Lec20-mwf

Monday, February 11, 2008 10:56 PM

Coverage - 15min

Friday, February 9, 2018 11:10 AM

- Let us first study the coverage probability, i.e. the SINR at the mobile is greater than a threshold T.

- First, condition on r (the distance to the serving BS).

$$
\begin{aligned}
& P[\, SINR > T \mid r \,] \\
&= P\left[\frac{h r^{-n}}{I_r + \sigma^2} > T \,\middle|\, r \right] \\
&= E\left(P\left[\frac{h r^{-n}}{I_r + \sigma^2} > T \,\middle|\, I_r, r \right] \right) \\
&= E\left[P\left[h > T r^n (I_r + \sigma^2) \,\middle|\, I_r, r \right] \right] \\
&= E\left[e^{-\mu T r^n (I_r + \sigma^2)} \,\middle|\, r \right] \\
&= e^{-\mu T r^n \cdot \sigma^2} \cdot \underbrace{E\left[e^{-\mu T r^n \cdot I_r} \,\middle|\, r \right]}_{\text{the m.g.f of } I_r}
\end{aligned}
$$

- Define $L_X(s) = E[e^{-sX}]$
- This term is $L_{I_r}(\mu T r^n)$

- In other words, if we know the Laplace transform of I_r, we can express the coverage probability in closed-form.

<u>Simplifying</u> the Laplace transform

- $I_r = \sum_{i \neq b_0} g_i R_i^{-n}$

- [illegible] the location of the other BSs : Φ

— Condition on the location of the other BSs : Φ

$$E[e^{-sI_r} \mid \Phi]$$

$$= E[e^{-s\sum_{i \neq b_0} g_i R_i^{-n}} \mid \Phi]$$

$$= \prod_{i \neq b_0} E[e^{-sR_i^{-n} \cdot g_i} \mid \Phi]$$

— And then we need to take the integral over R_i

— When g_i is also exponential with mean $\frac{1}{\mu}$, the above expression can be greatly simplified.

$$E[e^{-sg}] = \int_0^{+\infty} e^{-s} \cdot e^{-\mu a} \mu \, da$$

$$= \frac{\mu}{s+\mu}.$$

$$\Rightarrow E[e^{-sI_r} \mid \Phi] = \prod_{i \neq b_0} \frac{\mu}{\mu + sR_i^{-n}}$$

Integrating over R_i's

— Each R_i is independent and is only outside r.

— $$L_{I_r}(s) = E\left[\prod_{i \neq b_0} \frac{\mu}{\mu + sR_i^{-n}}\right]$$

$$= \exp\left\{-\lambda \cdot \int_r^{+\infty} \left(1 - \frac{\mu}{\mu + sR^{-n}}\right) 2\pi R \, dR\right\}$$

$$\Rightarrow L_{I_r}(\mu T r^n) = \exp\left\{-\lambda \int_r^{+\infty} \left(1 - \frac{\mu}{\mu + \mu T\left(\frac{r}{R}\right)^n}\right) \pi \, dR^2\right\}$$

$$\Rightarrow L_{I_r}(\mu T \ldots) = \exp\left\{-\lambda \int_r \ldots \quad \mu + \mu T\left(\frac{r}{R}\right)^{\ldots} \right\}$$

$$= \exp\left\{ -\lambda \int_r^{+\infty} \frac{1}{\frac{1}{T}\left(\frac{R}{r}\right)^n + 1} \pi \, dR^2 \right\}$$

– The integral can be written as

$$\int_r^{+\infty} \frac{1}{\frac{1}{T}\left(\frac{R}{r}\right)^n + 1} \, d\left(\frac{R}{r}\right)^2 \cdot r^2$$

$$= \underbrace{\int_1^{+\infty} \frac{1}{\frac{1}{T} u^n + 1} \cdot du^2}_{\rho(T,n) \text{ a number}} \cdot r^2$$

Then $L_{I_r}(\mu T r^n) = e^{-\pi\lambda r^2 \rho(T,n)}$

– $P[\text{SINR} > T)$

$$= \int_0^{+\infty} 2\pi\lambda r e^{-\lambda\pi r^2} dr.$$

$$e^{-\mu T r^n \cdot \sigma^2} \cdot e^{-\pi\lambda r^2 \rho(T,n)}$$

$\sigma^2 = 0$: noise is negligible compared to signals.

