Outline

1) Square law/ simplified bulk charge theory
2) Velocity saturation in simplified theory
3) Few comments about bulk charge theory, small transistors
4) Flat band voltage - What is it and how to measure it?
5) Threshold voltage shift due to trapped charges
6) Conclusion

Ref: Sec. 16.4 of SDF Chapter 18, SDF
Post-Threshold MOS Current ($V_G > V_{th}$)

$$I_D = -\frac{W}{L_{ch}} \mu_{eff} \int_{0}^{V_{DS}} Q_i(V) dV$$

Formula overview – derivation to follow

1) Square Law
$$Q_i(V) = -C_G \left[ V_G - V_T - V \right]$$

2) Bulk Charge
$$Q_i(V) = -C_G \left[ V_G - V_{TB} - 2\psi_B - V - \sqrt{\frac{2q\varepsilon_S N_A (2\phi_B + V)}{C_D}} \right]$$

3) Simplified Bulk Charge
$$Q_i(V) = -C_G \left[ V_G - V_T - mV \right]$$

4) “Exact” (Pao-Sah or Pierret-Shields)

Effect of Gate Bias

Gated doped or p-MOS with adjacent n$^+$ region
a) gate biased at flat-band
b) gate biased in inversion
The Effect of Drain Bias

2D band diagram for an n-MOSFET

a) device
b) equilibrium (flat band)
c) equilibrium ($\psi_S > 0$)
d) non-equilibrium with $V_G$ and $V_D > 0$ applied

Pao and Sah.

Effect of a Reverse Bias at Drain

Gated doped or p-MOS with adjacent, reverse-biased n$^+$ region
a) gate biased at flat-band
b) gate biased in depletion
c) gate biased in inversion

Inversion Charge in the Channel

\[ Q_i = -C_{ox} (V_G - V_{th} - V) + qN_A (W_T(V) - W_T(V = 0)) \]

Due to drain bias, additional gate voltage (as compared to threshold in MOSCAP) is now needed to invert the channel throughout its length.

Inversion Charge at one point in Channel

\[ V_{th} = 2\phi_F - \frac{qN_A W_T(V = 0)}{C_{ox}} \]

\[ V_{th}^* = (2\phi_F + V) - \frac{qN_A W_T(V)}{C_{ox}} \]

\[ V_{th}^* = V_{th} + V - \frac{qN_A (W_T(V) - W_T(V = 0))}{C_{ox}} \]

\[ Q_i = -C_{ox} (V_G - V_{th}^*) \]

Threshold voltage in the presence of drain bias.
Approximations for Inversion Charge

\[ Q_i = -C_o(V_G - V_{th} - V) + qN_A(W_f(V) - W_f(V = 0)) \]

\[ = -C_o(V_G - V_{th} - V) + \left[ \sqrt{2q\kappa_s\varepsilon_o N_A(2\phi_B + V)} - \sqrt{2q\kappa_s\varepsilon_o N_A(2\phi_B)} \right] \]

Approximations:

\[ Q_i = -C_{ox}(V_G - V_{th} - V) \quad \text{Square law approximation …} \]

\[ Q_i = -C_{ox}(V_G - V_{th} - mV) \quad \text{Simplified bulk charge approximation …} \]

The MOSFET

\( F_n = F_p = E_F \)

\( F_n \) increasingly negative from source to drain (reverse bias increases from source to drain)
Charge along the channel ....

\[ n = N_c e^{-(E_c - F_n) \beta} \quad p = N_c e^{(E_p - F_p) \beta} \]
Depletion into the channel...

\[ n = N_C e^{-(E_C - F_n) \beta} \quad p = N_C e^{(E_V - F_p) \beta} \]

Depletion into the channel....

\[ n = N_C e^{-(E_C - F_n) \beta} \quad p = N_C e^{(E_V - F_p) \beta} \]
Depletion into the channel ...

