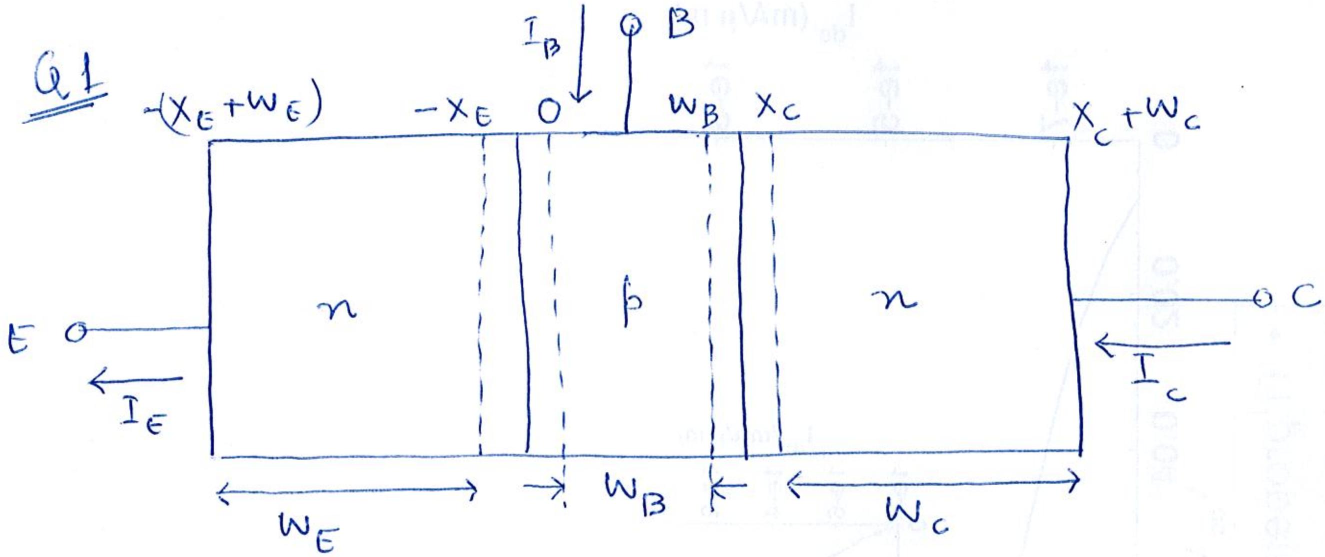
ECE 606

HW. 8 Solutions.

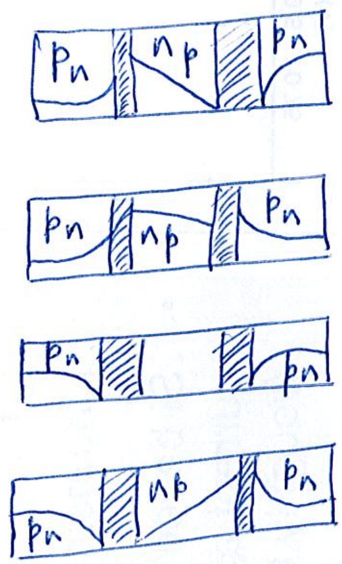
Fall 2012

Q1



There are four modes.

1. Normal active
2. Saturation
3. Cutoff
4. Inverse active



Minority carrier concentration profiles.

```
%Base Parameters
  NB=logspace(log10(0.1*NB0),log10(10*NB0));
  µB=µpmin+µp0./(1+(NB./NAref).^ap);
  DB=kT.*µB;
  TauB=TauB0;
  LB=sqrt(DB.*TauB);
%Beta Calculation
  Y=1 ./((DE./DB).*(NB./NE).*(W./LE)+(0.5).*(W./LB).^2);
  X=NB/NB0;
%Plotting
  subplot(2,2,3);  loglog(X,Y);  grid
  xlabel('NB/NB0');  ylabel('Beta')
%Beta vs. TauB/TauB0
 %Revised Base Parameters
   NB=NB0;
   µB=µpmin+µp0./(1+(NB./NAref).^ap);
   DB=kT.*µB;
   TauB=logspace(log10(0.1*TauB0),log10(10*TauB0));
   LB=sqrt(DB.*TauB);
 %Beta Calculation
   Y=1 ./((DE./DB).*(NB./NE).*(W./LE)+(0.5).*(W./LB).^2);
   X=TauB/TauB0;
 %Plotting
   subplot(2,2,4);  loglog(X,Y);  grid
   xlabel('TauB/TauB0');  ylabel('Beta')
```

