Solutions

HW #5

ECE 606

9.09.92
Problem 47:
The continuity equation for the excess hole is given by:
\[
\frac{\partial \Delta p}{\partial t} = \frac{1}{q} \frac{\partial J_p}{\partial x} - r_p + g_p
\]
For steady state, dark room and low level recombination, we have
\[
\frac{1}{q} \frac{\partial J_p}{\partial x} = 0
\]
The steady state current is given by:
\[
J_p = q \mu E \Delta p - q D_p \frac{\partial \Delta p}{\partial x}
\]
Substitute back into continuity equation we get,
\[
D_p \frac{\partial^2 \Delta p}{\partial x^2} - \mu E \frac{\partial \Delta p}{\partial x} - \frac{\Delta p}{\tau_p} = 0
\]
Using the hint, we have the following roots,
\[
\mu E = \sqrt{(\mu E)^2 + \frac{4D_p}{\tau_p}}
\]
\[
r = \frac{\sqrt{(\mu E)^2 + \frac{4D_p}{\tau_p}}}{2D_p}
\]
Hence
\[
r_1 = \frac{\mu E + \sqrt{(\mu E)^2 + \frac{4D_p}{\tau_p}}}{2D_p} \quad \text{and} \quad r_2 = \frac{\mu E - \sqrt{(\mu E)^2 + \frac{4D_p}{\tau_p}}}{2D_p}
\]
\[
\Delta p(x) = c_1 \exp(r_1 x) + c_2 \exp(r_2 x)
\]
To determine \(c_1\) and \(c_2\) we make use of boundary condition. 
\[
\Delta p(0) = c_1 + c_2
\]
\[
\Delta p(L) = c_1 \exp(r_1 L) + c_2 \exp(r_2 L) = 0
\]
\[
\Rightarrow c_1 = -c_2 \exp((r_2 - r_1) L)
\]
Therefore the final solution is:
\[
\Delta p(x) = \frac{\Delta p(0)}{1 - \exp((r_1 - r_2) L) \exp(r_1 x)} \exp(r_2 x)
\]

Problem 48:
\[
D_p = \frac{kT}{q \mu_p} = 10 \text{cm}^2 / \text{s}
\]
We have also the following constants for \(\Delta p(x) = c_1 \exp(r_1 x) + c_2 \exp(r_2 x)\)
\[ r_i = 1.22 \times 10^5 \]
\[ r_i = -8.2 \times 10^6 \]
\[ c_i = -1.5 \times 10^{20} \]
\[ c_0 = 1.15 \times 10^{31} \]

The drift current at \( x=0 \) is given by:
\[ q \mu E \Delta \rho = 6.41 \times 10^4 \text{ A/m} \]

The diffusion current at \( x=0 \) is given by:
\[ -qD_D \frac{\partial \Delta \rho}{\partial x} = 1.8 \times 10^4 \text{ A/m}^2 \]

Supposed now we assume \( E=0 \),
\[ r_i = 1.0 \times 10^5 \]
\[ r_i = -1.0 \times 10^5 \]
\[ c_i = -1.17 \times 10^{20} \]
\[ c_0 = 1.157 \times 10^{31} \]
\[ -qD_D \frac{\partial \Delta \rho}{\partial x} = 2.3 \times 10^4 \text{ A/m}^2 \]

The fraction error introduced is approximately 0.167.

**Problem 4C:**
False, True, False, False
(5,10)

(b) Reconversion... We note first of all that \( \frac{3}{2}I_0 = \frac{3}{2}I_c \).

