Lecture 37

Saturday, April 4, 2020 3:21 PM

Opportunistic beamforming

Wednesday, March 21, 2018 12:31 PM

Paper: P. Viswanath, D. N. C. Tse, R. Laroia, "Opportunistic Beamforming Using Dumb Antennas," IEEE Transactions on Information Theory, JUNE 2002

- The previous discussions have all focused on single-antenna systems.
- As we know, MIMO has been a key mechanism to improve spectrum efficiency for 3G/4G/5G.
- The question is whether one can exploit similar spatial multiplexing gains with random access.
- The difficulty is that most MIMO transmission schemes require the knowledge of CSI (channel state information). Estimating and communicating CSI incur additional overhead.
- This overhead may not be compatible with random access because random access is usually the first step of establishing communications.
- This overhead is also problematic for IoT when the payload is very short.
- We note that unlike opportunistic scheduling for fading channels, MIMO gain exists even when the channel is static.
- Interestingly, the schemes that I will discuss below introduce artificial randomness so that the effective channel becomes dynamic even though the real channel is static.
  - Then, the MIMO gain is attained through opportunistic

scheduling over the random effective channel.

Opportunistic Beamforming

- Random beamforming is an idea that aims to exploit the MIMO gain without full CSI knowledge
- Consider the down-link first
  - Assume that the BS has N antennas
  - K mobiles. Each only has one antenna.
  - The received signal at user k can be written as

$$y_k(t) = \sum_{n=1}^{N} h_{nk}(t) \cdot \sqrt{\alpha_n(t)}\, e^{j\theta_n(t)} \cdot x(t) + z_k(t)$$

vector of channel gains from all antennas to user k.

precoding/beamforming factor

source symbol

noise

- Letting $h_k(t) = \sum_{n=1}^{N} \sqrt{\alpha_n(t)}\, e^{j\theta_n(t)} \cdot h_{nk}(t)$, the resulting channel is

$$y_k(t) = h_k(t)\, x(t) + z_k(t)$$

  - To maintain fixed transmission power, need

$$\sum_{n=1}^{N} \alpha_n(t) = 1$$

- If the BS knows full CSI, i.e., $h_k(t)$, it can choose the beam-forming factor as

$$\alpha_n = \frac{|h_{nk}|^2}{\sum_{n=1}^{N} (h_{nk})^2}, \text{ so that } \sum_n \alpha_n^2 = 1$$

$$\theta_n = -\arg(h_{nk})$$

- This is called the beam-forming configuration

- In this case

$$h_k(t) = \sum_{n=1}^{N} |h_{nk}(t)|^2$$

which is the highest possible.

- However, getting full CSI is expensive, esp. if there are K users.

- In contrast, in random beam-forming, the precoding factor $\sqrt{\alpha_n(t)}\, e^{j\theta_n(t)}$ is chosen <u>randomly</u>.

  - This random beam may be good for some users, and likely bad for other users.

- We then let the users feedback the quality of the resulting effective channel $|h_k(t)|$

  - No need for per-antenna gains.

- Then, the BS picks the users with good $|h_k(t)|$

- Intuitively, the user chosen will likely be the one whose channel is aligned with the current beam-forming factor.

- Thus, we get the beam-forming gain without per-antenna CSI.

Random beamforming in uplink

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- Can we use this idea in the uplink?
  - BS chooses the receiving beam-forming vector.

$$y_k(t) = \sum_{n=1}^{N} h_{nk}(t) \cdot \sqrt{\alpha_n(t)}\, e^{j\theta_n(t)} \cdot x_k(t) + z_k(t)$$

received signal

vector of channel gains from user k to all antennas

receiving beamforming vector

source symbol xmitted by user k

noise

- Each user faces ~~the~~ the effective (uplink) channel

$$h_k(t) = \sum_{n=1}^{N} \sqrt{\alpha_n(t)}\, e^{j\theta_n(t)} \cdot h_{nk}(t)$$

- can use channel-aware Aloha at the UE
- UE needs to know the effective channel.