Q2

11.11

(a) Like in the standard ideal-transistor analysis, the general solution for $\Delta n_E(x'')$ is

$$\Delta n_E(x'') = A_1 e^{-x''/L_E} + A_2 e^{x''/L_E}$$

However, because of the finite width of the emitter, $A_2 \neq 0$. Rather

$$\Delta n_E(x''=0) = n_{E0}(e^{qV_{EB}/kT} - 1) = A_1 + A_2$$

$$\Delta n_E(x''=W_E) = 0 = A_1 e^{-W_E/L_E} + A_2 e^{W_E/L_E}$$

Solving for A_1 and A_2 yields

$$A_1 = \frac{\Delta n_E(0)\, e^{W_E/L_E}}{e^{W_E/L_E} - e^{-W_E/L_E}} \quad \text{and} \quad A_2 = -\frac{\Delta n_E(0)\, e^{-W_E/L_E}}{e^{W_E/L_E} - e^{-W_E/L_E}}$$

Substituting back into the general solution then gives

$$\Delta n_E(x'') = \Delta n_E(0)\left[\frac{e^{(W_E-x'')/L_E} - e^{-(W_E-x'')/L_E}}{e^{W_E/L_E} - e^{-W_E/L_E}}\right]$$

or

$$\boxed{\Delta n_E(x'') = n_{E0}\frac{\sinh[(W_E-x'')/L_E]}{\sinh(W_E/L_E)}\left(e^{qV_{EB}/kT} - 1\right)}$$

Applying Eq. (11.7), we obtain the revised I_{En} expression

$$\boxed{I_{En} = qA\frac{D_E}{L_E} n_{E0}\frac{\cosh(W_E/L_E)}{\sinh(W_E/L_E)}\left(e^{qV_{EB}/kT} - 1\right)}$$

(b) Relative to the standard result (Eq. 11.20), the finite width of the emitter leads to the modification

$$n_{E0} \rightarrow n_{E0}\frac{\cosh(W_E/L_E)}{\sinh(W_E/L_E)}$$

or equivalently, since $n_{E0} = n_i^2/N_E$,

$$\frac{1}{N_E} \rightarrow \frac{1}{N_E}\frac{\cosh(W_E/L_E)}{\sinh(W_E/L_E)}$$

Revised expressions for the performance parameters analogous to Eqs. (11.31)–(11.34) are therefore obtained by simply making the above noted substitution. Specifically,

$$\boxed{\left(\frac{D_E}{D_B}\frac{L_B}{L_E}\frac{N_B}{N_E}\right) \rightarrow \left(\frac{D_E}{D_B}\frac{L_B}{L_E}\frac{N_B}{N_E}\right)\frac{\cosh(W_E/L_E)}{\sinh(W_E/L_E)}}$$

in the γ, α_{dc}, and β_{dc} expressions. α_T remains unchanged.

(c) With $W_E/L_E << 1$,

$$\frac{\cosh(W_E/L_E)}{\sinh(W_E/L_E)} \cong \frac{L_E}{W_E}$$

and L_E in the Eq. (11.41)–(11.44) expressions is to first order simply replaced by W_E. This yields, for example,

$$\boxed{\gamma = \frac{1}{1 + \frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{W_E}}}$$

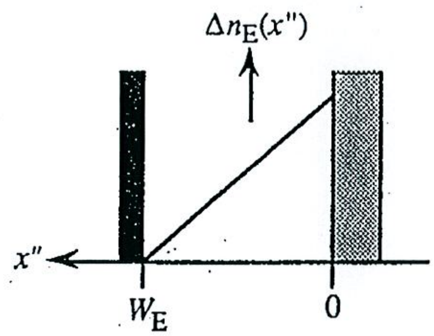
A similar modification applies to α_{dc} and β_{dc}. Being independent of L_E, the α_T expression remains unchanged.

(d) A systematic decrease in W_E obviously causes a monotonic decrease in γ and a corresponding decrease in β_{dc}. Reducing the width of the emitter has the negative effect of degrading the gain of the BJT (which is the point of the problem).

