11.1 Review

Similar to the NBTI physics from our previous study, HCI mechanism can be successfully explained using the reaction diffusion (RD) framework. Through the last two lectures, we have quantified the basic time dependency of HCI degradation and derived a time exponent of 0.5 (1/3 for H₂ model) under constant DC stress as follows,

\[
N_{IT} = \left( \frac{\pi k_f N_D}{3 k_e} \right)^{1/2} (D_{Ht} t)^{1/2} \quad \text{(for H model)}
\]

\[
= \left( \frac{\pi k_f N_D}{6 k_e} \right)^{2/3} (D_{Ht} t)^{1/3} \quad \text{(for H₂ model)}.
\]

In this relationship, we have noted that the forward reaction coefficient \( k_f \) is the key parameter of HCI physics which affects the temperature and field dependency of the degradation (\( D_{Ht} \) and \( D_{Ht} \) for neutral species have no field dependences and \( k_e \) is assumed to be field-independent to the first order). In this lecture, we will quantify the field dependency of \( k_f \) and further derive the lifetime (\( \tau \)) of transistor under different voltage stress. Since \( \tau \) is defined as time required to reach a certain level of degradation, \( N_{IT}^{crit} \), therefore Eq. (1a) requires that

\[
k_f^m \tau^n \propto N_{IT}^{crit} \quad \text{so that} \quad \tau(V_D) = \text{const} \left[ \frac{1}{k_f(V_D)} \right]^{-m/n} \quad \text{(1b)}
\]

Eq. 1(b) suggests that the voltage-dependence of \( \tau \) is encapsulated in voltage-dependence of \( k_f \), therefore, we now discuss the dependence of \( k_f \) on voltage (or electric field) in the channel region.

11.2 Field dependent \( k_f \)

We assume that the field dependency of \( k_f \) parameter linearly depends on the gate tunneling current as follows,

\[
k_f = C \times I_G^{(e)}, I_G^{(h)},
\]

where \( I_G^{(e)} \) and \( I_G^{(h)} \) denote electron and hole tunneling current due to impact ionization in the channel (near the drain). \( C \) is a constant scale factor.

Fig. 1 illustrates energy-band diagram during HCI degradation. The electrons in the pinch-off region (x=0~l) experiences high lateral field which induces impact ionization. Impact ionization generates \( I_G^{(e)} \) and \( I_G^{(h)} \) tunneling currents which break Si-H bonds near this region. Therefore, to
compute the gate current, we need to know the number of electrons that can surmount the Si/SiO₂ potential barrier at various locations in the channel. The calculation proceeds as follows:

The number of electrons in the channel is proportional to the drain-current, \( I_D \), given by

\[
I_D = \frac{C_{ox}}{L_{ch}}(V_G - V_{TH})^\alpha
\]

where \( \alpha \approx 1-2 \). Remember that for HCI degradation, we are only interested in voltage regime of \( V_D \geq V_G \), i.e., the saturation regime, therefore we need not write the corresponding expression for \( I_D \) for \( V_G \geq V_D \).

In order to determine the temperature of these electrons along the channel, we must first know the field accelerating the electrons along the channel by solving the Poisson equation. Although, potential in MOSFET is inherently 2D, we may remember from EE606 that in the linear regime, gradual channel approximation is appropriate and the 1D potential along the channel is easily determined by requiring that the current be continuous. The potential is given by

\[
\frac{\alpha}{L_{ch}}(V_G - V_{TH})V - \frac{V^2}{2} = \frac{V_D^2}{2}
\]

In the current-saturation regime, (i.e. \( V_D > V_G \)), however, the potential profile in the channel can no longer be described by a 1D Poisson equation. Lateral field in the pinch-off region can be derived from 2D solution of Poisson equation. Lateral field in the pinch-off region can be derived from 2D solution of Poisson equation (covered in ECE612 class) as follows,

\[
V(x) = V_{Dsat} + U_g e^{\frac{x}{l}}, \tag{3}
\]

where \( V_{Dsat} \) is the voltage at the pinch-off point (Fig. 1). \( l \) is the length from pinch-off point to the drain region. The corresponding electric field, obtained by taking derivative of (3) can now be used to calculate the electron temperature along the channel.

Some of you may remember from ECE656 that accurate estimation of the electron temperature (and thereby \( I_G \)’s) can only be obtained using Monte-Carlo (MC) simulation based on solution of the Boltzmann equation. However, approximate solution based on solution of moments of the Boltzmann equation (i.e. hydrodynamic model) can be remarkably instructive. In the following context, we will explain the derivation steps of \( I_G \) using Hydrodynamics equations.

