

Lec11

Saturday, January 27, 2018 10:15 AM

ECE 547 Lectures: on queueing analysis

<https://engineering.purdue.edu/~ee547/lectures/index.html>

Lectures 13-15

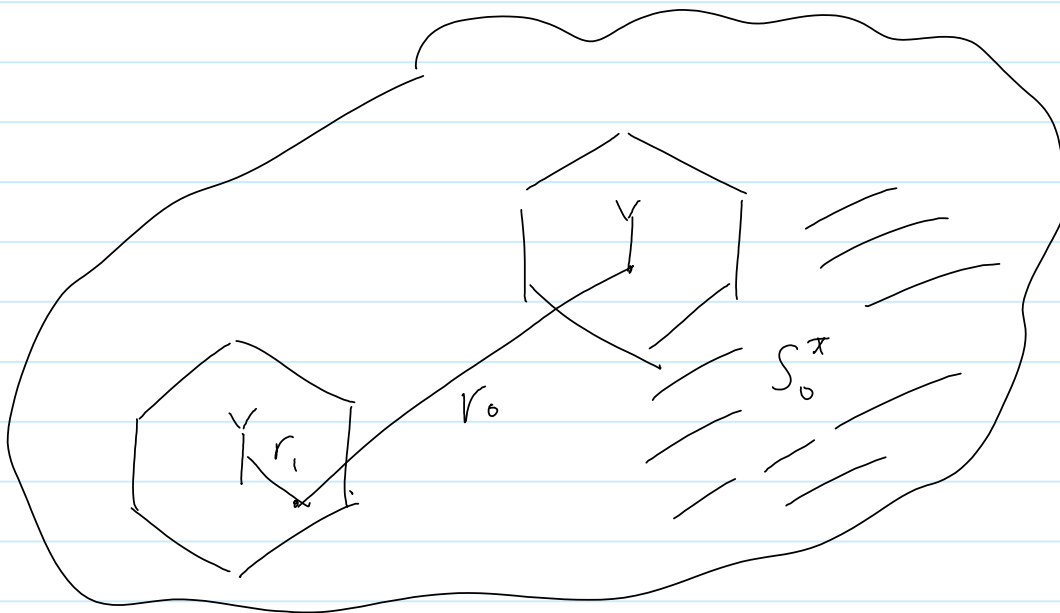
Ask instructor for password.

Effect of location term -5 min

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To derive the location term

$$\iint_{S_0^*} \left(\frac{r_1}{r_0} \right)^n dA,$$



we can integrate numerically. (see Schwartz p152)
p156)

For $n=4$, we have

$$\frac{2}{3\sqrt{3}R^2} \iint_{S_0^*} \left(\frac{r_1}{r_0} \right)^n dA = 0.44$$

(40)

CDMA capacity - 15min

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The shadow fading term is typically quite large

Example:

$$\sigma = 8 \text{ dB} \Rightarrow \sigma^2 = 64$$

$$\text{Let } b^2 = \frac{1}{2}$$

- half of the shadow fading is contributed by the independent terms

$$\begin{aligned} \Rightarrow E \left[10^{(z_1 - z_0)/10} \right] &= e^{\frac{1}{2} \cdot 2 \cdot \frac{1}{2} \cdot 64 \cdot \left(\frac{\ln 10}{10} \right)^2} \\ &= 5.42 \end{aligned}$$

$$\Rightarrow I_{S_0}^* = P_R \cdot K \cdot 0.44 \times 5.42 = 2.38 P_R \cdot K$$

We can thus derive the CDMA Capacity

$$\frac{E_b}{I_0} = \frac{P_R}{\underbrace{(P_R (K-1))}_{\substack{\uparrow \\ \text{interference} \\ \text{from the}}} + \underbrace{2.38 P_R \cdot K}_{\substack{\uparrow \\ \text{interference} \\ \dots}}) / W}$$

interference
from the
same cell

interference
from other
cells

$$= \frac{W}{3.38K - 1}$$

The effect of outside cell interference is quite strong!

$$\Rightarrow K = \frac{\frac{W}{E_b/I_0} + 1}{3.38}$$

For IS-95,

$$- W = 12.8$$

$$- E_b/I_0 = 5$$

$$\Rightarrow K = 7.8$$

For 25MHz band, we have

$$7.8 \times \frac{25\text{MHz}}{1.25\text{MHz}} = 156 \text{ users/cell}$$

compared with 248 users/cell in GSM.

- GSM capacity has not taken into account fading.

Note: I_{SS}^* is the expected amount of interference from neighboring cells

- averaged over node locations & shadow fading
- the actual amount of interference can vary up or down

$K=4$
is not
very large

- However, when K is large, a statistical multiplexing effect occurs, such that the instantaneous amount of interference will not be substantially different from the mean.
- In GSM systems, interference comes from a small number of co-channel cells,
 - the variations from the mean tend to be larger.