(e) If $W_E << L_E$ the part (a) result simplifies to

$$\Delta n_E(x'') = \Delta n_E(0)\left(1 - \frac{x''}{W_E}\right)$$

or one obtains a linear distribution analogous to the situation in the base. Moreover, since the emitter is forward biased under active mode biasing, $\Delta n_E(0) > 0$. Thus we conclude



```
Is=1.0e-8;
RS=[0,0.1,1.0,10];
VJ=linspace(0,0.6);
%Calculate I versus VA
I=Is.*(exp(VJ/kT)-1);
VA=[];
for i=1:4,
VA=[VA;VJ+I.*RS(i)];
end
%Plot result
semilogy(VA,I,'w');
axis([0,0.6,1.0e-9,1.0e-1]);   grid
xlabel('VA (volts)');  ylabel('I (amps)')
text(0.34,5.0e-2,'RS=0');  text(0.41,2.0e-2,'RS=1 ohm')
text(0.41,4.0e-3,'RS=10 ohms')
```

Q3

14.9

For a p^+-n junction...

$$I_{\mathrm{DIFF}} = I_0(e^{qV_A/kT} - 1) = qA\frac{D_P}{L_P}\frac{n_i^2}{N_D}(e^{qV_A/kT} - 1)$$

and employing Eqs. (14.24/14.25),

$$I_{\mathrm{TE}} = I_s(e^{qV_A/kT} - 1) = A\mathcal{A}^*T^2e^{-\Phi_B/kT}(e^{qV_A/kT} - 1)$$

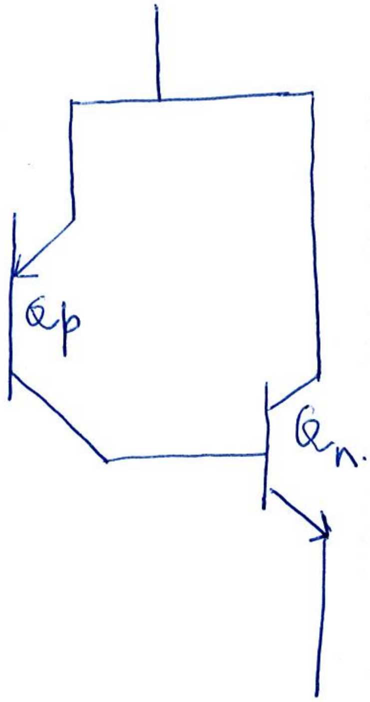
Thus noting

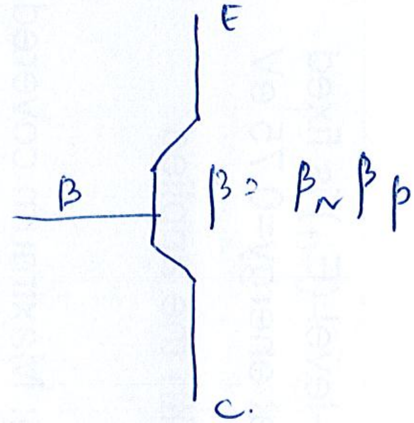
$$\frac{D_P}{L_P} = \sqrt{\frac{D_P}{\tau_p}} = \sqrt{\frac{(kT/q)\mu_p}{\tau_p}}$$

$$\frac{I_{\mathrm{DIFF}}}{I_{\mathrm{TE}}} = \frac{I_0}{I_s} = \frac{qA\dfrac{D_P}{L_P}\dfrac{n_i^2}{N_D}}{A\mathcal{A}^*T^2e^{-\Phi_B/kT}} = \frac{q\sqrt{\dfrac{(kT/q)\mu_p}{\tau_p}}\,\dfrac{n_i^2}{N_D}}{\mathcal{A}^*T^2e^{-\Phi_B/kT}}$$

$$= \frac{(1.6\times10^{-19})\left[\dfrac{(0.0259)(437)}{(10^{-6})}\right]^{1/2}\left(\dfrac{10^{20}}{10^{16}}\right)}{(140)(300)^2e^{-(0.72)/(0.0259)}} = 5.05\times10^{-7}$$

Q4





p-n-p and

n-p-n X-sistor pair

Equivalent

p-n-p.

Reason for this configuration is because it is difficult to make high β. p-n-p transistors.