$P(\text{SINR} > T)$

$$= \int_0^{+\infty} \pi\lambda e^{-\pi\lambda r^2 (1+\rho(T,n))} dr^2$$

$$= \frac{1}{1+\rho(T,n)}$$

Implication

Sunday, February 11, 2018 11:39 AM

- When fading is Raleigh and $\sigma^2 = 0$, the coverage probability is independent of λ
- In other words, the network can be arbitrarily dense or sparse, but the coverage probability remains the same.
- Even if $\sigma^2 \neq 0$, likely hold when $\lambda \to +\infty$.

Generalizations-10min

Sunday, February 11, 2018 11:50 AM

1. Frequency allocation
   a. Assume each BS choose each frequency band independently
   b. Compared to typical cellular channel reuse patterns, this may be a worse case.
2. Sectoring
3. Other channel distributions?
4. Shadow fading?

Average achievable rate -10min

Sunday, February 11, 2018 11:42 AM

- A similar derivation can be done for the achievable rate, assuming that

$$\text{rate} = \ln(1 + \text{SINR})$$

- $E[\text{rate} \mid I_r, r]$

$$= E\left[\ln\left(1 + \frac{h r^{-n}}{\sigma^2 + I_r}\right) \middle| I_r, r\right]$$

$$= \int_{t>0} \underbrace{P\left[\ln\left(1 + \frac{h r^{-n}}{\sigma^2 + I_r}\right) > t \middle| I_r, r\right]}\, dt$$

$$1 + \frac{h r^{-n}}{\sigma^2 + I_r} > e^t$$

$$\frac{h r^{-n}}{\sigma^2 + I_r} > e^t - 1$$

$$h > (e^t - 1)\, r^n (\sigma^2 + I_r)$$

$$\Rightarrow P(\qquad | I_r, r] = e^{-\mu (e^t - 1) r^n (\sigma^2 + I_r)}$$

- Hence,

$$E(\text{rate} \mid r)$$

$$= \int_{t>0} e^{-\mu (e^t - 1) r^n \cdot \sigma^2} \cdot \mathcal{L}_{I_r}(\mu (e^t - 1) r^n)$$

- For details, see reference paper

– When $\sigma^2 = 0$, the average rate is given by

$$\int_{\tau>0} \frac{1}{1+(e^{\tau}-1)^{2/n}\int_{(e^{t}-1)^{-2/n}}^{+\infty} \frac{1}{1+x^{n/2}}\,dx}\,dt$$

which is again independent of λ

HetNets

Tuesday, March 3, 2020 10:05 AM

- A key capacity-improving mechanism in 4G is the use of small cells (pico cells, femto cells).
- They leads to higher spatial reuse. However, because small cells are usually owned by the customers, their operation is not completely under the control of the service provider.
- The use of small cells thus leads to many new issues.

