Lec22-mwf

Saturday, February 17, 2018 2:05 PM

HW4 is on the web.

Model - 15min

Tuesday, February 05, 2008 2:53 PM

We will focus on one of the simplest models:

Reference: *The capacity of Wireless Networks* by Piyush Gupta and P.R. Kumar, IEEE Transactions on Information Theory, Vol. 46, No.2, Mar 2000

1. n nodes. Their locations are uniformly distributed in a unit area.

2. Each node can transmit at the rate of W bits/sec
   - No adaptive coding/modulation

3. Nodes may transmit simultaneously if they are "sufficiently" apart.

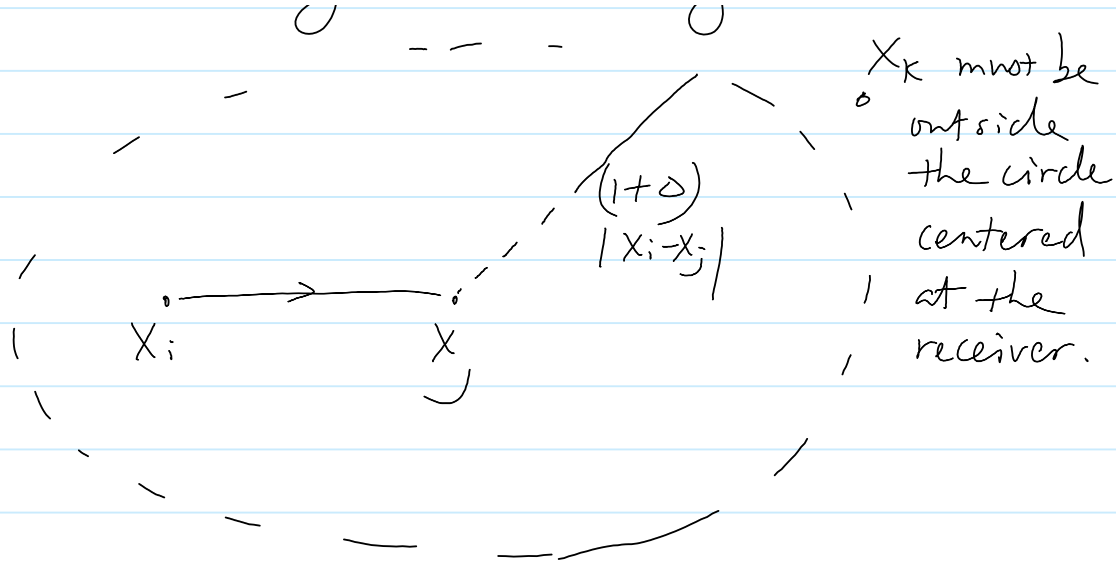
<u>Protocol model:</u>

Node X_i can transmit to node X_j if

$$|X_k - X_j| \geq (1+\Delta)\,|X_i - X_j|$$

for every other node X_k that is simultaneously transmitting.

X_k must be



- Δ related to the SINR threshold.

④ Each node randomly picks a node uniformly from all other nodes as the destination, and send data to the destination at the same rate λ.

Question: What is the largest λ that the network can support?

How to design MAC, routing schemes to achieve this rate?

As we know, this is a cross-layer problem, which could possibly involves control at the MAC, routing, transport layer, and is often very complex.

Instead, we are contented with "order-optimal" results.

Notations:

$$f(n) = O(g(n))$$
$$\Leftrightarrow \quad f(n) \le c\, g(n) \text{ when } n \text{ is large. } (n \ge N_0)$$

The constant c can differ substantially.

$$f(n) = \Omega(g(n)) \Leftrightarrow g(n) = O(f(n))$$

$$f(n) = \Theta(g(n)) \Leftrightarrow f(n) = O(g(n)) \ \& \ g(n) = O(f(n)).$$

Main results:

$$\lambda \le O\left(\frac{1}{\sqrt{n}}\right)$$

Note: Perhaps it is not surprising that $\lambda \to 0$ as $n \to +\infty$

Although $O\left(\frac{1}{\sqrt{n}}\right)$ is not very obvious

For example, if each node can hear every other node, then the throughput will be $O\left(\frac{1}{n}\right)$.