(JF)

Other capacity increasing features - 15min

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We have so far ignored a number of capacity increasing factors of CDMA

① Soft handoff

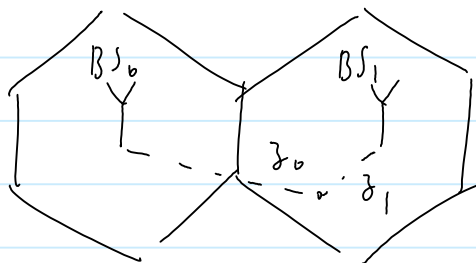
In the above analysis with hard handoff, shadow fading contributes to significant increase in the amount of interference

$$\text{When } \sigma = 0 \text{ dB, } E[10^{(\gamma_1 - \gamma_0)/10}] = 1$$

$$\text{When } \sigma = 8 \text{ dB, } E[10^{(\gamma_1 - \gamma_0)/10}] = 5.42$$

② Why does shadow fading tend to increase interference?

④



Interference will increase when the fading coefficient γ_0 to BS₀ is strong,

and the fading coefficient δ_1 to BS_1 is small.

- Mobile uses a large power to communicate with BS_1 , and creates large interference at BS_0 .

Instead, the mobile may very well communicate with BS_0 rather than BS_1 .

Soft-handoff: allows a mobile to measure signal strength from two or more BS, and pick the BS with the strongest signal to communicate.

Two-cell hand-off reduces the outside-cell interference from $2.38 P_R K$ to $0.77 P_R K$

- Increase capacity from $\frac{1}{3.38}$ to $\frac{1}{1.77}$, a factor of 1.90.

- In practice, due to measurement errors a factor of 1.25 is typically achieved.

② Silence detection.

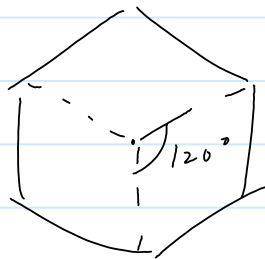
In normal speech, users tend to alternate between silence & talk spur.

On average, speech is active about 40% of the time.

⇒ Reduces both in-cell & outside-cell interference to 40%

⇒ Increase capacity by a factor of 2.5.

③ 120° sectional antennas



— Reduces both in-cell & outside-cell interference to $1/3$

— Increase capacity by a factor of 3.

In reality, power control, silence detection or sectional antennas are not perfect,

After accounting for potential losses, the # of users per cell for IS-95 in each 1.25 MHz band is found to be 84 if two-cell handoff is used, or 96 if three-cell handoff is used.

(Again: these numbers are obtained with $\sigma = 8 \text{ dB}$, $b^2 = 1/2$, $n = 4$, $E_b/I_0 = 7 \text{ dB}$)

For 25 MHz band, the # of users per cell is $84 \times 20 = 1680$ compared with 248 users/cell in GSM.

(70)

Summary

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Main capacity improvement is not due to CDMA itself, but rather due to

- silence detection
- sectional antennas
- soft handoffs

CDMA allows us to deal with the above features easily:

- capacity limited by interference
 - exploits statistical multiplexing
-

Q) Can we use similar capacity-improvement features in GSM systems?

A) Not always easy.

- sectional antennas: yes
 - power control: yes
 - silence detection: putting 3 calls into 2 channels?
 - soft handoff?
-

CDMA is a perfect example of physical-layer advances integrating with network-layer advances to attain significant throughput. When I am studying this subject, I cannot stop wondering how the researchers realize the potential of CDMA, esp. when the physical-layer attribute itself does not directly contribute to the capacity increase.

Perhaps they notice first the inefficiencies of GSM, and then looking for ways to overcome such

inefficiency, and find CDMA?

The reuse factor C immediately determines the # of available channels in each cell.

The size of the cell should then depend on the traffic intensity in the region.

Performance measures of interest are:
call blocking probability, delay, etc

Simple One-cell Model without Handoff

Assumptions:

- ① Calls arrive to a cell according to a Poisson process with rate λ .
— which in turn depends on the traffic density per area and the size of cell.

— For very small interval Δt ,
prob. of one arrival is $\lambda \Delta t$.
prob. of two or more arrivals is $o(\Delta t)$.