\( I_G \) sources from thermonic emission of heated channel carriers along the channel. Hence, first, localized temperature of electron carriers are obtained using energy conservation equation. Total energy \( W \) of electrons in the pinched-off region is expressed as follows,

\[
W = \frac{3}{2}nk_BT_e + \frac{1}{2}nm^2v^2 \approx \frac{3}{2}nk_BT_e \tag{4}
\]
where $n$ is the total number of electrons and $T_e$ is the local temperature of electrons. In addition, temporal energy equation can be written as follows,

$$\frac{\partial W}{\partial t} = -\frac{d}{dx} \left[ (W + nk_BT_e) \nu - \kappa \frac{dT_e}{dx} \right] + J_e \mathcal{E} - \left( \frac{W - W^0}{\tau_e} \right). \tag{5}$$

Since we are concerned in the steady-state behaviors, the time dependent left-hand-side (LHS) term can be approximated to 0. First term in the square bracket represents electron heating due to energy diffusion. Second term in the square bracket is the heat conduction expression, and can be neglected for our analysis. Third term in the right-hand-side (RHS) is the energy input (generation) due to conduction drift current ($J_d$). Last term in the RHS is the energy relaxation term, which represents the rate at which electrons lose their energy. Here, $W^0$ is negligibly small compared to $W$. By applying all of aforementioned simplifications, inserting Eq. (4) for $W$, we can simplify Eq. (5) into,

$$-\frac{d}{dx} \left[ \frac{5}{2} nk_BT_e \nu \right] + qn \nu \mathcal{E} - \frac{3}{2} n \frac{\kappa T_e}{\tau_e} = 0. \tag{6}$$

The electric field in the pinch-off region is so high that $\nu = \nu_{sat} = (h\omega_0 / m^*)^{1/2}$ is a position independent variable ($\nu_{sat}$ is the saturation velocity of electrons and $h\omega_0$ is the optical phonon frequency). Also, since $\nu = \text{const}$, therefore $n = n_0 = \text{constant by current continuity}$. This allows one to take $n\nu$ product outside the derivative in Eq. (5). After simplification, Eq. (6) is a first-order differential equation for $T_e$, and can be solved by integrating factor as,

$$T_e = \frac{2q}{5k_B} \int_0^x \mathcal{E}(x) e^{\frac{-3(x-x')}{5\tau_e \nu \omega}} dx', \tag{7}$$
Eq. (7) conceptually shows how electron gains temperature. Electron gains more energy as they experience more electric field (integration of $\varepsilon_e$). This energy gain is restricted by the mean-free path term of electron (denominator of exponential term $1/\tau_{evsat}$).

Having derived the local electron temperature $T_e$ in Eq. (7), we can now write the thermionic emission current $I_G$ by integrating over all positions along the channel as follows,

$$I_G = \int_0^\prime qn_0 \left( \frac{k_BT_e(x)}{2\pi m^*} \right)^{1/2} e^{\frac{-\Phi_e}{e^xT_e(x)}} dx. \quad (8)$$

Here, $\Phi_e$ is the barrier height which is approximately 3.1eV in silicon. Using Eqs. (7) and (8), one can now derive (numerically) an exact temperature dependency of $I_G$ and hence the $k_f$ (Eq. (1)). Further, from Equation 1(b), device lifetime $\tau$ is inverse proportional to $k_f$. Hence, device lifetime can be also numerically computed using this framework.

### 11.3 Simplified expression for device lifetime $\tau$

Although Eqs. (7) and (8) provide an reasonable estimate of device lifetime, one can further simplify them to derive an intuitive expression. Considering the fact that impact ionization occurs in a localized region within the channel, we can assume that the electron temperature does not vary with position and can apply an average temperature. Under this assumption the first term in Eq. (6) can be removed. This results in a simplified relationship of

$$k_BT_e = \frac{2}{3} (\tau_{evsat})e^x, q = \lambda_e e^x. \quad (9)$$

Also $qn_0$ term in Eq. (8) can be replaced with $I_D/V_{sat}$ (i.e., current continuity). Applying these simplifications, Eq. (8) can be rewritten as,

$$I_G = \frac{I_D}{V_{sat}} \int_0^\prime \left( \frac{k_BT_e}{2\pi m^*} \right)^{1/2} e^{\frac{-\Phi_e}{e^xT_e}} dx. \quad (10)$$

