Lec34

Saturday, April 4, 2020 3:17 PM

Infinite node

Thursday, March 15, 2018 11:14 AM

Guy Fayolle, Erol Gelenbe and Jacques Labetoulle, "Stability and Optimal Control of the Packet Switching Broadcast Channel," *Journal of ACM*, 1977

- Let us first take a closer look at Aloha
- Let us use a slotted model
- Assume that the # of new packets arriving at a time slot is Poisson with mean ν
- Each packet immediately starts its own random access procedure
    - Can be seen as infinite # of nodes
    - Each node has no more than one packet.
- In Aloha, assume that each node transmits in each time slot with probability $\frac{1}{r_0}$, $r_0 > 1$.
- Let $X(t)$ be the # of packets in the system

$$X(t+1) = X(t) + \underbrace{A(t)}_{\text{Poisson}(\nu)} - \underbrace{D(t)}_{\text{1 if a successful transmission takes place}}$$

$$P\{D(t) = 1\} = X(t) \cdot \frac{1}{r_0} \left(1 - \frac{1}{r_0}\right)^{X(t)-1}$$

- This is a Markov chain, and we can ask questions like

- Is $X(t)$ stable?
- $E(X(t))$, Delay, etc.

- It turns out that it is very easy to see Aloha is unstable. (Implies $E(X) \to +\infty$, $E(D) \to +\infty$).

- The problem is that, if $X(t)$ is large, $D(t)$ will be 0 with high probability

$$E(D(t) \mid X(t)) = X(t) \frac{1}{r_0} \left(1 - \frac{1}{r_0}\right)^{X(t)-1}$$

- if $X(t) = r_0 \cdot n$, then

$$E(D(t) \mid X(t)) = n \cdot \left(1 - \frac{1}{r_0}\right) \left(1 - \frac{1}{r_0}\right)^{r_0 n}$$

$$\to 0 \quad \text{as } n \to +\infty$$

- Hence, there is X_0 such that if $X(t) > X_0$,

$$E(D(t) \mid X(t)) < \lambda = E(A(t))$$

- The system will grow to infinity with high probability
- $E(D(t) \mid X(t)) \to 0$, no packets go through eventually

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- In summary, if the backoff is fixed, the system is unstable when an infinite # of nodes exist.
- Thus, some kind of adaptive backoff is crucial

Best Retransmission Policy

Tuesday, March 20, 2018 1:52 PM

- Suppose that we know current # of backlogged packets in the system: $X(t)$.
- How to control the (re-)transmission prob. as a function of $X(t)$
- This is equivalent to maximizing

$$X(t)\, f \cdot (1-f)^{X(t)-1}$$

- differentiate w.r.t. f

$$X(t)\,(1-f)^{X(t)-1} - X(t)\cdot f\,(X(t)-1)(1-f)^{X(t)-2} = 0$$

$$1-f - f(X(t)-1) = 0$$

$$f \cdot X(t) = 1$$

$$f = \frac{1}{X(t)}$$

$$\Rightarrow \quad X(t)\, f \cdot (1-f)^{X(t)-1}$$

$$= X(t)\cdot \frac{1}{X(t)} \cdot \left(1 - \frac{1}{X(t)}\right)^{X(t)-1}$$

$$= \left(1 - \frac{1}{X(t)}\right)^{X(t)-1} \rightarrow \frac{1}{e} \quad \text{as } X(t) \to +\infty$$

- Hence, the system should be stable if

$$\lambda < \frac{1}{e}$$

- same as the maximum throughput under Poisson assumption

– The above assume that new packets also transmit with prob. $\frac{1}{X(t)}$.

– If new packets always transmit, the retransmission prob. should be even lower. See paper

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What if we do not know $X(t)$?

– One can show that any adaptive backoff that is slower than exponential will also be unstable

– On the other hand, exponential backoff seems pretty fast in adapting

– Is the system going to be stable or not?

