23.0 Introduction

Time dependent dielectric breakdown (TDDB) results from the creation of charge traps inside the oxide. The charge traps are created by the 'hot'-carriers tunneling across the oxide. The details of the physical processes, by which the breakdown takes place, have been discussed in the previous two lectures. In this lecture we will discuss the statistics pertaining to that breakdown phenomenon. First we will review some experimental observations related to the statistical nature of TDDB. Then we will discuss two theoretical models that rationalize these experimental observations and give insight into the statistical aspect of the problem.

23.1 Review

As discussed in the previous two lectures the tunneling current through the oxide under certain conditions create oxide charge traps (OTs) due to the breaking of Si-O bonds. One can imagine that when an OT is created it affects the total current $I_G$ through the oxide as the electrons can now hop through the OTs in addition to directly tunneling through the oxide. With the passage of time $t$, density of OT ($N_{OT}$) increases to a critical value when a direct short-circuit path through the oxide is established and a breakdown is said to have taken place. At that instant the current through the oxide rises sharply. This characteristic time is called the breakdown time $T_{BD}$.

Figure 1: Left: Top view of an ensemble of oxides. The elements of the ensemble are denoted by 1*, 2*, ..., N*. Right: Schematic temporal variation of current for different breakdown phenomena (belonging to the same or different oxide sample). The numbering 1, 2, ..., M are assigned according the ascending order of the breakdown times $T_{BD,1}$, $T_{BD,2}$, ..., $T_{BD,M}$.

characteristic time is not exactly the same for each element in an ensemble of oxide samples and has statistical attributes. The statistical attributes are revealed when one performs stress tests on the ensemble whereby a fixed
voltage $V_g$ is applied across the oxides and different breakdown occur at different times $T_{BD,1}, T_{BD,2}, \ldots$ and leads to sudden increases in $I_g$ (Fig. 1). One can carry out an extensive experiment on such an ensemble and can estimate a cumulative distribution $F$ of the breakdown time. Such experiments have consistently yielded a Weibull distribution (Fig. 2). As per the observation of Jack Lee et. al. (1988) the distribution shows two characteristics: The distribution scales with the area (Fig. 2) and it is independent of stress voltage $V_g$. If $V_g$ is reduced/increased, the whole curve shifts to the right/left but slope remains the same.

### 23.2 Models of Weibull Distribution

We will now look at two models that address the issue of statistical distribution of the breakdown time.

#### 23.2.1 Model 1: Defect Model (Intrinsic defect model):

This model attributes the statistical distribution of $T_{BD}$ to the defects that are already present in the substrate before the oxide is grown on it. These defects give rise to a variation $\Delta t_{ox}$ in the oxide thickness $t_{ox}$ (Fig. 3). The breakdown time can be written as:

$$T_{BD} = \tau_0 e^{\beta t_{ox}} = \tau_0 e^{B(t_{ox} - \Delta t_{ox})/\sqrt[3]{V_g}}$$

(1)

where, $E_{ox}$ is the electric field due to stress voltage and $\tau_0, B$ are some constants. This gives us:

$$\frac{T_{BD}}{\tau_0} \approx C e^{-B\Delta t_{ox}/\sqrt[3]{V_g}}$$

(2)
where, \( C = e^{B_{t_{ox}}/N_G} \). Now, it is assumed that no. of defects per unit area causing a thickness reduction of \( \Delta t_{ox} \) is given by:

\[
D = a_i e^{-b_i \Delta t_{ox}}
\]

(3)

where, \( a_i, b_i \) are empirical constants. The above expression means that the no. of defects causing higher thickness reduction decreases exponentially. From Eq. (2), it can be seen that \( T_{BD} \) has a one to one correspondence with \( \Delta t_{ox} \) such that \( T_{BD} \) decreases with increasing \( \Delta t_{ox} \). As a result, the fraction of oxides having a breakdown time smaller than a particular value \( T_{BD}^* \) is equal to the fraction having a thickness reduction larger than the corresponding value \( \Delta t_{ox}^* \):

\[
F \left( T_{BD}^* \right) = F \left( T_{BD} < T_{BD}^* \right) = F \left( \Delta t_{BD} > \Delta t_{ox}^* \right)
\]

(4)

where, \( T_{BD}^* \) and \( \Delta t_{ox}^* \) are related by Eq. (2). It is also assumed that:

\[
F \left( \Delta t_{ox} > \Delta t_{ox}^* \right) = 1 - e^{-AD(\Delta t_{ox}^*)} = F \left( T_{BD}^* \right)
\]

(5)

where, \( A \) is the area. From Eq. (4) and (5) one gets:

\[
1 - F \left( T_{BD}^* \right) = e^{-AD(\Delta t_{ox}^*)}
\]

\[
- \ln \left( 1 - F \left( T_{BD}^* \right) \right) = AD(\Delta t_{ox}^*)
\]

\[
W = - \ln \left( 1 - F \right) = \ln A + \ln a_i - b_i \Delta t_{ox}
\]

(6)

Now, combining Eq. (2) and (6) one gets:

\[
W = \ln A + \beta_{def} \ln T_{BD} + C_i
\]

(7)

where, slope of the distribution \( \beta_{def} = \frac{b_i V_G}{B} \) and \( C_i = \ln a_i - \frac{b_i V_G}{B} \ln \left( C \tau_0 \right) \). Equation (7) is the essential result of this model. From this equation we can see that the distribution is going to scale with the area as:
\[ W_2 - W_1 = \ln \left( \frac{A_2}{A_1} \right) \]

where, \( W_1 \) and \( W_2 \) are the distribution for two oxides having areas of \( A_1 \) and \( A_2 \) respectively.