Integration term in Eq. (10) can be further simplified through ‘saddle point integration’ by considering the fact that tunneling current is mostly generated near the peak electrical field region (www.planetmath/encyclopedia/SaddlePointApproximation.html). By applying this fact, we have,

$$I_G \approx \frac{I_D}{V_{sat}} \left[ \frac{q\varepsilon_m\lambda_e}{m^*} U_{Dq} \frac{\lambda_e}{\Phi_e} \right] e^{\frac{-\Phi_e}{e^xT_e}} \quad (11a)$$

$$\approx \frac{I_D}{V_{sat}} e^{\beta(V-D_{sat})} \approx \frac{I_D}{V_{sat}} e^{\beta(V_D)}, \quad (11b)$$

where $E_{sat}=(V_D-V_{D,sat})/l$ by taking derivative of (3) and realizing that maximum field occurs at the drain junction with $V(x)=V_D$. Eq. (11a) is also known as Shockley’s lucky electron model for gate current – because the equation can be interpreted as follows: Only those electrons lucky enough to gain energy of $\Phi_e$ without losing energy to various energy-relaxation mechanisms (e.g.,
phonons, impact ionizations, etc.) can contribute to gate current. Since the average energy gain by the electrons, accelerated by an electric field of $E_m$ for a distance of $\lambda_e$, is $\sim q\lambda_e E_m$, therefore Boltzmann distribution requires that the number of lucky electrons having energy higher that $\Phi_e$ be exponentially reduced by the factor $\Phi_e / q\lambda_e E_m$, as is the case in Eq. 11a. To put it in another way, one could also say that ‘lucky electrons’ are those that do not scatter up to a distance of $d=\Phi_e / q\lambda_e E_m$, allowing it to gain more energy beyond the mean-free-path $\lambda_e$, however the number of such electrons reduce exponentially with distance $\sim \exp(-d/\lambda_e)$. The ‘Lucky electron model’ (LEM) is often used as a starting point of HCI calculations, here we derive the LEM from Hydrodyanmic equations to illustrate the approximations and limitations of the model.

Finally, Eq. (11) can be used to derive voltage dependency of device lifetime as follows,

$$\tau \sim \frac{1}{k_f} = \frac{A}{I_G} = \frac{A}{\left(\frac{I_D}{V_{sat}}\right)} e^{-B/V_D} \Rightarrow \tau I_D \sim e^{B/V_D}. \quad (12)$$

Eq. (12) provides a direct relationship of device lifetime $\tau$ with applied voltage stress $V_D$ and can be useful in projecting the actual device lifetime from the high voltage test data.

### 11.4 $I_{sub}$-measurement based lifetime projection

In reality, measuring $I_G$ and determining exponential term in Eq. (12) can be difficult because we do not know $\lambda_m$ and $E_m$ accurately. As a result, there exists an alternative method of projecting device lifetime by a substrate current $I_{sub}$ measurement. $I_{sub}$ can be written as follows,

$$I_{sub} = \int_0^1 I_D \cdot \alpha_{II} (x) \cdot dx, \quad \alpha_{II} (x) = \alpha_0 e^{\Phi_e T_e (x)} \approx \alpha_0 e^{q\epsilon_e \lambda_e}$$

$$= I_D e^{\Phi_e \lambda_e} \Rightarrow \log \left(\frac{I_{sub}}{I_D}\right) = \frac{\Phi_e}{q\epsilon_e \lambda_e}, \quad (13)$$

where we applied a same simplification as used in Eq. (11) for the integration. The form of $\alpha_{II}$ may appear unfamiliar, but note that if we define $A^* = \Phi_e / q\lambda_e$ to be a constant, then we get the familiar form, $\alpha_{II} = \alpha_0 e^{-A^* / \lambda_e}$. (In practice, Ridley, Hess, and Baraff have shown full MC simulation gives $\alpha_{II} = \alpha_0 e^{-A^* / \lambda_e}$ where $n\sim1-2$).

Now Eq. (12) can be simplified as,

$$\log \left(\frac{I_G}{I_D}\right) = \frac{\Phi_e}{q\epsilon_e \lambda_e}. \quad (14)$$

By merging Eqs. (13) and (14) we have

$$I_G = \left(\frac{I_{sub}}{I_D}\right)^{\Phi_e / \Phi_e} \cdot I_D \quad (15)$$

Eq. (15) established a simple relationship between a measurable parameter $I_D$ and $I_{sub}$ with $I_G$. Eq. (15) can be used with Eq. (12) to predict lifetime. This has been extensively verified by measurement and appears to be efficient estimator of device lifetime under HCI degradation.