- There are many possibilities, depending
  - whether any entity (BS or UE) can measure the CSI
  - whether the effective channel resulting from random beamforming can be measured or must be predicted.
  - whether the UE knows the random beamforming vector chosen by the BS.

<u>One possibility:</u>

- Neither BS nor UE know the CSI.
- BS chooses random beamforming
  BS send pilots using the beamforming vector, so

that UE can measure the effective (downlink) channel.

- Assuming channel reciprocity, UE transmits via channel-aware Aloha (i.e., only when the effective channel is good.)

- The main overhead is the measurement of the effective channel (downlink).

Alternatively.

- UE knows the CSI.
- BS chooses the receiving beam-forming vector according to a pseudo-random sequence
- UE knows the pseudo-random sequence of the BS, and therefore knows the receiving beam-forming vector
- UE uses CSI to calculate the effective channel gain
- UE uses channel-aware Aloha to transmit.
- Main overhead is to measure the per-antenna CSI at the UE
    - OK when channel changes very slowly, and downlink channel the same as uplink channel.

- Compared to standard MIMO uplink:
    - No need to communicate the per-user CSI to the BS.
    - No need for BS to estimate uplink CSI.

Caveat:

- Problematic if mobiles move, so channel changes.
- Or if the channel cannot be estimately very accurately
- Or when # of antennas is large

Performance gain

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— Apply to both downlink & uplink

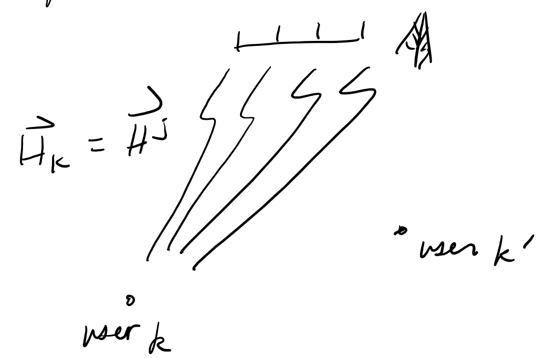
Slow fading

— Channel is random but otherwise fixed in time

— Model the situation when channel changes slowly compared to the time-scale of interest for performance (e.g. throughput, fairness, etc.)

— Specifically, suppose that the channel vector $\vec{H}_k = [h_{1k}, \dots, h_{Nk}]$ can only take one of M possible values. The j-th possible value is $\vec{H}^j$

  — Each user's actual channel is in state $\vec{H}^j$ with probability p_j, $j=1, \dots, M$



Centralized BF

— If this channel state $\vec{H}^j$ of user is known to the BS, BS should choose the beamforming vector to maximize the effective channel of the user

$$h_j = \sum_{n=1}^{N} h_{nj} \sqrt{\alpha_n} \cdot e^{j\theta_n}$$

— Best when $\theta_n = -\arg h_{nj}$

$$h_j = \sum_{n=1}^{N} |h_{nj}| \sqrt{\alpha_n}$$

$$\leq \sqrt{\left(\sum_{n=1}^{N} |h_{nj}|^2\right) \cdot \left(\underbrace{\sum \alpha_n}_{\leq 1}\right)} \quad \text{by (Cauchy-Schwartz)}$$

$$\leq \sqrt{\sum_{n=1}^{N} |h_{nj}|^2}$$

— The best h_j is achieved when

$$\frac{\alpha_n}{|h_{nj}|^2} \text{ is the same for all } n$$

$$\Rightarrow \quad \alpha_n = \frac{|h_{nj}|^2}{\sum_{n=1}^{N} |h_{nj}|^2}, \quad n = 1, \cdots, N$$

— Since users' channels are random, some users have larger $\sqrt{\sum |h_{nj}|^2}$ than others.

 — Only serving the best users will be unfair.