Also, following a line-like argument, the recombination rate is expected to be proportional to both \( n \) and \( p \), the electron and hole concentrations. Setting \( c_b \) be the proportionality constant, we conclude

\[
\frac{3}{2}I_0 = \frac{3}{2}I_c = -c_b np \quad (c_b > 0)
\]

Generation... The generation rate, \( \frac{3}{2}I_0 = \frac{3}{2}I_c \), should depend on the number of filled states in the valence band and on the number of empty states in the conduction band. If the semiconductor is taken to be nondegenerate, both of the cited quantities are approximately constant. Thus

\[
\frac{3}{2}I_0 = \frac{3}{2}I_c = \beta = \text{constant}
\]

Combining the above relationships yields

\[
\beta = -\left( \frac{3}{2}I_0 + \frac{3}{2}I_c \right) = -\left( \frac{3}{2}I_c + \frac{3}{2}I_c \right) = c_b np - \beta
\]

Next invoking the equilibrium simplification based on the principle of detailed balance gives

\[
\beta_{\text{equilibrium}} = 0 = c_{b\text{e}} m_p n - \beta_0
\]

or

\[
\beta_0 = c_{b\text{e}} m_p n - \beta_{b\text{e}} \beta
\]

Making the usual assumption that \( \beta_0 \) and \( \beta_{b\text{e}} \) do not change significantly under non-equilibrium conditions then yields

\[ \beta_{b\text{e}} \]

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\[ \beta = \beta_0 = c_6 m^4 = c_6 m^4 \]

\[ \left[ r_b = c_6 (m c - m_b^2) \right] \]

(c) Setting \( m = m_b + \Delta m \)
\( p = p_b + \Delta p \)
and substituting into the \( r_b \) expression gives

\[ r_b = c_6 (m c + m_b \Delta m + p b \Delta p) \]
\[ \approx c_6 (m c + m_b) \Delta m \]

\[ r_b \approx \frac{\Delta m}{c_6} \]
\[ r_b = \frac{1}{c_6 (m_b + p_b)} \]

(a) A nondegenerate, doping in silicon at 300K corresponds to a doping of \( N_0 + N_A \geq 10^{19} \text{cm}^{-3} \). Thus the typical range of interest is \( m_b + p_b \in 10^{16} \text{cm}^{-3} \) to \( 10^{18} \text{cm}^{-3} \).

Computing \( r_b \) from the part (c) result with \( c_b = 5 \times 10^{-10} \text{cm/sec} \), and comparing against the worst case scenario (\( c_b_{\text{max}} = 1 \text{ms/cm} \) for \( p-n \) Si) as deduced from Fig. 5.11, we obtain:

<table>
<thead>
<tr>
<th>Nondegenerate (cm^-3)</th>
<th>( r_b ) (sec)</th>
<th>( \tau \times \tau_{\text{inter}} ) (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{19} )</td>
<td>2</td>
<td>( 1 \times 10^{-2} )</td>
</tr>
<tr>
<td>( 10^{18} )</td>
<td>( 2 \times 10^{-1} )</td>
<td>( 1 \times 10^{-3} )</td>
</tr>
<tr>
<td>( 10^{17} )</td>
<td>( 2 \times 10^{-2} )</td>
<td>( 1 \times 10^{-4} )</td>
</tr>
<tr>
<td>( 10^{16} )</td>
<td>( 2 \times 10^{-3} )</td>
<td>( 6 \times 10^{-5} )</td>
</tr>
<tr>
<td>( 10^{15} )</td>
<td>( 2 \times 10^{-4} )</td>
<td>( 1 \times 10^{-5} )</td>
</tr>
</tbody>
</table>

For this worst case scenario, \( \tau_b = 3 \times 10^{-2} \), over most of the doping range, only changes to \( \sim 3 \times 10^{-2} \) min. \( N_b > 10^{16} \text{cm}^{-3} \). Clearly, the silicon has to worry about band electron recombination in nondegenerately doped Si at room temperature.
Intrinsic silicon at 300 K.

