Lec36

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Fading

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X. Qin and R. Berry, "Exploiting Multiuser Diversity for Medium Access Control in Wireless Networks," IEEE INFOCOM, 2003.

- Our discussion so far has mostly assume that channel is fixed.
- In reality, channel changes
- As we have studied in opportunistic scheduling, how to exploit channel variation is a big consideration to improve overall throughput/capacity.
- If the random access mechanism does not respond to channel variations, it will likely miss the opp. scheduling gain.
- Therefore, it is desirable to come up with channel-aware random access mechanisms.
- However, the challenge is that most opp. scheduling mechanisms require the BS to know the channel of All users
- For random access, such a centralized knowledge seems unrealistic.
- Can we still get the opp. scheduling gain with only decentralized information.

Model.

- uplink: n users all transmitting to the BS.

- time is slotted. Channel is fixed in a time-slot, but change independently in the next time-slot.

- If user i is the only user transmitting in a given time slot, the received signal, $y_i(t)$, is given by

$$y_i(t) = \underbrace{\sqrt{H_i}}_{\text{channel gain}}\ \underbrace{X_i(t)}_{\text{transmitter signal}} + \underbrace{Z(t)}_{\text{noise}}$$

- Let P_t denote the transmission power of the user.
  - Thus the received power is $P_r = H_i P_t$

- We assume that H_i is i.i.d. across users & time, with pdf given by $f_H(h)$.
  - e.g. for Raleigh

    $$f_H(h) = e^{-\frac{h}{h_0}} \cdot \frac{1}{h_0}$$

    - mean is h_0

- If more than one users transmit, all packets will fail.
  - a collision.

<u>Key to decentralized operation</u>:

- At the start of each time-slot, each user knows its own channel gain during the slot, but not the gain of any other users

– However, the common distribution $f_H(h)$ is known

<u>Channel-Aware Aloha</u>

– In standard Aloha, each user can be thought of transmitting with prob. p.

– In our system, conceptually there is also a prob. that each user transmits.

– Given p, it seems intuitive that each user should only transmit when its channel gain is high

 – The other users' channel gains are i.i.d.

– This suggests that each user should use a threshold policy. There is a value H_0 such that a user only transmits when its channel gain is above H_0.

– Then, the transmission probability is simply

$$p = \int_{H_0}^{+\infty} f_H(h)\,dh = \underset{\substack{\uparrow \\ \text{cdf}}}{F_H(H_0)}$$

$$H_0 = F_H^{-1}(p).$$

<u>Power:</u>

– Let $R(P_r)$ be the achieved rate when the received power is P_r.

 – e.g. $R(P_r) = W \log\left(1 + \frac{P_r}{N_0 W}\right)$

- Assume that, whenever a user transmits, it aims for a constant received power P_r
  - rate becomes also a constant
  - the transmission power needs to be controlled

$$P_t = \frac{P_r}{h}$$

- Suppose that there is a long term power constraint $\bar{P}$

$$\int_{F_H^{-1}(p)}^{+\infty} f_H(p) \cdot \frac{P_r}{h}\, dh \le \bar{P}$$

$$\Leftrightarrow \quad P_r \le \frac{\bar{P}}{\int_{F_H^{-1}(p)}^{+\infty} f_H(p) \cdot \frac{1}{h}\, dh}$$

Throughput

- Each user transmits with prob. p, indep. of others
- success occurs with prob. $np(1-p)^{n-1}$
- rate when success is $R(P_r)$ with

$$P_r = \frac{\bar{P}}{\int_{F_H^{-1}(p)}^{+\infty} f_H(h) \frac{1}{h}\, dh}$$

- Hences the total throughput of the system is

$$s(p,n) = np(1-p)^{n-1} R\left(\frac{\bar{P}}{\int_{F_H^{-1}(p)}^{+\infty} f_H(h) \cdot \frac{1}{h}\, dh}\right)$$

Throughput scaling

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- Let us look at the resultant throughput and focus on the scaling when n is large

$$s(p,n) = np(1-p)^{n-1} R\left(\frac{P}{\int_{F_H^{-1}(p)}^{+\infty} f_H(h)\cdot\frac{1}{h}\,dh}\right)$$

- Note that the first term $np(1-p)^{n-1}$ is maximized when $p=\frac{1}{n}$
  - Let $s(n) = s(\frac{1}{n}, n)$, i.e., when $p=\frac{1}{n}$
  - Not necessarily optimal though!
- If $p=\frac{1}{n}$, $F_H^{-1}(p) \nearrow$ with n
  - Threshold for transmission $\nearrow$
  - The rate should then $\nearrow$
- This suggests that the throughput $s(n)$ of system should increase with n
  - Consistent with the intuition that there are more "opportunity" of good channels when n is large.