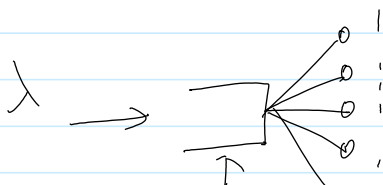
- ② Call holding time is exponentially distributed with mean $1/\mu$

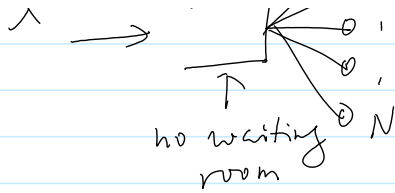
— $\mu \Delta t$.
— $P\{\tau \geq t\} = e^{-\mu t}$

- ③ N channels in a cell. A call is immediately dropped if all N channels are in use.

- ④ No handoff. Calls complete in the same cell.

\Rightarrow Simple $M/M/N/N$ N -server queue with no waiting room.



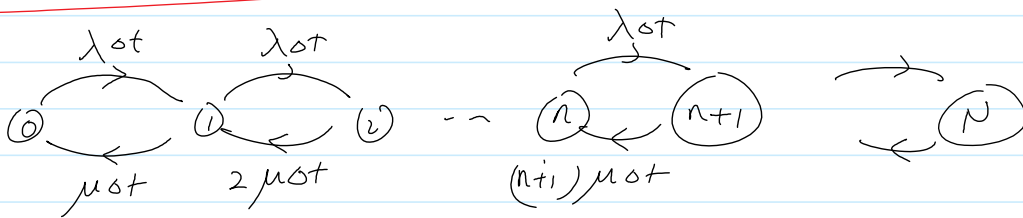


Traffic intensity (or offered load)

$$\rho = \frac{\lambda}{\mu} \quad (\text{in Erlangs})$$

Probability of blocking is given by the Erlang-B formula

$$P_B = \frac{\frac{\rho^N}{N!}}{\sum_{n=0}^N \frac{\rho^n}{n!}}$$



$$\begin{aligned} P\{n \rightarrow n+1\} &\approx P_n \cdot \lambda \quad \backslash \\ P\{n+1 \rightarrow n\} &\approx P_{n+1} \cdot (n+1)\mu \quad / \end{aligned} \quad \text{must be equal}$$

$$\Rightarrow P_n \cdot \lambda = P_{n+1} (n+1)\mu \quad (\text{local balance eqn.})$$

$$\Rightarrow P_{n+1} = \frac{\rho}{n+1} P_n$$

$$P_n = \frac{\rho}{n} P_{n-1} = \frac{\rho^2}{n \cdot (n-1)} P_{n-2} = \dots = \frac{\rho^n}{n!} P_0$$

$$\Rightarrow 1 = \sum_{n=0}^N P_n = P_0 \left(\sum_{n=0}^N \frac{\rho^n}{n!} \right) \quad P_0 = \frac{1}{\sum_{n=0}^N \frac{\rho^n}{n!}}$$

$$\Rightarrow \text{Loss prob. } P_N = \frac{\rho^N}{N!} \cdot P_0 \rightarrow \text{Erlang-B formula}$$

Example 1:

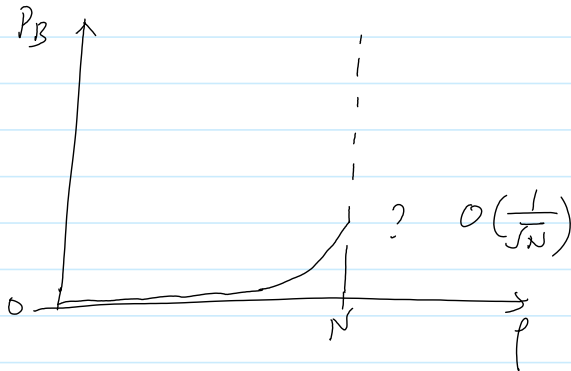
$N = 100$ channels, $\rho = 84$ Erlangs

$$\Rightarrow P_B = 1\%$$

$$N = 100$$

$$\rho = 95 \text{ Erlangs}$$

$$\Rightarrow P_B = 5\%$$



Example 2:

A call lasts 200 seconds on average.

A user makes a call every 15 minutes, on average.

$N = 100$, desired $P_B = 1\%$

How many users can the cell accommodate?

①

$$\frac{1}{\mu} = 200$$

$$\lambda = \frac{n}{15 \times 60}$$

$$\rho = \frac{\lambda}{\mu} = \frac{2}{9} n$$

$$\text{At } N=100, P_B=1\% \Rightarrow \rho = 84$$

$$\Rightarrow n = 378 \text{ users.}$$

If user density is 2 terminals per km^2
the cell can cover area

$$\frac{378}{2} = 189 \text{ km}^2$$

$$\Rightarrow \text{radius} = 7.75 \text{ km}$$

If user density is 1000 terminals per km^2
the cell can cover area

$$\frac{378}{1000} = 0.378 \text{ km}^2$$

$$\Rightarrow \text{radius} = 0.35 \text{ km.}$$