ is large.}}}$$

Important: Shadow fading occurs over distances of tens/hundreds of wavelengths ($\gg \lambda$) -> large-scale fading

see Fig 2.4, Fig 2.5 in Schwartz

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Shadow fading-supp

Wednesday, January 18, 2012 11:49 AM

lec4-supp2

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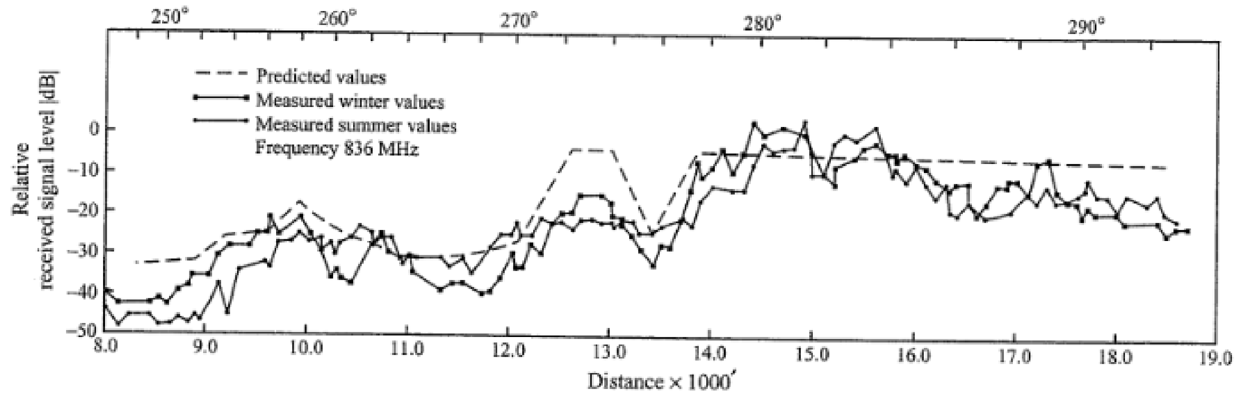


Figure 2.4 Measured signal level, fixed distance from transmitter, 836 MHz (from Jakes, 1974: Fig. 2.2–24)

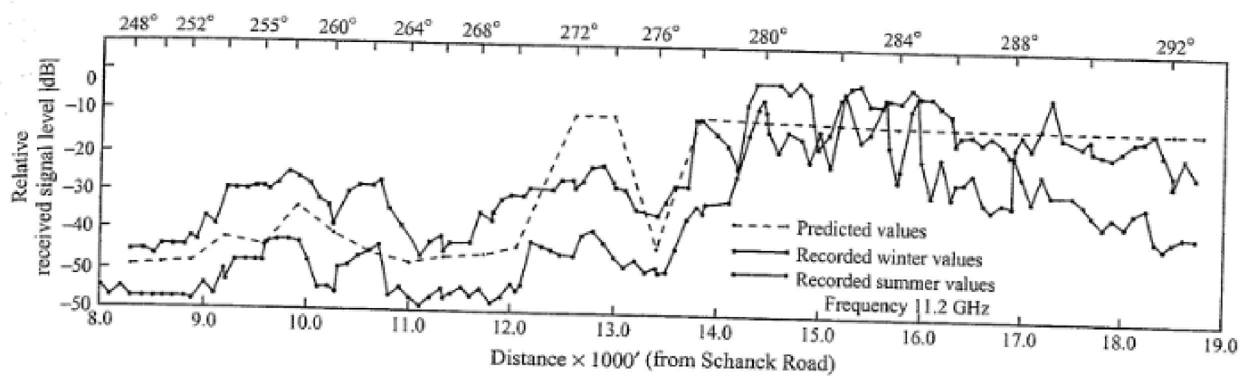


Figure 2.5 Measured signal level, fixed distance from transmitter, 11.2 GHz (from Jakes, 1974: Fig. 2.2–25)

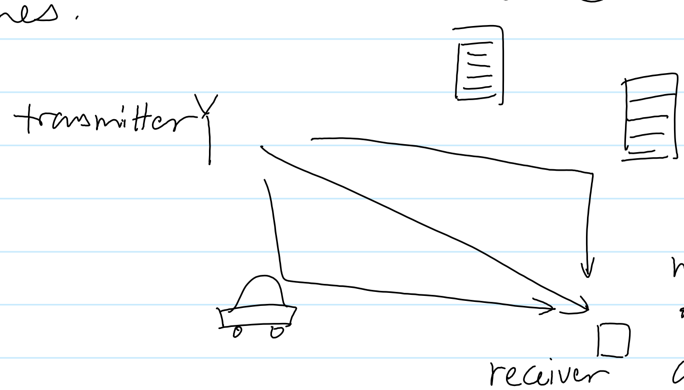
Multi-path fading - 10min

Sunday, January 13, 2008 2:25 PM

Multi-path fading (small-scale fading) is used to describe the rapid fluctuation of the amplitude of a radio signal over a short period of time or over a short travel distance (on the order of wavelengths).

- Variations of path-loss or large-scale fading should be negligible over such short distances

Multipath fading is caused by interference between 2 or more versions of the transmitted signal, which arrive at the receiver at slightly different times.



multiple versions of the signal arrive at slightly different times (phases).

These multiple paths combine at the receiver antenna to give a resultant signal that varies wildly in amplitude & phase

- Similar to what happens in the

2-ray model

Multipath fading is modelled by the α^2 term in

$$P_R = \alpha^2 10^{X/10} g(d) P_T G_T G_R$$

When # of multipath signals is large (> 6), and none of them dominate, α follows Raleigh distribution with pdf

$$f_\alpha(a) = \frac{a}{\sigma_R^2} e^{-\left(\frac{a^2}{2\sigma_R^2}\right)}$$

When only one component dominates (e.g. when there is a strong direct path), α follows Ricean distribution

$$f_\alpha(a) = \frac{a}{\sigma_R^2} e^{-\frac{a^2+A^2}{2\sigma_R^2}} I_0\left(\frac{aA}{\sigma_R^2}\right)$$

where A : amplitude of the dominant signal

$I_0(x)$: Bessel function of the first kind & 0-order

$$I_0(x) = \frac{1}{2\pi}\int_0^{2\pi} e^{x\cos\theta}\, d\theta$$

Note: If $A \to 0$, Ricean $\to$ Raleigh.

- More recent models assume 2 dominant paths. (see suggested readings), which may match better with the pdf of fast-fading components measured from field trials!