HW # 11.

Solutions

ECE 606, Fall 2012

Q1

a $$V_{fb} = \phi_{ms} - \frac{Q_F}{C_{ox}}$$

$$= -1 - \frac{8 \times 10^{-8} \ c/cm^2}{2 \times 10^{-7} \ F/cm^2} = \underline{\underline{-1.40V}}$$

b Electric field in oxide $\mathcal{E}_{ox} = -\frac{Q_{semi}}{t_{ox}}$

Q_{semi} = total charge density on semiconductor side of oxide

$\therefore Q_{semi} = Q_F + Q_{IT} + Q_D + Q_N$

$Q_{IT} = 0$ (stated)

At flat-band $Q_D = Q_N = 0$

$\therefore Q_{semi} = Q_F$

$$E_{ox} = \frac{-Q_F}{\mathcal{E}_{ox}} = -2.318 \times 10^5 \ V/cm$$

c To make $\mathcal{E}_{ox} \longrightarrow 0$, the total charge on the semiconductor side of oxide must be zero.

In general $Q_{semi} = Q_N + Q_D + Q_F = 0$

If we assume inversion.

$$Q_D = -qN_A x_d = -qN_A\sqrt{2\varepsilon_s \phi_s / qN_A}$$

$$\phi_s = 2\phi_F, \quad \phi_F = \frac{kT}{q}\ln\left(\frac{N_A}{n_i}\right) = 0.3578\ \text{V}$$

$$Q_D = -\sqrt{2q\varepsilon_s N_A (2\phi_F)} = -4.890\times10^{-8}\ \text{c/cm}^2$$

Since this charge is less than Q_F, the remainder of the semiconductor charge must be in the inversion layer

$\therefore |Q_N| > 0$ and surface potential $\phi_s = 2\phi_F = 2\times 0.3578$
$= 0.7156\ \text{V}$

Q2

17.2

(a)

$$\phi_F = \frac{kT}{q}\ln(N_A/n_i) = 0.0259\ \ln(10^{15}/10^{10}) = 0.298\text{V}$$

$$V_T = 2\phi_F + \frac{K_S x_o}{K_O}\sqrt{\frac{4qN_A}{K_S\varepsilon_0}\phi_F} \qquad \text{...(17.1a)}$$

$$= (2)(0.298) + \frac{(11.8)(5\times10^{-6})}{(3.9)}\left[\frac{(4)(1.6\times10^{-19})(10^{15})(0.298)}{(11.8)(8.85\times10^{-14})}\right]^{1/2}$$

$$\boldsymbol{V_T = 0.800\ \text{V}}$$

(b) In the square-law theory

$$I_{Dsat} = \frac{Z\bar{\mu}_n C_o}{2L}(V_G - V_T)^2 \qquad \text{...(17.22)}$$

$$C_o = \frac{K_O\varepsilon_0}{x_o} = \frac{(3.9)(8.85\times10^{-14})}{(5\times10^{-6})} = 6.90\times10^{-8}\ \text{F/cm}^2$$

$$I_{Dsat} = \frac{(5\times10^{-3})(800)(6.9\times10^{-8})(2-0.8)^2}{(2)(5\times10^{-4})} = 0.397\text{mA}$$

(c) In the bulk-charge theory we must first determine V_{Dsat} using Eq.(17.29). We know ϕ_F and V_T from part (a), but must compute V_W before substituting into the V_{Dsat} expression.

$$W_T = \left[\frac{2K_S\varepsilon_0}{qN_A}(2\phi_F)\right]^{1/2} = \left[\frac{(2)(11.8)(8.85\times10^{-14})(2)(0.298)}{(1.6\times10^{-19})(10^{15})}\right]^{1/2} = 0.882\mu\text{m}$$

$$V_W \equiv \frac{qN_A W_T}{C_o} = \frac{(1.6\times10^{-19})(10^{15})(8.82\times10^{-5})}{(6.90\times10^{-8})} = 0.205\text{V}$$

Noting that $V_G - V_T = 1.20\text{V}$, substituting into Eq.(17.29) then gives

$$V_{Dsat} = 1.20 - 0.205\left\{\left[\frac{(1.20)}{(2)(0.298)} + \left(1 + \frac{(0.205)}{(4)(0.298)}\right)^2\right]^{1/2} - \left[1 + \frac{(0.205)}{(4)(0.298)}\right]\right\}$$

or

$$V_{Dsat} = 1.06\text{V} \qquad \text{...smaller than } V_{Dsat} \text{ of square-law theory as expected}$$

Now

$$\frac{Z\bar{\mu}_n C_o}{L} = \frac{(5\times10^{-3})(800)(6.90\times10^{-8})}{(5\times10^{-4})} = 5.52\times10^{-4}\ \text{amps/V}^2$$

Finally, substituting into Eq.(17.28) gives I_{Dsat} if $V_D = V_{Dsat}$. Thus

$$I_{Dsat} = (5.52\times10^{-4})\left\{(1.20)(1.06) - \frac{(1.06)^2}{2} - \frac{4}{3}(0.205)(0.298)\left[\left(1+\frac{(1.06)}{(2)(0.298)}\right)^{3/2} - \left(1+\frac{(3)(1.06)}{(4)(0.298)}\right)\right]\right\}$$

$I_{Dsat} = \mathbf{0.349mA}$ ⇐bulk charge result (smaller than the square-law result as expected)

(d) Clearly here the device is biased below pinch-off. From Table 17.1 we note that both the square-law and bulk-charge theories reduce to the same result if $V_D = 0$.