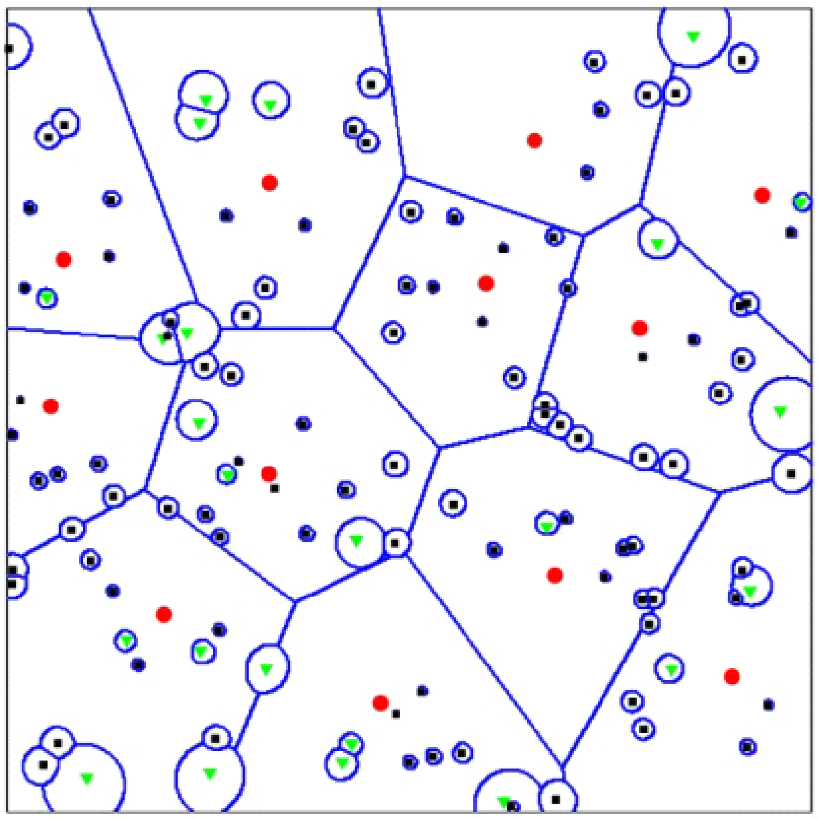


Fig. 5. Coverage regions in a three-tier network where macro BS locations (large circles) now correspond to actual 4G deployment. Other parameters are same as Fig. 4

Modeling and Analysis of K-Tier Downlink Heterogeneous Cellular Networks
Harpreet S. Dhillon, Radha Krishna Ganti, Franc¸ois Baccelli and Jeffrey G. Andrews,

1. With so many small cells overlay with macro-cell, will the performance of the macro-cell users degrade due to the interference from these small cells?
2. Small cells usually have much lower transmission power. While BS should the user be served at? The closest BS? Or the one with the strongest signal?
    a. Even if we still let users connect to the BS with the strongest power, the users served by the small cells will be very limited. Should we allow users to connect to the small-cell BS even when the signal is weaker than to other BS?

3. Should small cells be open (allowing anyone in the vicinity to use them) or closed (only allowing the owner to use them)?

It turns out the stochastic geometry method can again provide useful guidance.

K-Tier HetNet - 15min

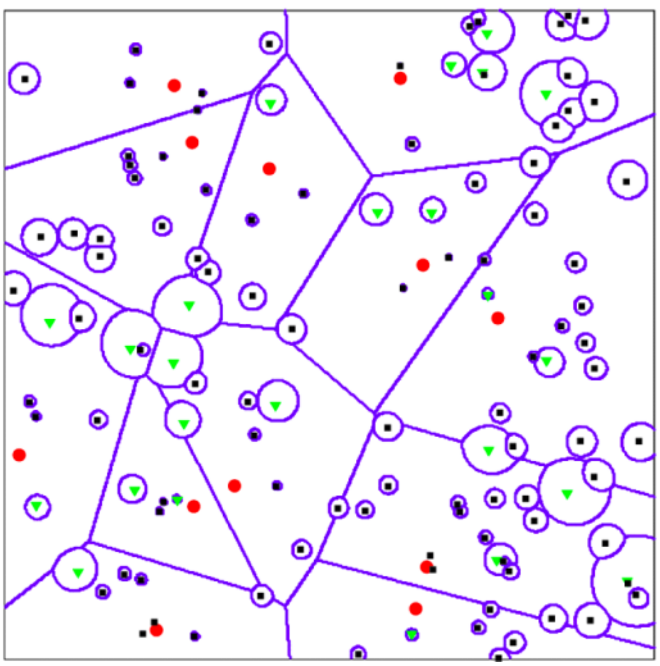
Tuesday, February 13, 2018 3:18 PM

- Now let us look at heterogeneous networks
- There are k tiers (macro-, micro-, pico-, femto-)
- Focus again on the downlink, and consider a mobile at the origin
- The BSs at tier i is distributed according to PPP with intensity λ_i
  - Each of them uses power P_i
  - λ_i, P_i vary across tiers

Determine the serving BS

- Since the transmission powers are different, the serving BS is usually not the closest BS
- May decide based on power & distance

  $P_i \cdot r^{-n}$



Modeling and Analysis of K-Tier Downlink Heterogeneous Cellular Networks
Harpreet S. Dhillon, Radha Krishna Ganti, Franc¸ois Baccelli and Jeffrey G. Andrews,

- This complicates the calculation of both the received signal and the location of the interferers.
- The above reference consider an alternate model.