— Instead, we may impose a fairness contrain that each user is served $\frac{1}{K}$ fraction of time

— Since the channel is the constant over time, the average rate that the user in state j gets will be

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— ...

rate that the user in state j gets will be

$$\frac{1}{K} R_j^{BF}$$

— where R_j^{BF} is the instantaneous rate at the channel gain $\sqrt{\sum_{n=1}^{N} |h_{n,j}|^2}$

Opportunistic BF

— We now see whether opportunistic BF can attain similar throughput / fairness goals

— First, the BF vector is random.

— However, note that each state j has a desirable BF vector in the centralized case.

— It is intuitive that opportunistic BF should also only choose the BF vectors from this set.

— Let q_j be the probability that the BF vector for state j is chosen

— Consider the following scheduling algo (not optimal)

  — If the BF vector for state j is chosen by the BS, randomly serve one user from those in state j

— Their rate will also be R_j^{BF}

- Let ψ_j be the fraction of users on state j

Then the throughput of each such user is

$$\frac{q_j}{\psi_j K} R_j^{BF}$$

- If we instead uses random access, then each user in state j transmits with prob. $\frac{1}{\psi_j K}$, only when its effective channel is the best possible.

$\Rightarrow$ a $\frac{1}{e}$ loss will occur due to collision.

- Note ~~that~~ when K is large, $\psi_j \to p_j$.

Therefore, if we let $q_j = p_j$, then

$$\frac{q_j}{\psi_j K} R_j^{BF} \to \frac{1}{K} R_j^{BF}$$

- which is exactly the rate of centralized BF

Theorem 1: Suppose that the slow fading states of the users are i.i.d., and the joint distribution of the BF vectors is the same as that of

$$\alpha_n = \frac{|h_{nj}|^2}{\Sigma |h_{nj}|^2}, \quad \theta_n = -\arg h_{nj}$$

for the slow-fading state of the users. Then,

$$\lim_{K \to +\infty} K \bar{T}_k^{(k)} = R_k^{BF} \quad \text{for all users } k$$

where R_k^{BF} is its best rate with centralized BF, i.e., when its instantaneous channel is

N . 2

its instantaneous channel is

$$\sum_{n=1}^{N} |h_{nk}|^2 .$$

- In summary, if the OF process "matches" the slow-fading distribution of the users, the performance of opp. BF approaches that of centralized BF when K is large

- Important to note that, although our derivation assumes the BS knows the CSI of each state j (i.e. each group of users), the beam-forming operation of the BS only needs the distribution of the states (e.g. sufficient to know that the channel is Raleigh

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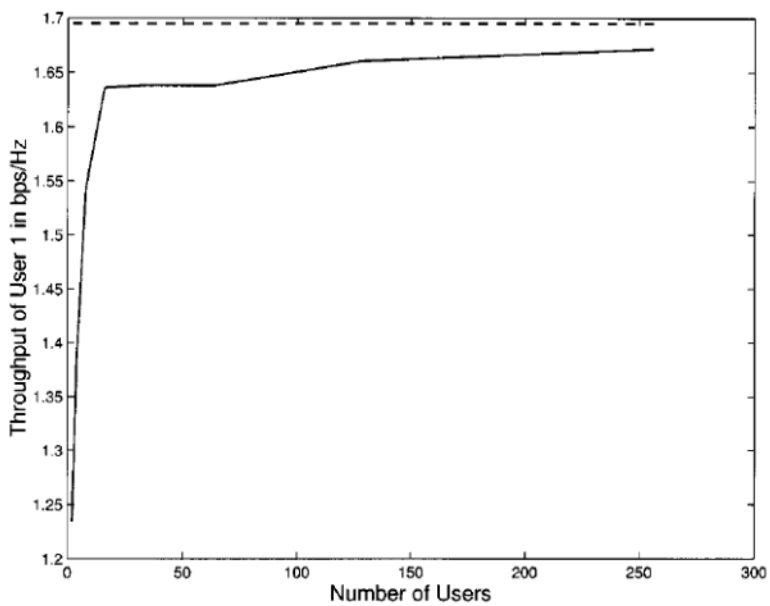

Fig. 7. Throughput in b/s/Hz for user 1 multiplied by number of users scheduled for slow Rayleigh fading at 0-dB SNR with the proportional fair scheduling algorithm. Performance of coherent beamforming for user 1 and scheduled at all time is plotted as a dotted line. We have chosen two antennas.

