EE 649: SPEECH PROCESSING BY COMPUTER

Homework #3

1. The short-time autocorrelation function is defined as

\[ R_n(k) = \sum_{m=-\infty}^{\infty} x(m)w(n-m)x(m+k)w(n-k-m) \]

(a) Show that

\[ R_n(k) = R_n(-k) \]

i.e., show that \( R_n(k) \) is an even function of \( k \).

(b) Show that \( R_n(k) \) can be expressed as

\[ R_n(k) = \sum_{m=-\infty}^{\infty} x(m)x(m-k)h_k(n-m) \]

where

\[ h_k(n) = w(n)w(n+k) \]

2. This problem examines some of the issues introduced by computing autocorrelation function on windowed signals.

The short time autocorrelation function is defined as

\[ R_n(k) = \sum_{m=0}^{N-1-k} [x(n+m)w'(m)] [x(n+m+k)w'(k+m)] \]

(a) Consider the periodic impulse train

\[ x(m) = \sum_{r=-\infty}^{\infty} \delta(m-rP) \]

Let \( w'(m) \) be a rectangular window whose length \( N \) satisfies

\[ QP < N - 1 < (Q + 1)P \]

Where \( Q \) is an integer. Sketch \( R_n(k) \) for \( 0 \leq k \leq N-1 \).
For symmetric window functions (i.e., \( w(m) = w(-m) \)), the modified short time autocorrelation function is defined as

\[
\hat{R}_n(k) = \sum_{m = -\infty}^{\infty} x(n + m)\hat{w}_1(m)x(n + m + k)\hat{w}_2(m + k)
\]

(b) Define the window functions \( \hat{w}_1(m) \) and \( \hat{w}_2(m) \) as follows:

\[
\hat{w}_1(m) = 1 \quad 0 \leq m \leq N - 1 \\
0 \quad \text{otherwise}
\]

\[
\hat{w}_2(m) = 1 \quad 0 \leq m \leq N - 1 + K \\
0 \quad \text{otherwise}
\]

where \( K \) is the greatest lag for which the autocorrelation function \( \hat{R}_n(k) \) is to be computed (i.e., \( 0 \leq k \leq K \)).

For this \( \hat{w}_1(m) \) and \( \hat{w}_2(m) \), simplify the expression for \( \hat{R}_n(k) \).

(c) For the \( \hat{R}_n(k) \) of part (b) and the impulse train of part (a), sketch \( \hat{R}_n(k) \) for the same value of \( N \) as you used in part (a). Let \( K = QP \).

(d) Describe in words what problems the modified short time autocorrelation function is attempting to address.

3. Using windows similar to those used for the modified autocorrelation function, the short-time average magnitude difference function (AMDF) of the signal \( x(n) \) is defined as

\[
D_n(k) = \frac{1}{N} \sum_{m = 0}^{N - 1} |x(n + m) - x(n + m - k)|
\]

(a) Using the inequality

\[
\frac{1}{N} \sum_{m = 0}^{N - 1} |x(m)| \leq \left( \frac{1}{N} \sum_{m = 0}^{N - 1} |x(m)|^2 \right)^{1/2}
\]

show that

\[
D_n(k) \leq \left[ 2(\hat{R}_n(0) - \hat{R}_n(k)) \right]^{1/2}
\]
4. Consider the signal
\[ x(n) = A \cos(\omega_0 n) \]
as input to a three-level center clipper which produces an output
\[
y(n) = \begin{cases} 
1 & x(n) > C_L \\
0 & |x(n)| \leq C_L \\
-1 & x(n) < -C_L 
\end{cases}
\]
(a) Sketch \( y(n) \) as a function of \( n \) for \( C_L = 0.5A \), \( C_L = 0.75A \), and \( C_L = A \).
(b) Sketch the autocorrelation function for \( y(n) \) for the values of \( C_L \) in (a).
(c) Discuss the effect of the setting of \( C_L \) as it approaches \( A \). Suppose that \( A \) varies with time such that
\[
0 < A(n) \leq A_{\text{max}}
\]
Discuss problems that this can cause if \( C_L \) is close to \( A_{\text{max}} \).

5. One proposed method for detecting pitch based on LPC processing is to use the autocorrelation function of the LPC error signal \( e(n) \). Recall that \( e(n) \) can be written as
\[
e(n) = s'(n) - \sum_{k=1}^{p} a_k s'(n-k)
\]
and if we define \( a_0 = -1 \), then
\[
e(n) = - \sum_{k=1}^{p} a_k s'(n-k)
\]
where the windowed signal \( s'(n) = s(n)w(n) \) is nonzero for \( 0 \leq n \leq N-1 \), and zero everywhere else.
(a) Show that the autocorrelation function of \( e(n) \), \( R_e(m) \) can be written in the form
\[
R_e(m) = \sum_{l=-\infty}^{\infty} R_{a(l)} R_{s'}(m-l)
\]
where \( R_{a(l)} \) is the autocorrelation function of the LPC coefficients and \( R_{s'}(l) \) is the autocorrelation function of \( s'(n) \).
(b) For a speech sampling rate of 10 kHz, how much computation (i.e., multiplies and adds) is required to evaluate \( R_e(m) \) for values of \( m \) in the interval 3 to 15 msec?