  Let $\beta \geq 1$ be the SINR threshold.

  When $\beta \geq 1$, there can be at most one BS with SINR $\geq \beta$.

  Then, the mobile is served by the BS with the largest SINR.
- Note that Raleigh fading is part of the SINR calculation. (May be unrealistic when multi-path fading changes quickly.)

Coverage Probability

SINR from BS x_i

$$
\begin{aligned}
P\{\text{coverage}\} &= P\Big\{ \bigcup_{\substack{i=1,\dots K,\\ x_i \in \Phi_i}} \{ \mathrm{SINR}(x_i) \geq \beta_i \} \Big\} \\
&= \sum_{i=1}^{K} P\Big\{ \bigcup_{x_i \in \Phi} \{ \mathrm{SINR}(x_i) \geq \beta_i \} \Big\} \\
&= \sum_{i=1}^{K} E\Big[\sum_{x_i \in \Phi_i} \mathbb{1}\{ \mathrm{SINR}(x_i) \geq \beta_i \} \Big] \\
&= \sum_{i=1}^{K} \lambda_i \iint_{\mathbb{R}^2} dx_i^2 \cdot P\Big[\frac{P_i h_{x_i} \|x_i\|^{-n}}{I_{x_i} + \sigma^2} \geq \beta_i \Big] \\
&= \sum_{i=1}^{K} \lambda_i \iint_{\mathbb{R}^2} dx_i^2 \, \underbrace{P\Big[h_{x_i} \geq \frac{\beta_i (I_{x_i} + \sigma^2)}{P_i \|x_i\|^{-n}} \Big]}_{= E\left[P\left\{ h_{x_i} \geq \frac{\beta_i (I_{x_i} + \sigma^2)}{P_i \|x_i\|^{-n}} \,\middle|\, I_{x_i} \right\} \right]}
\end{aligned}
$$

$$= E\left[e^{-\mu \cdot \frac{\beta_i (I_{x_i} + \sigma^2)}{P_i \|x_i\|^{-n}}} \right]$$

$$= e^{-\frac{\mu \beta_i \sigma^2}{P_i \|x_i\|^{-n}}} \cdot \underbrace{\mathcal{L}_{I_{x_i}}\left(\frac{\mu \beta_i}{P_i \|x_i\|^{-n}}\right)}_{\substack{\text{can be calculated} \\ \text{as before since } I_{x_i} \\ \text{is the total} \\ \text{interference of} \\ \text{all BSs.}}}$$

- In particular, for any x_i, the rest of the BSs still follow a PPP with the same rate

$$\mathcal{L}_{I_{x_i}}(s) = E\left[e^{-s \sum_{j=1}^{K} \sum_{x_j \in \Phi_j} g(x_j) \|x_j\|^{-n}} \right]$$

$$= E\left[\prod_{j=1}^{K} \prod_{x_j \in \Phi_j} \frac{\mu}{\mu + s P_j \|x_j\|^{-n}} \right]$$

$$= \prod_{j=1}^{K} E\left[\prod_{x_j \in \Phi_j} \frac{\mu}{\mu + s P_j \|x_j\|^{-n}} \right]$$

$$= \prod_{j=1}^{K} \exp\left(-\lambda_j \underbrace{\iint_{R^2} \left(1 - \frac{\mu}{\mu + s P_j \|x_j\|^{-n}}\right) dx_j^2}_{\left(\frac{sP_j}{\mu}\right)^{\frac{2}{n}} \iint_{R^2} \left(1 - \frac{1}{1 + \frac{sP_j}{\mu} \|x_j\|^{-n}}\right) d\left(\frac{x_j}{\sqrt[n]{\frac{sP_j}{\mu}}}\right)^2} \right)$$