How fast is the growth of $s(n)$?

- Suppose ~~that~~ $p=\frac{1}{n}$.
- A lower bound for $s(n)$ can be easily established

$$\int_{F_H^{-1}(p)}^{+\infty} f_H(h)\cdot\frac{1}{h}\,dh$$

$$\le \int_{F_H^{-1}(p)}^{+\infty} f_H(h)\frac{1}{F_H^{-1}(p)}\,dh \qquad (\text{since } h \ge F_H^{-1}(p))$$

$$\leq \int_{F_H^{-1}(p)}^{+\infty} f_H(h) \frac{1}{F_H^{-1}(p)} dh \qquad (\text{since } h \geq F_H^{-1}(p))$$

$$= \frac{1}{F_H^{-1}(p)} \cdot \underbrace{\int_{F_H^{-1}(p)}^{+\infty} f_H(h) dh}_{= p}$$

$$= \frac{p}{F_H^{-1}(p)} = \frac{1}{n F_H^{-1}(\frac{1}{n})}$$

$$\Rightarrow s(n) \geq \left(1 - \frac{1}{n}\right)^{n-1} \cdot R\left(\bar{P} \cdot n \cdot F_H^{-1}\left(\frac{1}{n}\right)\right) \qquad (*)$$

- For the special case of Raleigh distribution

$$f_H(h) = \frac{1}{h_0} e^{-\frac{h}{h_0}}$$

$$F_H(h) = e^{-h/h_0} = \frac{1}{n}$$

$$\Rightarrow h = h_0 \cdot \log n$$

$$\Rightarrow s(n) = \left(1 - \frac{1}{n}\right)^{n-1} R(\bar{P} \cdot n \cdot h_0 \log n)$$

- Further, if $R(\bar{p}) = W \log\left(1 + \frac{\bar{P}}{N}\right)$, then

$$s(n) \propto \log n$$

- It turns out that the expression in $(*)$ is fairly tight

Prop. 1: Assume that $R(x)$ is a monotonically increasing and concave function of x, and $\frac{F_H(h)}{h} = O(f_H(h))$, then as $n \to +\infty$

$$s(n) = \Theta\left[R\left(\bar{P} \cdot n \cdot F_H^{-1}\left(\frac{1}{n}\right)\right)\right]$$

- The concavity assumption of $R(x)$ is reasonable, e.g. see the Shannon formula $R(x) = W \log\left(1 + \frac{x}{N_0}\right)$

- The assumption that $\frac{F_H(h)}{h} = O(f_H(h))$ holds when the tail of $f_H(h)$ is exponential

  - $f_H(h) \propto e^{-ch}$

$$F_H(h) \propto \int_h^{+\infty} e^{-cu} du = \frac{1}{c}\left[-e^{-cu}\right]_h^{+\infty}$$

- $f_H(h) \propto e$

$$\bar{F}_H(h) \propto \int_h^{+\infty} e^{-cu}\,du = \frac{1}{c}\left[-e^{-cu}\right]_h^{+\infty}$$

$$= \frac{1}{c} e^{-ch}$$

$$\frac{\bar{F}_H(h)}{h} = O(f_H(h))$$

<u>To see why</u> this results is true:

- Start from

$$S(n) = S\left(\tfrac{1}{n}, n\right) \quad \swarrow p = \tfrac{1}{n}$$

$$= \left(1 - \frac{1}{n}\right)^{n-1} R\left(\underbrace{\frac{\bar{p}}{\int_{\bar{F}_H^{-1}(\frac{1}{n})}^{+\infty} f_H(h)\frac{1}{h}\,dh}}_{\text{only need to focus on this part}}\right)$$

- lower bound: (just a repeat of the earlier argument)