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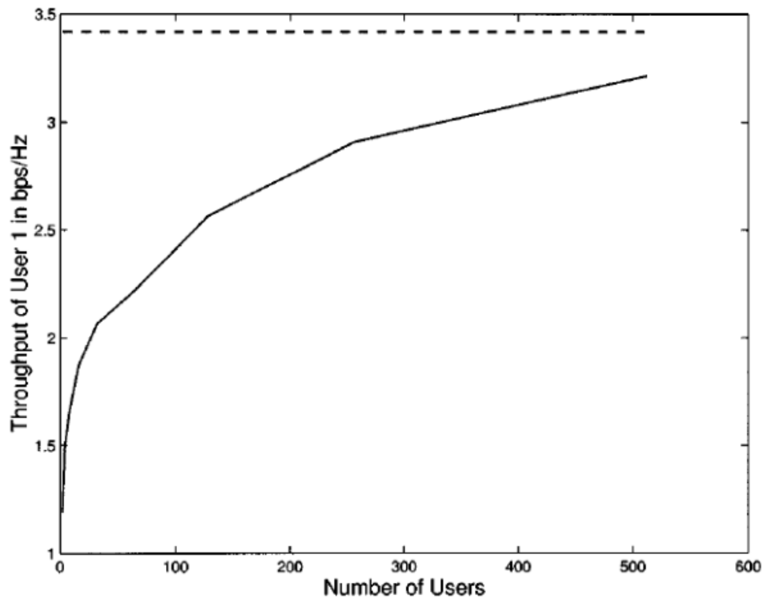

Fig. 8. Throughput in b/s/Hz for user 1 multiplied by number of users scheduled for slow Rayleigh fading at 0-dB SNR with the proportional fair scheduling algorithm. Performance of coherent beamforming for user 1 and scheduled at all time is plotted as a dotted line. There are 10 antennas in this experiment.

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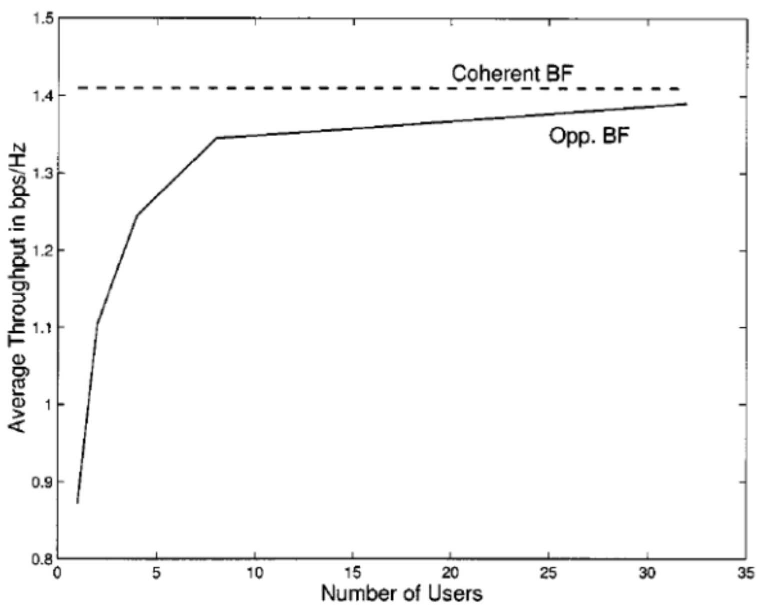

Fig. 9. Total throughput in b/s/Hz averaged over slow Rayleigh fading at 0-dB SNR with the proportional fair scheduling algorithm. Performance of coherent beamforming is also plotted.

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Fast fading

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- The situation is a little bit different when fading is fast
  - In addition to the increasing overhead due to the change of the channel (e.g. to measure either the CSI or the effective channel), the gain due to random beamforming may also be limited.
- The reason is, even if one antenna, there is already opportunistic scheduling gains (e.g. the throughput of the system will increase with the # of users)
- We will need to ask whether multiple antenna and opp. BF will further improve the throughput.