$$= \prod_{j=1}^{K} \exp\left(-\lambda_j \left(\frac{sP_j}{\mu}\right)^{2/n} \underbrace{\iint_{R^2} \left(1 - \frac{1}{1 + \|x_j\|^{-n}}\right) dx_j}_{C(n)} \right)$$

$$= \exp\left\{ -\left(\frac{s}{\mu}\right)^{2/n} \cdot C(n) \sum_{j=1}^{K} \lambda_j P_j^{2/n} \right\}$$

— since $s = \frac{\mu \beta_i}{P_i \|x_i\|^{-n}}$

$$\mathcal{L}_{I_{x_i}}\left(\frac{\mu \beta_i}{P_i \|x_i\|^{-n}}\right)$$

$$= \exp\left\{ -\left(\frac{\beta_i}{P_i \|x_i\|^{-n}}\right)^{2/n} C(n) \sum_{j=1}^{K} \lambda_i P_j^{2/n} \right\}$$

— Go back to the probability of coverage

$P\{\text{coverage}\}$

$$= \sum_{i=1}^{K} \lambda_i \iint_{R^2} dx_i^2 \; e^{\frac{-\mu \beta_i \sigma^2}{P_i \|x_i\|^{-n}}} \cdot$$

$$\exp\left\{ -\left(\frac{\beta_i}{P_i \|x_i\|^{-n}}\right)^{2/n} C(n) \sum_{j=1}^{K} \lambda_i P_j^{2/n} \right\}.$$

$\sigma^2 = 0$

— If $\sigma^2 = 0$

$P\{\text{coverage}\}$

$$= \sum_{i=1}^{K} \lambda_i \iint_{R^2} dx_i^2 \exp\left\{ -\left(\frac{\beta_i}{P_i}\right)^{2/n} \cdot \|x_i\|^2 \; C(n) \cdot \sum_{j=1}^{K} \lambda_j P_j^{2/n} \right\}$$

— Note that

$$\iint_{R^2} dx^2 \, e^{-s\|x\|^2}$$

$$= \int_0^{+\infty} 2\pi R \, dR \, e^{-sR^2}$$

$$= \pi \int_0^{+\infty} e^{-sR^2} \cdot dR^2$$

$$= \frac{\pi}{s}$$

S

- Hence

P{coverage}

$$= \sum_{i=1}^{K} \lambda_i \cdot \lambda \cdot \frac{1}{\left(\frac{\beta_i}{P_i}\right)^{2/n} C(n) \cdot \sum_{j=1}^{K} \lambda_j P_j^{2/n}}$$

$$= \frac{\lambda}{C(n)} \frac{\sum_{i=1}^{K} \lambda_i P_i^{2/n} \beta_i^{-2/n}}{\sum_{j=1}^{K} \lambda_j P_j^{2/n}}$$

Implication:

- If $\beta_i = \beta$, the coverage probability simplifies to

$$\frac{\lambda}{C(n)} \frac{1}{\beta^{2/n}}$$

- This is independent of the density and power of all tiers! Why?

Extensions

(1) Closed access: the mobile can only access a subset B of Tiers. When $\sigma^2 = 0$

$$P\{\text{coverage}\} = \frac{\lambda}{C(n)} \cdot \frac{\sum_{i \in B} \lambda_i P_i^{2/n} \beta_i^{-2/n}}{\sum_{j=1}^{K} \lambda_j P_j^{2/n}}$$

- closed access typically reduce coverage

—

(2) Load: when $\sigma^2 = 0$, the fraction N_j of mobiles served by tier j is (open access)

$$\overline{N_j} = \frac{P\left\{\bigcup_{X_j \in \phi_j} SINR(X_j) \geq \beta_j\right\}}{P\left\{\bigcup_{i=1}^{K} \bigcup_{X_i \in \phi_i} SINR(X_i) \geq \beta_i\right\}}$$

$$= \frac{\lambda_j P_j^{2/n} \beta_j^{-2/n}}{\sum_{i=1}^{K} \lambda_i P_i^{2/n} \beta_i^{-2/n}}$$

- Load per tier-j BS scales as $P_j^{2/n} \beta_j^{-2/n}$
- At the same β_j, smaller power tends to have smaller load.
- May decrease β_j to attract more mobiles to small cells

$\rightarrow$ cell biasing or range extension.

