EE 649: SPEECH PROCESSING BY COMPUTER

Homework 2: Cepstrum and LPC Analysis

1. The system function $H(z)$ evaluated at $N$ equally spaced points on the unit circle is

$$H \left[ e^{-j\frac{2\pi}{N}k} \right] = \frac{G}{1 - \sum_{n=1}^{p} a_n e^{-j\frac{2\pi}{N}kn}} \quad 0 \leq k \leq N - 1$$

Describe a procedure for using an FFT algorithm to evaluate $H \left[ e^{-j\frac{2\pi}{N}k} \right]$

2. For formant analysis using cepstrum analysis, the cepstrum $\{c(n)\}$ of signal $\{s(n)\}$ is windowed and then Fourier transformed as shown below:

![Figure 1: Processing of cepstrum signal to perform formant analysis.](image)

(a) Write an expression for the Fourier transform of $\{c(n)\}$ in terms of $\{s(n)\}$ and/or $S(e^{j\omega})$.

(b) Write an expression relating $Y(e^{j\omega})$ to $C(e^{j\omega})$ and $L(e^{j\omega})$, where $L(e^{j\omega})$ is the Fourier transform of $\{\ell(n)\}$ and $C(e^{j\omega})$ is the fourier transform of $\{c(n)\}$.

(c) For formant analysis, what type (e.g., shape) of window should $\{\ell(n)\}$ be? How long should the window by? Why?

3. Person #1 has an LPC vocoder system, but would like to transmit vocoded speech to person #2, who has a cepstrum vocoder system, and vice versa.

(a) Devise a method for converting the LPC coefficients ($a_k$'s) to the low-time cepstrum (the $y(n) = c(n)\ell(n)$ products shown in problem 2). Describe clearly the steps performed in doing the conversion.

(b) Device a method for converting the low-time cepstrum to the LPC coefficients.
4. Consider the first order \((p = 1)\) linear predictor:

\[
\hat{s}_n = \alpha s_{n-1}
\]

Assume that signal \(\{s_n\}\) is zero mean and stationary (i.e., the statistics are not affected by a shift of time origin). Let the prediction error be

\[
e_n = s_n - \hat{s}_n
\]

**Derive** the expression for \(\alpha\) which will minimize

\[
E = \sum_{n=-\infty}^{\infty} e_n^2
\]