ECE 606
HW 8 Solutions
Fall 2012
There are four modes:

1. Normal active
2. Saturation
3. Cut off
4. Inverse active

\{ \text{Minority Carrier Concentration Profiles} \}
% Base Parameters
NB=logspace(log10(0.1*NB0),log10(10*NB0));
µB=µpmin+µp0./(1+(NB./NAref).^ap);
DB=kT.*µB;
TauB=TauB0;
LB=sqrt(DB.*TauB);

% Beta Calculation
Y=1 ./((DE./DB).* (NB./NE).*(W./LE)+(0.5).*(W./LB).^2);
X=NB/NB0;

% Plotting
subplot(2,2,3); loglog(X,Y); grid
xlabel('NE/NB0'); ylabel('Beta')

% Revised Base Parameters
NB=NB0;
µB=µpmin+µp0./(1+(NB./NAref).^ap);
DB=kT.*µB;
TauB=logspace(log10(0.1*TauB0),log10(10*TauB0));
LB=sqrt(DB.*TauB);

% Beta Calculation
Y=1 ./((DE./DB).* (NB./NE).*(W./LE)+(0.5).*(W./LB).^2);
X=TauB/TauB0;

% Plotting
subplot(2,2,4); loglog(X,Y); grid
xlabel('TauB/TauB0'); ylabel('Beta')

11.11
(a) Like in the standard ideal-transistor analysis, the general solution for $Δn_E(x'')$ is

$$Δn_E(x'') = A_1 e^{-x''/LE} + A_2 e^{x''/LE}$$

However, because of the finite width of the emitter, $A_2 \neq 0$. Rather

$$Δn_E(x''=0) = n_{E0}(eqV_{EB}/kT - 1) = A_1 + A_2$$

$$Δn_E(x''=V_E) = 0 = A_1 e^{-V_E/LE} + A_2 e^{V_E/LE}$$

11-14
Solving for $A_1$ and $A_2$ yields

$$A_1 = \frac{\Delta n_E(0) e^{W_E/L_E}}{e^{W_E/L_E} - e^{-W_E/L_E}} \quad \text{and} \quad A_2 = \frac{-\Delta n_E(0) e^{-W_E/L_E}}{e^{W_E/L_E} - e^{-W_E/L_E}}$$

Substituting back into the general solution then gives

$$\Delta n_E(x') = \Delta n_E(0) \left[ \frac{e^{(W_E-x')/L_E} - e^{-(W_E-x')/L_E}}{e^{W_E/L_E} - e^{-W_E/L_E}} \right]$$

or

$$\Delta n_E(x') = n_{E0} \frac{\sinh[(W_E-x')/L_E]}{\sinh(W_E/L_E)} \left( e^{qV_{EB}/kT} - 1 \right)$$

Applying Eq. (11.7), we obtain the revised $I_{En}$ expression

$$I_{En} = qA \frac{D_E}{L_E} n_{E0} \frac{\cosh(W_E/L_E)}{\sinh(W_E/L_E)} \left( e^{qV_{EB}/kT} - 1 \right)$$

(b) Relative to the standard result (Eq. 11.20), the finite width of the emitter leads to the modification

$$n_{E0} \rightarrow n_{E0} \frac{\cosh(W_E/L_E)}{\sinh(W_E/L_E)}$$

or equivalently, since $n_{E0} = n_i^2 / N_E$,

$$\frac{1}{N_E} \rightarrow \frac{1}{N_E} \frac{\cosh(W_E/L_E)}{\sinh(W_E/L_E)}$$

Revised expressions for the performance parameters analogous to Eqs. (11.31)–(11.34) are therefore obtained by simply making the above noted substitution. Specifically,

$$\begin{bmatrix} D_E & L_B & N_B \\ D_B & L_E & N_E \end{bmatrix} \rightarrow \begin{bmatrix} D_E & L_B & N_B \\ D_B & L_E & N_E \end{bmatrix} \frac{\cosh(W_E/L_E)}{\sinh(W_E/L_E)}$$

in the $\gamma$, $\alpha_{dc}$, and $\beta_{dc}$ expressions. $\alpha_T$ remains unchanged.
(c) With $W_E/L_E << 1$,

$$\frac{\cosh(W_E/L_E)}{\sinh(W_E/L_E)} \approx \frac{L_E}{W_E}$$

and $L_E$ in the Eq. (11.41)–(11.44) expressions is to first order simply replaced by $W_E$.

This yields, for example,

$$\gamma = \frac{1}{1 + \frac{D_E}{D_B} \frac{N_B}{N_E} \frac{W}{W_E}}$$

A similar modification applies to $\alpha_{dc}$ and $\beta_{dc}$. Being independent of $L_E$, the $\alpha_T$ expression remains unchanged.

(d) A systematic decrease in $W_E$ obviously causes a monotonic decrease in $\gamma$ and a corresponding decrease in $\beta_{dc}$. Reducing the width of the emitter has the negative effect of degrading the gain of the BJT (which is the point of the problem).