K-Tier HetNet - handout

Tuesday, February 13, 2018 3:18 PM

Coverage Probability

SINR from BS x_i

$$P\{\text{coverage}\}$$

$$= P\Big\{ \bigcup_{\substack{i=1,\dots K,\\ x_i \in \Phi_i}} \{ SINR(x_i) \geq \beta_i \} \Big\}$$

$$=$$

$$= \sum_{i=1}^{K} E\Big[\sum_{x_i \in \Phi_i} \mathbb{1}\{ SINR(x_i) \geq \beta_i \} \Big]$$

$$= \sum_{i=1}^{K} \lambda_i \iint_{R^2} d^2 x_i \cdot P\left[\frac{P_i h_{x_i} \|x_i\|^{-n}}{I_{x_i} + \sigma^2} \geq \beta_i \right]$$

$$=$$

$$= e^{-\frac{\mu \beta_i \sigma^2}{P_i \|x_i\|^{-n}}} \cdot \underbrace{\mathcal{L}_{I_{x_i}}\left(\frac{\mu \beta_i}{P_i \|x_i\|^{-n}} \right)}$$

can be calculated as before since I_{x_i} is the total interference of all BSs.

- In particular, for any x_i, the rest of the BSs still follow a PPP with the same rate

$$\mathcal{L}_{I_{x_i}}(s) = \mathbb{E}\left[e^{-s \sum_{j=1}^{K} \sum_{x_j \in \Phi_j} g(x_j) \|x_j\|^{-n}} \right]$$

$$=$$

$$= \prod_{j=1}^{K} \mathbb{E}\left[\prod_{x_j \in \Phi_j} \frac{\mu}{\mu + s P_j \|x_j\|^{-n}} \right]$$

$$=$$

$$= \prod_{j=1}^{K} \exp\left(-\lambda_j \left(\frac{sP_j}{\mu}\right)^{2/n} \underbrace{\iint_{\mathbb{R}^2} \left(1 - \frac{1}{1+\|x_j\|^{-n}}\right) dx_j^2}_{C(n)} \right)$$

$$= \exp\left\{ -\left(\frac{s}{\mu}\right)^{2/n} \cdot C(n) \sum_{j=1}^{K} \lambda_j P_j^{2/n} \right\}$$

- since $s = \frac{\mu \beta_i}{P_i \|x_i\|^{-n}}$

$$\mathcal{L}_{I_{x_i}}\left(\frac{\mu \beta_i}{P_i \|x_i\|^{-n}}\right)$$

$$=$$

— Go back to the probability of coverage

$P\{\text{coverage}\}$

$$= \sum_{i=1}^{K} \lambda_i \iint_{R^2} dx_i^2 \; e^{\frac{-\mu \beta_i \sigma^2}{P_i \|x_i\|^{-n}}} \cdot$$

$$\exp\left\{ -\left(\frac{\beta_i}{P_i \|x_i\|^{-n}}\right)^{2/n} C(n) \sum_{j=1}^{K} \lambda_j P_j^{2/n} \right\}.$$

$\underline{\underline{\sigma^2 = 0}}$

— If $\sigma^2 = 0$

$P\{\text{coverage}\}$

$$= \sum_{i=1}^{K} \lambda_i \iint_{R^2} dx_i^2 \exp\left\{ -\left(\frac{\beta_i}{P_i}\right)^{2/n} \cdot \|x_i\|^2 \, C(n) \cdot \sum_{j=1}^{K} \lambda_j P_j^{2/n} \right\}$$