Binary exponential backoff

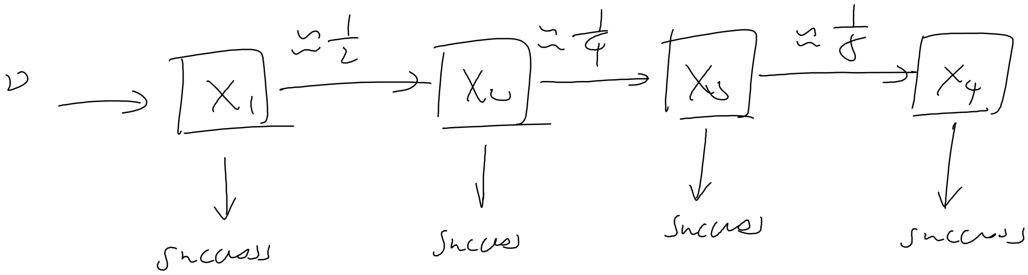
Thursday, March 15, 2018 11:35 AM

Paper: D. J. Aldous, "Ultimate Instability of Exponential Back-Off Protocol for Acknowledgment-Based Transmission Control of Random Access Communication Channels," IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. IT-33, NO. 2, MARCH 1987

- We are going to show that in the infinite-node case, binary exponential backoff is also unstable for any $\nu > 0$

- To be specific, let $r_0 = 1$, $r = 2$
  - first transmission occurs with prob. $\frac{1}{r_0} = 1$
  - every subsequent transmission occurs with prob.
    $$\frac{1}{r_0 r^i} = \frac{1}{2^i}$$
    - i is # of collisions that have occurred.

- To model this system, let $X_i(t)$ be the # of packets whose backoff stage is i at time t.



<u>Intuition for instability</u>

- Let us consider a system with no success

$$\nu \rightarrow \boxed{Y_1} \xrightarrow{\simeq \frac{1}{2}} \boxed{Y_2} \xrightarrow{\simeq \frac{1}{4}} \boxed{Y_3} \xrightarrow{\frac{1}{8}} \boxed{Y_4} \longrightarrow \cdots$$

- Surprisingly, there is a stationary distribution for $\vec{Y} = [Y_i]$

- We can calculate the mean easily

$$\nu = \frac{1}{2} E[Y_1] \Rightarrow E[Y_1] = 2\nu$$

$$\frac{1}{2} E[Y_1] = \frac{1}{4} E[Y_2] \Rightarrow E[Y_2] = 2E[Y_1] = 2^2 \nu$$

- Similarly

$$E(Y_i) = 2^i \cdot \nu$$

- Further, using the fact that each packet evolves independently, one can show that Y_i is a Poisson r.v. with the mean $2^i \cdot \nu$ (when no success)

- The observation is that this "no success" paradigm is self-sustaining.

- Given Y_i, the probability of having at least one transmission is

$$1 - \left(1 - \frac{1}{2^i}\right)^{Y_i} \simeq 1 - \left(1 - \frac{1}{2^i}\right)^{\nu 2^i}$$

$$\simeq 1 - e^{-\nu}$$

- which is independent of i

- Thus, for an infinite # of stages, the probability of success will go to zero

- Of course, in the real system there will be success, and hence $X_i(t) \le Y_i$
  - However, if for some reason $X_i(t) \ge 2^i \nu$ for a large number of stages, ~~then~~ very few success will occur, and thus the system will become close of Y_i for the rest of time.
- Similar to the way Aloha becomes unstable
- We only need to show ~~that~~ such "no-return" occurs with positive probability. See paper for details.

Discussions

Friday, March 16, 2018 8:29 AM

- What if the # of backoff stages is finite
  - e.g. 802.11, max contention window 1024
  - unstable. Like Aloha
- What if the # of stations is finite
  - instability in the # of nodes cannot occur

Alternative adaptive schemes

Wednesday, April 8, 2020 4:42 PM

V. A. Milkhailov, "Geometrical analysis of the stability of markov chains in rn+ and its application to throughput evaluation of the adaptive random multiple access algorithm," Probl. Inform. Transm., pp. 47–56, 1988

In the above paper, a scheme is proposed to estimate the # of contending stations

- Let the true number of active stations be $n(t)$
- Let $s(t)$ be the estimate.
- Each node transmits with probability $\frac{1}{s(t)}$
  - if $s(t) = n(t)$, this would be the best attempt prob.
- The key is to update $s(t)$ based on the outcome the channel: idle, success, collision
  - If $s(t) < n(t)$, the attempt prob. is too high, we will likely see a lot of collision
  - If $s(t) > n(t)$, the attempt prob. is too low, we will likely see a lot of idle slots.
- Thus, $s(t)$ is updated by

$$s(t+1) = s(t) + \underset{\substack{\uparrow \\ a<0}}{a}\, 1_{\{idle\}} + \underset{\substack{\uparrow \\ >,<,=0}}{b}\, 1_{\{success\}} + \underset{\substack{\uparrow \\ c>0}}{c}\, 1_{\{collision\}}$$

- The authors show that for properly chosen values of a, b, c, the system is stable for any $\nu < \frac{1}{e}$.