(e) If $W_E << L_E$ the part (a) result simplifies to

$$\Delta n_E(x'') = \Delta n_E(0) \left(1 - \frac{x''}{W_E}\right)$$

or one obtains a linear distribution analogous to the situation in the base. Moreover, since the emitter is forward biased under active mode biasing, $\Delta n_E(0) > 0$. Thus we conclude

\[\Delta n_E(x'')\]

\[\begin{array}{c}
\Delta n_E(x'') \\
\end{array}\]

\[\begin{array}{c}
W_E \\
0 \\
\end{array}\]

11 - 16
\[ I_d = I_s \times (e^{qV_a/kT} - 1) \]

For a \( p^+\)-\( n \) junction...

\[ I_{\text{DIFF}} = I_0 (e^{qV_a/kT} - 1) = qA \frac{D_p \cdot \eta_I^2}{L_p N_D} (e^{qV_a/kT} - 1) \]

and employing Eqs. (14.24/14.25),

\[ I_{\text{TE}} = I_s (e^{qV_a/kT} - 1) = A_\alpha T^2 e^{-\Phi_B/kT} (e^{qV_a/kT} - 1) \]

Thus noting

\[ \frac{D_p}{L_p} = \sqrt{\frac{D_p}{\tau_p}} = \sqrt{\frac{(kT/q)\mu_p}{\tau_p}} \]

\[ \frac{I_{\text{DIFF}}}{I_{\text{TE}}} = \frac{I_0}{I_s} = \frac{qA \frac{D_p \cdot \eta_I^2}{L_p N_D}}{A_\alpha T^2 e^{-\Phi_B/kT}} = \frac{q\sqrt{(kT/q)\mu_p \frac{\eta_I^2}{\tau_p}}}{A_\alpha T^2 e^{-\Phi_B/kT}} \]

\[ = \left(1.6 \times 10^{-19} \right) \left( \frac{(0.0259)(437)}{(10^{-6})} \right)^{1/2} \left( \frac{10^{20}}{10^{16}} \right) = 5.05 \times 10^{-7} \]
p-n-p and
n-p-n X-sister pair

Reason for this configuration is because it is difficult to make high $\beta$ p-n-p transistors.
In saturation both junctions are forward biased. The emitter and collector currents are given in the N-Well model as

\[ I_E = I_{Es} \left( e^{q V_{EB}/kT} - 1 \right) - \kappa_R I_{Rs} \left( e^{q V_{CB}/kT} - 1 \right) \]  

(1)

\[ I_C = \kappa_F I_{Es} \left( e^{q V_{EB}/kT} - 1 \right) - I_{Rs} \left( e^{q V_{CB}/kT} - 1 \right) \]  

(2)

In these equations, under forward bias, the exponential terms are larger than unity.

\( \Rightarrow \) Solving (1) and (2)

\[ V_{EB} = \frac{kT}{q} \ln \frac{I_E - \kappa_R I_C}{I_{Es} \left( 1 - \kappa_R \kappa_F \right)} \]

\[ V_{CB} = \frac{kT}{q} \ln \frac{I_C - \kappa_F I_E}{I_{Rs} \left( \kappa_R \kappa_F - 1 \right)} \]

Voltages can be also written as

\[ V_{EC} = V_{EB} - V_{CB} \]

\( \Rightarrow \) \[ V_{EC} = \frac{kT}{q} \ln \left[ \frac{I_E - \kappa_R I_C}{I_{Es} \left( 1 - \kappa_R \kappa_F \right)} \right] \frac{I_{Rs} \left( \kappa_R \kappa_F - 1 \right)}{I_C - \kappa_F I_E} \]  

(3)
Eqn 3 can be simplified using

\[ I_F = I_B + I_C \]

\[ \beta_F = \frac{\alpha_F}{1 - \alpha_F} \]

\[ \beta_R = \frac{\alpha_R}{1 - \alpha_R} \]

\[ \frac{\alpha_F}{\alpha_R} = \frac{I_{es} (\alpha_F \alpha_R - 1)}{I_{es} (1 - \alpha_F \alpha_R)} \]

\[ V_{EC} \text{ after some algebra is:} \]

\[
\frac{K_T}{I} \left[ \ln \frac{1 \alpha_R + \frac{\beta_s}{\beta_R}}{1 - \frac{\beta_s}{\beta_R}} \right]
\]

where \( \beta_s = \frac{I_C}{I_B} \)
From the Nenni model:
\[ I_c = I_{CT} - \frac{I_{EC}}{\beta_R} \quad (1) \]

Current supplied by current source

Substituting expressions for \( I_{CT} \) and \( I_{EC} \) and by

setting \( I_c = 0 \) we have

\[ \left(1 + \frac{1}{\beta_R}\right) \exp\left(\frac{V_{BC}}{V_T}\right) = \exp\left(\frac{V_{BE}}{V_T}\right) + \frac{1}{\beta_R} \quad (V_T = kT/q) \]

\[ V_{BC} = \beta_R V_T \ln \left[ \frac{\beta_R \exp\left(V_{BE}/V_T\right) + 1}{\beta_R + 1} \right] \]

\[ = V_T \ln \left[ \alpha_R \left\{ \exp\left(V_{BE}/V_T\right) - 1 \right\} + 1 \right] \]

\[ = V_T \ln \left[ \alpha_R \exp\left(V_{BE}/V_T\right) \right] \]

\[ \alpha_R = \frac{\beta_R}{1 + \beta_R} \]
From solution of a) we see that $V_{BC}$ is positive.

BC junction is forward-biased.

Both junctions are forward-biased, transistor is in saturation stage.

$\exp\left(\frac{V_{BC}}{V_T}\right) = \frac{1}{R(V_{BE})} = \beta \exp\left(\frac{V_{BE}}{V_T}\right)$

$\exp\left[\frac{V_{BE} - V_{BC}}{V_T}\right] = \exp\left[-\frac{V_{CE}}{V_T}\right] = \frac{1}{\beta R}$

$V_{CE} = V_T \ln\left[\frac{1}{\beta R}\right]$