We can achieve $\lambda \geq \Omega\left(\frac{1}{\sqrt{n \log n}}\right)$ with high probability by:

① set the transmission range of each hop as $\Theta\left(\sqrt{\frac{\log n}{n}}\right)$

② Each source-destination pair takes on average $\Theta\left(\sqrt{\frac{n}{\log n}}\right)$ hops

③ Uses almost straight-line routing.

Note: ① Multi-hop is essential to achieve order-of-magnitude improvement in throughput.

② Since there are n nodes in a unit area, the # of neighbors in radius $O\left(\sqrt{\frac{\log n}{n}}\right)$ is $O(\log n)$.

This turns out to be the smallest possible in the sense that, if the transmission range is further reduced, some nodes may be isolated.

Although these results are asymptotics when $n \to +\infty$, they do provide important insights for the near-optimal mode of operation.

J5

Upper bound - 10min

Tuesday, February 05, 2008 3:17 PM

The upper bound is important because it reveals how key constraints in the system interact.

Proof of the upper bound ($O(\frac{1}{\sqrt{n}})$)

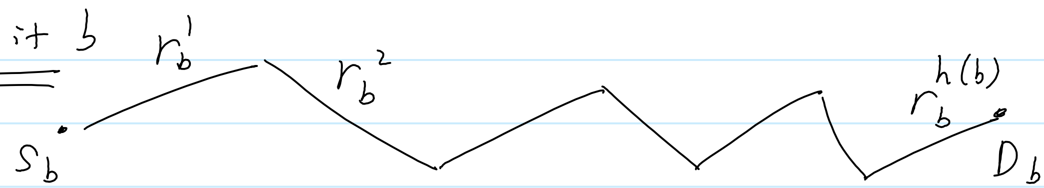
Let $\bar{L}$ denote the mean distance between source & destination.

Define the aggregate transport capacity as $\lambda n \bar{L}$ (in bits-meters-per-second).

We will first show an upper bound for $\lambda n \bar{L}$ for <u>arbitrary</u> placement of nodes.

Consider a large enough time T. There are $\lambda n T$ bits that are transmitted. Consider each bit b, $1 \le b \le \lambda n T$. Suppose that it moves from its source to its destination in a sequence of $h(b)$ hops, where the h-th hop traverses a distance of r_b^h

<u>A bit b</u>



Constraint ① (distance)

$$\sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} r_b^h \geq \lambda n T \bar{L}$$

(direct line is always the shortest.)

Next, suppose the system operates in a slotted system. The duration of a time-slot is τ. In any time-slot, only $\frac{n}{2}$ nodes can transmit. Hence, for any slot s

$$\sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} 1\{\text{The } h\text{-th hop of bit } b \text{ is scheduled on slot } s\}$$

$$\leq \frac{n}{2} W \cdot \tau$$

Summing over all time slots $s = 1, 2, \cdots \frac{T}{\tau}$, we have

Constraint ② (bandwidth)

$$\sum_{b=1}^{\lambda n T} h(b) \leq \frac{W T n}{2}$$

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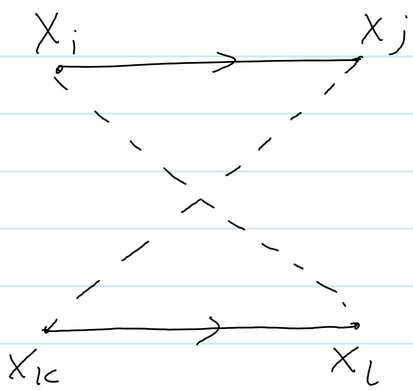
Interference - 15min

Tuesday, February 05, 2008 3:28 PM

Consider the protocol model. If X_i is transmitting to X_j, and X_k is transmitting to X_l at the same time



Then

$$|X_k - X_j| \geq (1+\Delta)\,|X_i - X_j|$$

$$|X_i - X_l| \geq (1+\Delta)\,|X_k - X_l|$$

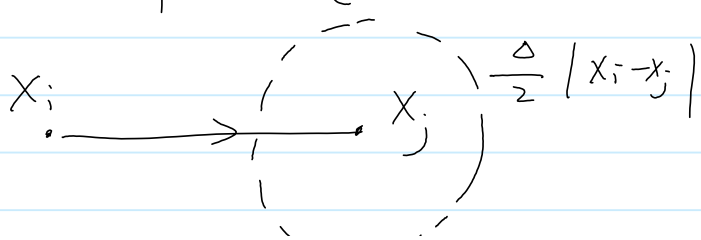
$$\Rightarrow \quad |X_j - X_l| \geq |X_k - X_j| - |X_k - X_l|$$