Raleigh fading

- If the fast fading is Raleigh
  It means that h_{nk} is complex Gaussian, circular in phase
- Given the BF vectors $\sqrt{\alpha_n} e^{j\theta_n}$

$$h_k = \sum_{n=1}^{N} h_{nk} \sqrt{\alpha_n} e^{j\theta_n}$$

  - Let us understand the distribution of the eff. channel h_k
  - Note that for our purpose, $\sqrt{\alpha_n} e^{j\theta_n}$ should be chosen randomly, without knowing the exact channel

h_{nk}

- Thus, it is reasonable to assume that $\sqrt{\alpha_k}\, e^{j\theta_k}$ is independent of h_{nk}.

- First, note that $h_{nk} e^{j\theta_n}$ has the same distribution as h_{nk}.

- Second, the sum of Gaussian r.v.'s is still Gaussian.

- Hence, h_k should also be complex Gaussian, circular

$$E[|h_k|^2] = \sum_{n=1}^{N} \alpha_n \cdot E[|h_{nk}|^2] = \sigma^2 \cdot \sum_{n=1}^{N} \alpha_k = \sigma^2$$

  - h_k has the same Raleigh distribution!

- Further, if we consider 2 users, conditioned on α_n, θ_n

$$E(h_k \cdot h_l^*) = E\left(\sum_{n=1}^{N} h_{nk} \cdot \sqrt{\alpha_n}\, e^{j\theta_n} \cdot \left(\sum_{n=1}^{N} h_{nl} \sqrt{\alpha_n}\, e^{j\theta_n} \right)^* \right)$$

$$= 0$$

- Hence, all h_k's are independent Raleigh

- In summary, even with multiple antennas, the effective channel seen by a user has the same i.i.d. Raleigh distribution as if there is only one antenna

  - No gain from opp. BF!

- Does it mean ~~that~~ there is no gain due to

— Does it mean ~~that~~ there is no gain due to MIMO?

— No, centralized BF can still pick $\alpha_n e^{j\theta_n}$ based on each user's channel.

— Require knowing the exact CSI.

— In other words, if the channel is already sufficiently ~~fast &~~ diffused, it is difficult for opp BF to attain the higher MIMO gain

— This conclusion will change if channel is not that "random".

— Ricean

— Correlated Raleigh

<u>Correlated Raleigh</u>

— The channel gain from all antennas to the user is the same magnitude except for a phase shift

$$h_{nk}(t) = l_k(t) \cdot e^{j\phi_{nk}}$$

$\uparrow$
same for all antennas

— Since the gain is the same across antennas, we also choose $\alpha_n = \frac{1}{\sqrt{N}}$.

— From $\cdot^{1}$ $\quad j\theta_n$

$$h_k = \sum_{n=1}^{N} h_{nk} \cdot \sqrt{\alpha_n}\, e^{\cdots}$$

$$= \left[\sum_{n=1}^{N} \frac{1}{\sqrt{N}} e^{j(\theta_n + \phi_{nk})} \right] l_k$$

$$\Rightarrow \quad |h_k|^2 = \left| \frac{1}{\sqrt{N}} \sum_{n=1}^{N} e^{j(\theta_n + \phi_{nk})} \right|^2 \cdot |l_k|^2$$

BF gain   Raleigh

- The system throughput depends on the distribution of $\max_{k=1,\dots K} |h_k|^2$

- For one antenna, $h_k = l_k$
  - $\max_k h_k$ scales like $\log k$

- For N antennas, the maximum value of

$$\left| \frac{1}{\sqrt{N}} \sum_{n=1}^{N} e^{j(\theta_n + \phi_{nk})} \right|^2$$

can be N, if all the BF phases are right.