  - for $h \ge \bar{F}_H^{-1}\left(\frac{1}{n}\right)$

$$\frac{f_H(h)}{h} \le \frac{f_H(h)}{\bar{F}_H^{-1}\left(\frac{1}{n}\right)}$$

$$\Rightarrow \int_{\bar{F}_H^{-1}(\frac{1}{n})}^{+\infty} \frac{f_H(h)}{h}\,dh$$

$$\le \frac{1}{\bar{F}_H^{-1}\left(\frac{1}{n}\right)} \int_{\bar{F}_H^{-1}(\frac{1}{n})}^{+\infty} f_H(h)\,dh$$

$$= \frac{1}{n\,\bar{F}_H^{-1}\left(\frac{1}{n}\right)}$$

$$\Rightarrow S(n) \ge \left(1 - \frac{1}{n}\right)^{n-1} R\left(\bar{p}\, n \cdot \bar{F}_H^{-1}\left(\tfrac{1}{n}\right)\right)$$

- Upper bound:

- Upper bound:
  Suppose
  - $\wedge \frac{\bar{F}_H(h)}{h} < M \cdot f_H(h)$ when $h \geq h_c$
  - Then

$$\frac{f_H(h)}{h} \geq \frac{1}{M} \frac{\bar{F}_H(h)}{h^2}$$

$$\Rightarrow \left(\frac{1}{M}+1\right)\frac{f_H(h)}{h} \geq \frac{1}{M}\left[\frac{\bar{F}_H(h)}{h^2} + \frac{f_H(h)}{h}\right]$$

$$\Rightarrow \frac{f_H(h)}{h} \geq C\left[\frac{\bar{F}_H(h)}{h^2} + \frac{f_H(h)}{h}\right]$$

$$C = \frac{\frac{1}{M}}{\frac{1}{M}+1}$$

- Thus

$$\int_{\bar{F}_H^{-1}(\frac{1}{n})}^{+\infty} \frac{f_H(h)}{h}\,dh$$

$$\geq C \cdot \int_{\bar{F}_H^{-1}(\frac{1}{n})}^{+\infty} \left[\frac{\bar{F}_H(h)}{h^2} + \frac{f_H(h)}{h}\right]$$

$$= -C \left.\frac{\bar{F}_H(h)}{h}\right|_{\bar{F}_H^{-1}(\frac{1}{n})}^{+\infty}$$

$$= C \cdot \frac{1}{n\,\bar{F}_H^{-1}(\frac{1}{n})}$$

- Substituting into $R(\cdot)$ gets us the result.

- In general, we have

$$\int_{h_0}^{+\infty} \frac{f_H(h)}{h}\,dh$$

scales like

$$\frac{F_H(H_0)}{H_0}$$

when H_0 is large.

- We will compare the throughput of channel aware Aloha with centralized scheduling

- However, we need to be careful of the power limits

Max instantaneous power

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- Earlier discussion focuses on average power constraint $\bar{P}$
- Another practical limitation is instantaneous power
- Let us look at how large the instantaneous power needs to be in order to achieve the above throughput scaling
- The highest transmission power is needed when the channel gain is the lowest, i.e.,

$$h = H_0 = F_H^{-1}\left(\frac{1}{n}\right)$$

$$\Rightarrow P_m = \frac{P_r}{H_0} = \frac{\bar{P}}{H_0 \underbrace{\int_{F_H^{-1}(\frac{1}{n})}^{+\infty} f_H(h) \frac{1}{h} \, dh}_{\text{about } \frac{1}{n F_H^{-1}(\frac{1}{n})} = \frac{1}{n H_0}}}$$

- Hence, $P_m = \Theta(n)$ when n is large
  - In other words, higher instantaneous power must be used to exploit the opportunistic gain.
  - This will eventually become problematic when n is large.
- If P_m is lower than $\Theta(n)$, then the throughput will be further limited.

- Assume that a fixed power P_m is used when $H > H_0$
- Instead of $R\left(\frac{\overline{P}}{\int_{F_H^{-1}(\frac{1}{n})}^{+\infty} f_H(h)\frac{1}{h}\,dh}\right)$

  we get

  $$R(P_m \cdot H_0)$$
- Since $H_0 = F_H^{-1}(\frac{1}{n})$

  we get

  $$S_m(n) = \left(1-\frac{1}{n}\right)^{n-1} R\Big(P_m \cdot \underbrace{F_H^{-1}\left(\tfrac{1}{n}\right)}_{\approx h_0 \log n}\Big)$$
- $S_m(n)$ grows as $\Theta(\log\log n)$!

- $S_m(n)$ is a lot lower than $S(n)$! Why?
  - With only average power constraint, it appears that the transmission power is comparable to the total power $n\overline{P}$, which increases with n.
  - With instantaneous power constraint, this growth disappears!