In saturation both junctions are forward-biased.

The emitter and collector currents are given in Ebers-Moll model as

$$I_E = I_{ES}\left(e^{qV_{EB}/kT} - 1\right) - \alpha_R I_{CS}\left(e^{qV_{CB}/kT} - 1\right) \quad (1)$$

$$I_C = \alpha_F I_{ES}\left(e^{qV_{EB}/kT} - 1\right) - I_{CS}\left(e^{qV_{CB}/kT} - 1\right) \quad (2)$$

In these equations, under forward bias, the exponential terms are larger than unity

$\Rightarrow$ Solving (1) and (2)

$$V_{EB} = \frac{kT}{q} \ln \frac{I_E - \alpha_R I_C}{I_{ES}(1 - \alpha_R . \alpha_F)}$$

$$V_{CB} = \frac{kT}{q} \ln \frac{I_C - \alpha_F I_E}{I_{CS}(\alpha_R \alpha_F - 1)}$$

Voltages can be also written as

$$V_{EC} = V_{EB} - V_{CB}$$

$$\Rightarrow V_{EC} = \frac{kT}{q} \ln \left[\frac{I_E - \alpha_R I_C}{I_{ES}(1 - \alpha_R \alpha_F)} \frac{I_{CS}(\alpha_R \alpha_F - 1)}{I_C - \alpha_F I_F} \right] \quad (3)$$

Eqn (3) can be simplified using

$$I_E = I_B + I_C$$

$$\beta_F = \alpha_F / 1 - \alpha_F$$

$$\beta_R = \alpha_R / 1 - \alpha_R.$$

$$\alpha_F / \alpha_R = \frac{I_{CS} (\alpha_F \alpha_R - 1)}{I_{ES} (1 - \alpha_F \alpha_R)}$$

$\therefore V_{EC}$ after some algebra is:

$$\boxed{\frac{KT}{q} \ln \frac{1/\alpha_R + \beta_S / \beta_R}{1 - \beta_S / \beta_R}}$$

where $\beta_S = I_C / I_B$

Q6

a) Open circuit voltage.

From Ebers-Moll model:

$$I_C = I_{CT} - \frac{I_{EC}}{\beta_R} \quad \text{——— (1)}$$

// Current supplied by current source.

∴ Substituting expression for I_{CT} and I_{EC} and by setting $I_C = 0$ we have

$$\left(1 + \frac{1}{\beta_R}\right) \exp\left(\frac{V_{BC}}{V_T}\right) = \exp\left(\frac{V_{BE}}{V_T}\right) + \frac{1}{\beta_R} \qquad (V_T = KT/q)$$

$$\therefore V_{BC} = \cancel{\beta_R}\, V_T \ln\left[\frac{\beta_R \exp(V_{BE}/V_T) + 1}{\beta_R + 1}\right]$$

$$= V_T \ln\left[\alpha_R \left\{\exp\left(V_{BE}/V_T\right) - 1\right\} + 1\right]$$

$$\approx V_T \ln\left[\alpha_R \exp\left(V_{BE}/V_T\right)\right]$$

$$\boxed{\alpha_R = \beta_R/1+\beta_R}$$

b From solution of a) we see that V_{BC} is positive

$\therefore$ BC junction is forward-biased.

$\because$ Both junctions are forward-biased, transistor is in saturation stage.

c From solution of part a)

$$\exp\left(\frac{V_{BC}}{V_T}\right) = \cancel{\alpha_R\left(\frac{V_{BE}}{V_T}\right)} \quad \alpha_R \exp\left[V_{BE}/V_T\right]$$

or $$\exp\left[\frac{V_{BE}-V_{BC}}{V_T}\right] = \exp\left[\frac{V_{CE}}{V_T}\right] = 1/\alpha_R$$

$$\therefore V_{CE} = V_T \ln\left[1/\alpha_R\right]$$

Q7 $$\frac{g_{CB}}{g_{CE}} = \frac{W_B}{L_n^2}\,\frac{W_B}{2} = \frac{W_B^2}{2L_n^2} \approx 1/\beta$$

We know that β (usually) $>> 1 \Rightarrow g_{CB} << g_{CE}$

$L_n = \sqrt{D_n \tau_n}$: Minority carrier diffusion length in base.