- For a fixed $\delta > 0$, when K is large, there will be some fraction εK of users for which their

$$\left| \frac{1}{\sqrt{N}} \sum_{n=1}^{N} e^{j(\theta_n + \phi_{nk})} \right|^2 > N - \delta.$$

- Choosing the best among these εK gives a maximum of $|h_k|^2$ that grows at least as fast as

$$(N-\delta) \log(\varepsilon K) = (N-\delta) \log K + O(1)$$

as $K \to +\infty$

- Since this is true for all δ, it suggests that opp BF will have a gain of N, i.e., the # of antennas, compared to one-antenna system.

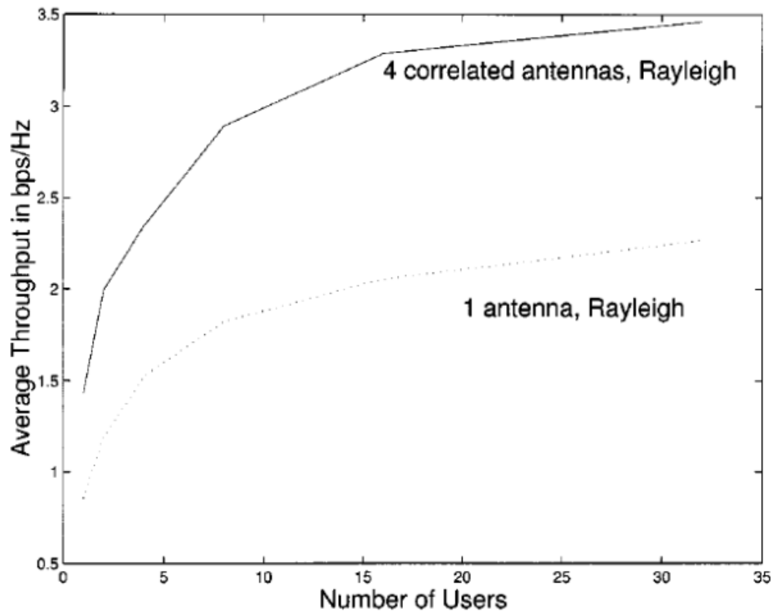

Fig. 13. Total throughput as a function of the number of users under completely correlated Rayleigh fading, with and without opportunistic beamforming.

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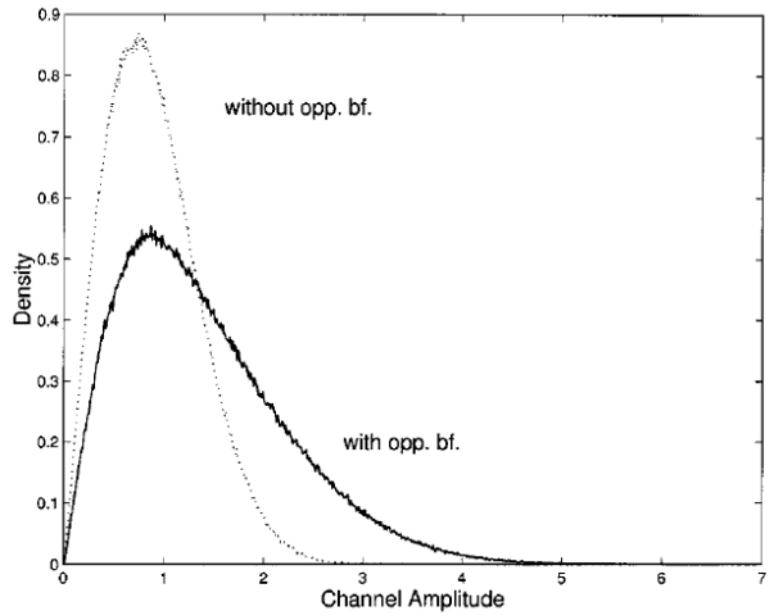

Fig. 12. Comparison of the distribution of the overall channel gain with and without opportunistic beamforming using 4 transmit antennas, completely correlated Rayleigh fading.

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- In summary, if the channel is not that "random", opp. BF introduces more randomness in the system,
- This makes the peak channel even higher,

which improves system throughput with multiple antennas
- Similar gains when channel is Ricean. See paper.