Comparison with centralized scheduling

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Comparison with centralized scheduling

- The above result suggests that channel-aware Aloha can also attain the opportunistic scheduling gain.
- We now compare it with a centralized scheduling who knows all channel conditions
  - H_i : channel of user i
- Assume that the scheduler uses a fixed power P_m. Its average throughput is then

$$S_{ct}(n) = E\left[R\left(P_m \cdot \max_i H_i\right)\right]$$

- Assume that channel-aware Aloha also uses fixed power P_m
  - otherwise, the max power grows with n, which is unfair for comparison with a centralized scheduler

$$S_m(n) \geq \left(1-\frac{1}{n}\right)^{n-1} R\left(\underbrace{P_m F_H^{-1}\left(\frac{1}{n}\right)}\right)$$

Fixed rate. Adaptive rate should only be better.

- The $\left(1-\frac{1}{n}\right)^{n-1}$ contributes to a $\frac{1}{e}$ loss.
- We will be interested in the ratio of the remaining terms

. . . .

$$\frac{R\left(P_m F_H^{-1}\left(\frac{1}{n}\right)\right)}{R\left(P_m \max_i H_i\right)}$$

- Consider the special case when the channel is Raleigh

$$f_H(h) = \frac{1}{h_0} e^{-\frac{h}{h_0}}$$

$$F_H(h) = e^{-\frac{h}{h_0}} = \frac{1}{n} \Rightarrow h = h_0 \log n$$

$$\Rightarrow F_H^{-1}\left(\frac{1}{n}\right) = h_0 \log n$$

- How about $\max_i H_i$

$$P\left[\max_i H_i > h_0 \log n + c\right]$$

$$\leq n \cdot P\left[H_i > h_0 \log n + c\right)$$

$$= n \cdot e^{-\frac{h_0 \log n + c}{h_0}} = e^{-\frac{c}{h_0}}$$

- This prob. goes down to zero at a rate independent of n. Thus, when $\log n$ is large, it is unlikely that $\max H_i$ will be much larger than $h_0 \log n$

Lemma 1 (extreme order statistics)

- Let $z_1, \cdots, z_n$ be i.i.d. random variables with a cdf $F(\cdot)$ & pdf $f(\cdot)$ satisfying

$$\lim_{z \to +\infty} \frac{F(z)}{f(z)} = c > 0$$

for some constant c

- holds for exp. tail, e.g. Raleigh

- Let l_n be given by $F(l_n) = \frac{1}{n}$

- Then $\max_i z_i - l_n$ converges in distribution to a limiting r.v. with cdf $\exp(-e^{-x}))$

- According to Lemma 1, $\max_i Z_i$ grows at about the same rate as l_n

- In our case, $l_n = F_{Z}^{-1}(\frac{1}{n})$

  Hence, the ratio above will approach 1

Prop. 5. If $R(\cdot)$ is strictly increasing, then

$$\lim_{n \to +\infty} \frac{S_m(n)}{S_{ct}(n)} = \lim_{n \to +\infty} \left(1 - \frac{1}{n}\right)^{n-1} = \frac{1}{e}$$

- The only penalty for channel-aware Aloha is due to contention.

- Distributed channel knowledge does not incur a loss when n is large

- Of course will be different when n is finite.

With arrival, infinite node case

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- Suppose that packets arrive with rate of λ
- Each new packet corresponds to a new user
- Suppose that we know the current # of packets in the system n.
- Choose $p = \frac{1}{n}$
  - In other words, the channel threshold H_0 also varies with n

$$H_0 = \bar{F}_H^{-1}\left(\frac{1}{n}\right)$$

- Let

$$C_n = \underbrace{\left(1-\frac{1}{n}\right)^{n-1}}_{\text{approaches } \frac{1}{e}} R\left(\underbrace{\frac{\bar{P}}{\int_{\bar{F}_H^{-1}(\frac{1}{n})}^{+\infty} f_H(h)\frac{1}{h}\,dh}}_{\text{scales as } \bar{P}\cdot n\,\bar{F}_H^{-1}(\frac{1}{n})}\right)$$

approaches $\frac{1}{e}$

scales as $\bar{P} \cdot n \bar{F}_H^{-1}\left(\frac{1}{n}\right)$
- increases indefinitely with n

- Hence $\lim_{n\to+\infty} C_n = +\infty$
- The system is stable at any arrival rate!
- Not true if the # of users is finite