Although this model gives the correct qualitative variation it was abandoned later on when people discovered that using a very clean substrate (with very low amount of defects) did not improve the breakdown condition. Moreover, the model predicts that Weibull slope scales with gate voltage \( V_G \) (i.e. \( \beta_{def} = \frac{bV_G}{B} \)) – a conclusion not supported by subsequent experiments. We are now going to discuss another model now that seems to be more consistent with experimental observations.

### 23.2.2 Model 2: Percolation Model

This model considers the stochastic nature of a direct conducting path formation through the oxide. The oxide is divided into a grid-like structure having \( M \) rows and \( N \) columns as shown in Fig. 4. The probability \( P_n \) of \( n \) columns being short is given by binomial distribution:

\[ P_n = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n} \]  

(8)

where, \( p \) is the probability of an entire column being shorted and is given by:

\[ p = q^M \]  

(9)

where, \( q \) is the probability of a single cell being failed which itself is related to time \( t \) as:

\[ q = at^\alpha \]  

(10)

where, \( a \) is voltage and temperature dependent constant and \( \alpha \) is the power-law exponent of the order \( \sim 0.7-0.9 \) (Similar to \( n \sim 1/6 \) in NBTI). Equation (8) can be rewritten as:

![Figure 4: Creation of a direction conduction (short) path through the oxide](image)
\[ P_n = \frac{N(N-1)(N-2)\cdots(N-n+1)}{n!} p^n \left(1 - \frac{Np}{N}\right)^{N-n} \]

When \( N \gg n \) the above relation becomes:
\[ P_n \approx \frac{\chi^n}{n!} \left(1 - \frac{\chi}{N}\right)^N \]

where, \( \chi = Np \). Now when \( N \) becomes very large, i.e. \( N \to \infty \) we finally get:
\[ P_n = \frac{e^{-\chi}}{n!} \chi^n \quad (11) \]

We can now write the probability \( F_n \) of having \( n \) or more columns shorted as:
\[ F_n = 1 - \sum_{k=0}^{n-1} P_k = 1 - e^{-\chi} \sum_{k=0}^{n-1} \frac{\chi^k}{k!} \]

So the probability of the first occurrence of a short path, i.e. the probability of having 1 or more columns shorted is given by:
\[ F = F_1 = 1 - P_0 = 1 - e^{-\chi} \]

The above relation can be rewritten as:
\[ W = \ln\{-\ln(1-F)\} = \ln \chi \quad (12) \]

Now, using the definition of \( \chi \), Eq. (9) and (10) one can write that:

![Weibull Distribution](image)

**Figure 5: Slope of the Weibull decreases with the thickness of the oxide.**

\[ \chi = N \left(\frac{t}{\eta_0}\right)^{\alpha M} \quad (13) \]

where, \( \eta_0 = a^{-1/\alpha} \). Substituting Eq. (13) into Eq. (12) one gets:
\[ W = \beta_{per} \ln t + C_2 \quad (14) \]

where, the slope of distribution \( \beta_{per} = \alpha M \) and \( C_2 = \ln N - \alpha M \ln \eta_0 \).

Equation (14) is the essential result of this model and the predictions of Eq. (14) has repeatedly been confirmed by experiments. One can see that \( M \) being proportional to \( t_{ox} \) the slope of the distribution \( \beta_{per} \propto t_{ox} \) (Fig. 5). The consequence of this result is that thinner oxide has a larger probability of accidental breakdown. Also, since \( N \) is
proportional to transistor area, $A$, one can easily prove area scaling, i.e. $W_2 - W_1 = \ln \left( \frac{A_2}{A_1} \right)$ for the same observation time, $t$. Finally, unlike Model 1, $\beta_{\text{per}}$ does not depend on $V_G$ – an observation consistent with experiments.

23.3 Conclusion:

Today we have discussed the stochastic nature of the TDDB and explained the theory of Weibull distribution for gate dielectric breakdown. A key assumption of the theory is that the trap generation remains spatially and temporally uncorrelated regardless the number of traps generated. In other words, the presence of one trap does not affect the generation of the next trap. The Weibull distribution does not apply to very thick oxides, because this assumption of uncorrelated trap generation does not hold in that case (i.e. charge trapping modifies the band profiles and changes trap generation rates as a function of time). Even for thin oxides, the assumption of spatially uncorrelated trap generation is not easy to understand. In the next class we will discuss the measurement of the defect density (i.e. trap density created by tunneling holes) and in the process we will explore the origin of this spatially and temporally uncorrelated trap generation in thin oxides.