\[ n_0 = p_0 = n_i = 10^{10} \text{ cm}^{-3} \]

\[ \mu_n = 1360 \text{ cm}^2/\text{V.s} \]

\[ \mu_p = 460 \text{ cm}^2/\text{V.s} \]

\[ \phi = \frac{1}{q(\mu_n + \mu_p p_0)} = \frac{1}{1.6 \times 10^{-19} (1360 + 460) \times 10^{-10}} = 3.93 \times 10^5 \text{ A/cm} \]

n-doped silicon \( N_D = 10^{16} \text{ cm}^{-3} \) (300 K)

\[ \phi = \frac{1}{q \mu_n N_D} = \frac{1}{1.6 \times 10^{-19} \times 1360 \times 10^{16}} = 0.46 \text{ A/cm} \]
4. A semi-infinite p-type semiconductor bar is subject to uniform penetrating illumination resulting in a generation rate of \( G_0 \) electron-hole pairs per second per cm² throughout the bar. \( G_0 \) is such that sample remains in low-level injection. Minority carriers are extracted at the surface at \( x = 0 \). Obtain an expression for the maximum hole current that can be drawn from the bar in steady-state.

**MCDE:**
\[
\frac{\partial \Delta n}{\partial t} = 0 = D_p \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta P_n}{\partial x} + \Delta G_0
\]

**GENL. soln.:**
\[
\Delta P_n(x) = \Delta G_0 \frac{x}{L} + B e^{-x/L} + r P_0 G_0 L
\]

**BC:**
\[
\text{At } x = 0, \quad \Delta P_n \text{ must be finite, so } A = 0.
\]

\[
\Delta P_n(x) = - \frac{\Delta G_0}{L} x + r P_0 G_0 L e^{-x/L} - \frac{\Delta G_0}{L} x
\]

\[
J_p(0) = - \frac{8}{L} \frac{\Delta P_n}{\partial x} \bigg|_{x=0} = - \frac{8}{L} \left( \frac{\Delta G_0}{L} \right) P_0 G_0 L e^{-0/L} = -J_h
\]
Consider a region in a semiconductor that is totally depleted of carriers \((n = p = 0)\). Obtain an expression for the energy level of the RG centers relative to midgap \(\Delta E = (E_T - E_i)\) that results in the highest possible generation rate. Your answer should include the minority carrier lifetimes \(\tau_n\) and \(\tau_p\).

\[
R = \frac{P_n - N_i^2}{\gamma_n (P_n + P_i) + \gamma_p (P_n + P_i)} \left\{ \begin{array}{l}
N_i = N_i e^{(E_T - E_i)/kT} \\
P_i = N_i e^{(E_i - E_T)/kT}
\end{array} \right.
\]

If \(n = p = 0\), \(-R = G = \frac{N_i^2}{\gamma_p P_i + \gamma_n P_n}\)

\[
G = \frac{N_i^2}{\gamma_n N_i e^{(E_T - E_i)/kT} + \gamma_p N_i e^{(E_i - E_T)/kT}}
\]

Let \((E_T - E_i) = \Delta E\), then \(G = \frac{N_i}{\gamma_n e^{-\Delta E/kT} + \gamma_p e^\Delta E/kT}\)

Set \(\frac{dG}{d\Delta E} = 0\) to find the maximum generation rate.

\[
\frac{dG}{d\Delta E} = \left(\frac{\Delta E}{\gamma_n e^{\Delta E/kT} + \gamma_p e^{-\Delta E/kT}}\right)^2 \left(\frac{-\gamma_n e^{-\Delta E/kT} + \gamma_p e^{\Delta E/kT}}{\gamma_n kT + \gamma_p kT}\right) = 0
\]

\[
; \gamma_p e^{\Delta E/kT} - \gamma_n e^{-\Delta E/kT} = 0
\]

Multiply by \(e^{\Delta E/kT}\) gives \(\gamma_p e^{\Delta E/kT} = \gamma_n e^{\Delta E/kT}\).\

\[
\Delta E = \frac{kT}{\frac{\gamma_n}{\gamma_p}} \ln\left(\frac{\gamma_n}{\gamma_p}\right)
\]

\[
\Delta E = E_T - E_i = \frac{kT}{ \ln\left(\frac{\gamma_n}{\gamma_p}\right) }
\]