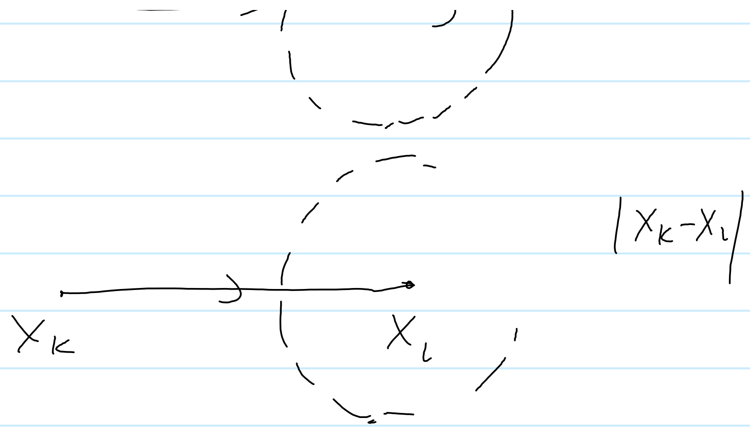
$$\geq (1+\Delta)\,|X_i - X_j| - |X_k - X_l|$$

$$|X_j - X_l| \geq |X_i - X_l| - |X_i - X_j|$$

$$\geq (1+\Delta)\,|X_k - X_l| - |X_i - X_j|$$

$$\Rightarrow \quad |X_j - X_l| \geq \frac{\Delta}{2}\left[\,|X_i - X_j| + |X_k - X_l|\,\right]$$





Hence, in each time slot, disks of radius $\Delta/2$ times the transmission range centered at the receivers are disjoint from each other.

Recall that the total area is 1. This then puts a constraint on the # (and ranges) of simultaneous transmissions we can schedule.

Assume a unit-square area. At least $\frac{1}{4}$ of each disk must lie side the area.

<u>hop h of bit b:</u>

- range r_b^h
- $\frac{1}{4}$ of disk : $\frac{1}{16}\pi\Delta^2(r_b^h)^2$

At each time-slot, each such disk can communicate $W\cdot\tau$ bits

$$\frac{1}{\quad} \sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} 1\{\text{the } h\text{-th hop of bit } b$$

$$\frac{1}{W\tau} \sum_{b=1} \sum_{h=1} 1\{\text{the } h\text{-th hop of bit } b \text{ is scheduled in slot } s\} \cdot \frac{\pi\Delta^2}{16}(r_b^h)^2$$

$$\leq 1 \quad \leftarrow \text{area of the unit-square}$$

Summing over all time slots $S = 1, \ldots, \frac{T}{\tau}$, we have

<u>Constraint ③</u> (Interference)

$$\sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} \frac{\pi\Delta^2}{16}(r_b^h)^2 \leq W \cdot T$$

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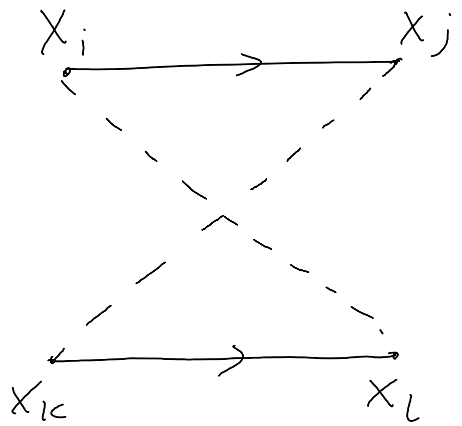
Interference - handout

Sunday, March 8, 2020 11:15 AM

Consider the protocol model. If X_i is transmitting to X_j, and X_k is transmitting to X_l at the same time