$$g_d = \frac{Z\bar{\mu}_n C_o}{L}(V_G - V_T) = (5.52\times10^{-4})(2-0.8) = \mathbf{0.662mS}$$

(e) In the square-law theory, $V_{Dsat} = V_G - V_T$. Thus $V_{Dsat} = 1.20\text{V}$ and $V_D = 2\text{V}$. Since $V_D > V_{Dsat}$, the device is saturation (above-pinch-off) biased, and from Table 17.1

$$g_m = \frac{Z\bar{\mu}_n C_o}{L}(V_G - V_T) = \mathbf{0.662mS} \qquad \text{...same as } g_d \text{ of part (d)}$$

(f) In part (c) we calculated the bulk-charge $V_{Dsat} = 1.06\text{V}$. Since $V_D > V_{Dsat}$, the device is above-pinch-off biased, and from Table 17.1

$$g_m = \frac{Z\bar{\mu}_n C_o}{L}V_{Dsat} = (5.52\times10^{-4})(1.06) = \mathbf{0.585mS}$$

(g) For the applied $V_G = 2\text{V}$, $V_{Dsat} = 1.20\text{V}$ in the square-law theory and $V_{Dsat} = 1.06\text{V}$ in the bulk-charge theory. Since in either case $V_D < V_{Dsat}$, we can utilize the second form of Eq.(17.37).

$$f_{max} = \frac{\bar{\mu}_n V_D}{2\pi L^2} = \frac{(800)(1)}{(2\pi)(5\times10^{-4})^2} = \mathbf{509MHz}$$

Q3

17.9

(a) From Fig. P17.9 we note in general that

$$V_G = V_D + V_B \qquad \text{or} \qquad V_D = V_G - V_B$$

In the square-law formulation $V_{Dsat} = V_G - V_T$. If $V_B = V_T/2$, then $V_D = V_G - V_T/2 > V_{Dsat}$ and the MOSFET is *always biased into saturation.* Noting $I_D = 0$ if $V_G < V_T$ or $V_D < V_T/2$, and using Eq.(17.22), we conclude

$$I_D = \frac{Z\bar{\mu}_n C_o}{2L}(V_G - V_T)^2 = \frac{Z\bar{\mu}_n C_o}{2L}(V_D - V_T/2)^2 \qquad \ldots V_D > V_T/2$$

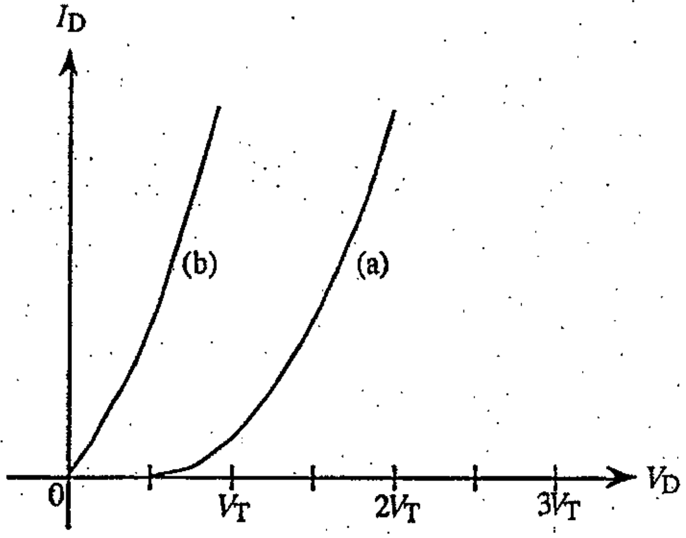
and

$$I_D = 0 \qquad \ldots V_D < V_T/2$$

(b) If $V_B = 2V_T$, then $V_D = V_G - 2V_T < V_G - V_T = V_{Dsat}$ and the MOSFET is always biased in the linear region of operation. The device turns on for $V_G > V_T$ or $V_D > -V_T$ and is therefore on for all $V_D \geq 0$. Using Eq.(17.17) we obtain

$$I_D = \frac{Z\bar{\mu}_n C_o}{L}\left[(V_G - V_T)V_D - V_D^2/2\right] = \frac{Z\bar{\mu}_n C_o}{L}\left[(V_D + V_T)V_D - V_D^2/2\right]$$

$$= \frac{Z\bar{\mu}_n C_o}{L}\left(V_D^2/2 + V_T V_D\right) = \frac{Z\bar{\mu}_n C_o}{2L}\left[(V_D + V_T)^2 - V_T^2\right] \qquad \ldots V_D \geq 0$$



Note that both curves have the same general shape; the part (b) curve is simply shifted to the left and displaced downward.

Q4 A smaller channel length is impacted by depletion region caused by drain-source bias. The channel length is shortened by the depletion region making it easier to turn-on the transistor. Use of shallow doping is one effective way to reduce this problem.

Q5 Sub-threshold swing can be calculated as follows.

$$\frac{10 I_D}{I_D} = \frac{e^{qV'_{GS}/\eta KT}}{e^{qV_{GS}/\eta KT}}$$

$$\Rightarrow 10 = e^{q\Delta V_{GS}/\eta KT}$$

$$\cancel{\Rightarrow V_{GS} =} \Rightarrow \Delta V_{GS} = \eta \frac{KT}{q} \ln 10 = \eta \times 60 \text{ mV/decade}$$

Since $\eta \geq 1$, the minimum value of sub-threshold swing must be at least 60 mV/decade.