Then

$$|X_k - X_j| \geq (1+\Delta)|X_i - X_j|$$

$$|X_i - X_l| \geq (1+\Delta)|X_k - X_l|$$

$$\Rightarrow \quad |X_j - X_l| \geq |X_k - X_j| - |X_k - X_l|$$

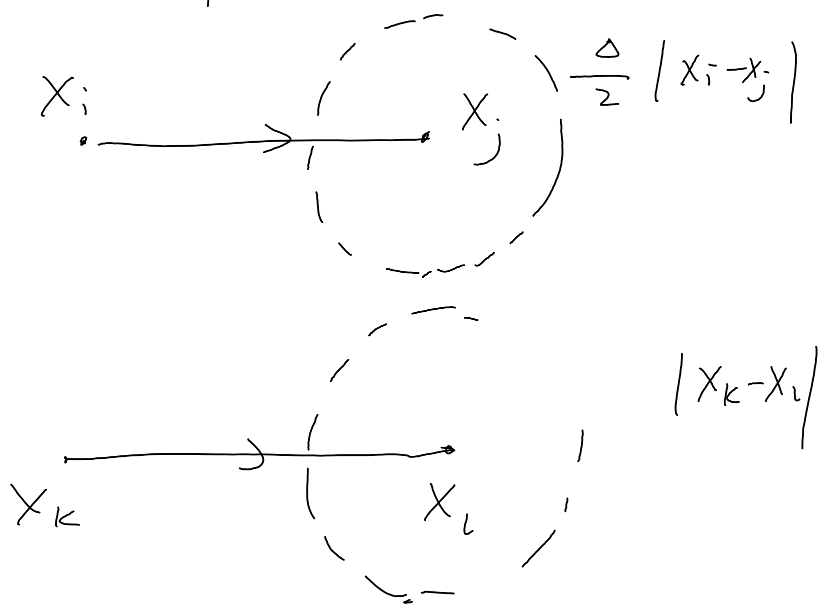
$$\geq$$

$$|X_j - X_l| \geq |X_i - X_l| - |X_i - X_j|$$

$$\geq$$

$$\Rightarrow \quad |X_j - X_l| \geq \frac{\Delta}{2}\left[\,|X_i - X_j| + |X_k - X_l|\,\right]$$

$$\Rightarrow |X_j - X_l| \geq \frac{\Delta}{2}\left(|X_i - X_j| + |X_k - X_l|\right)$$



Hence, in each time slot, disks of radius $\Delta/2$ times the transmission range centered at the receivers are disjoint from each other.

Rest of proof (upper bound) - 10min

Tuesday, February 05, 2008 3:40 PM

Three constraints

① (distance)

$$\sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} r_b^h \geq \lambda n T \bar{L}$$

② (bandwidth)

$$\sum_{b=1}^{\lambda n T} h(b) \leq \frac{W T n}{2}$$

③ (Interference)

$$\sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} \frac{\pi \Delta^2}{16} (r_b^h)^2 \leq W \cdot T$$

They are sufficient for deriving an upper bound on $\lambda n \bar{L}$

By Cauchy-Schwartz Inequality

$$\left(\sum_n a_n^2\right) \left[\sum_n b_n^2\right] \geq \left(\sum a_n b_n\right)^2$$

$$\left[\sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} (r_b^h)^2\right] \left[\sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} 1\right]$$

$$\Rightarrow \left[\sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} (r_b^h)^2 \right] \left[\sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} 1 \right]$$

$$\geq \left(\sum_{b=1}^{\lambda n T} \sum_{h=1}^{h(b)} r_b^h \right)^2 \qquad (4)$$

$$\Rightarrow \frac{16 W T}{\pi \Delta^2} \cdot \frac{W T n}{2} \geq (\lambda n T \bar{L})^2$$

$$\Rightarrow (\lambda n \bar{L}) \leq \sqrt{\frac{8}{\pi}} \frac{1}{\Delta} W \cdot \sqrt{n}$$

(bits-meters/sec)

Note that this is a deterministic bound regardless of the placements of the source-destination pairs.

For the case when the source & destination nodes are placed uniformly inside a unit area.

$$\bar{L} = O(1)$$

$$\Rightarrow \lambda \leq O\left(\sqrt{\frac{8}{\pi}} \frac{1}{\Delta} \frac{W}{\sqrt{n}} \right)$$

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