Another view of Channel Potential

Source
N+

X
P-doped

Drain
N+

E_C
E_F
E_V
**Square Law Theory**

\[ J_1 = Q_1 \mu \mathcal{E}_1 = Q_1 \mu \frac{dV}{dy} \]

\[ J_2 = Q_2 \mu \mathcal{E}_2 = Q_2 \mu \frac{dV}{dy} \]

\[ J_3 = Q_3 \mu \mathcal{E}_3 = Q_3 \mu \frac{dV}{dy} \]

\[ J_4 = Q_4 \mu \mathcal{E}_4 = Q_4 \mu \frac{dV}{dy} \]

\[ \sum_{i=1}^{N} J_{dy} \mu = \sum_{i=1}^{N} Q_i dV \]

\[ J_D = \frac{\mu C_m}{L_{ch}} \left[ (V_G - V_m)V_D - m \frac{V_D^2}{2} \right] \]

**Purdue** Klimeck - ECE606 Fall 2012 - notes adopted from Alam

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**Square Law or Simplified Bulk Charge Theory**

\[ I_D = W \frac{\mu C_m}{L_{ch}} \left[ (V_G - V_m)V_D - m \frac{V_D^2}{2} \right] \]

\[ \frac{dI_D}{dV} = 0 = (V_G - V_m) - mV_D \Rightarrow V_{D,sat} = \left( V_G - V_m \right)/m \]

\[ I_D = \frac{W \mu C_m}{2mL_{ch}} (V_G - V_T)^2 \]

\[ I_D = \mu C_m \frac{W}{L} (V_G - V_T) V_D \]

**Purdue** Klimeck - ECE606 Fall 2012 - notes adopted from Alam

Unphysical, since current does not decrease with increase in

Region of validity for the expression for currents
Why square law? And why does it become invalid

\[ V_{DSAT} = \frac{(V_{GS} - V_F)}{m} \]

\[ I_D = \frac{W \mu C_m}{2mL_c} (V_G - V_F)^2 \]

\[ Q_i \approx Q_m (V_G - V_{th} - mV) \]

This situation doesn’t arise since electrons travelling from left to right are swept into the drain under the effect of the reverse bias applied.

---

Linear Region (Low \( V_{DS} \))

\[ I_D = W \frac{\mu C_m}{L_c} \left[ (V_G - V_{th})V_D - m\frac{V_D^2}{2} \right] \]

\[ I_D = \frac{W \mu C_m}{L_c} (V_G - V_F) \]

Actual

Slope gives mobility

Mobility degradation at high \( V_{GS} \)

\[ V_{DS} \] small

Subthreshold Conduction

Intercept gives \( V_T \)

Can get \( V_T \) also from C-V
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Ref: Sec. 16.4 of SDF  Chapter 18, SDF

Velocity vs. Field Characteristic (electrons)

Velocity saturates at high fields because of scattering

\[ \nu = \mu E \]

\[ \nu = \nu_{sat} \]

\[ \nu_d = \frac{-\mu E}{1 + \left( \frac{E}{E_c} \right)^2}^{1/2} \]

\[ \nu_d = \frac{-\mu E}{1 + \left( \frac{E}{E_c} \right)} \]

\[ \nu_{d, sat} = \mu E_c \]

This expression can be used to re-derive the expression for current since since mobility is now, in principle, a function of distance
Recap - derivation for MOSFET current

\[ J_1 = Q_1 \mu_1 \xi_1 = Q_1 \mu_1 \frac{dV}{dy}_1 \]
\[ J_2 = Q_2 \mu_2 \xi_2 = Q_2 \mu_2 \frac{dV}{dy}_2 \]
\[ J_3 = Q_3 \mu_3 \xi_3 = Q_3 \mu_3 \frac{dV}{dy}_3 \]
\[ J_4 = Q_4 \mu_4 \xi_4 = Q_4 \mu_4 \frac{dV}{dy}_4 \]
\[ \Rightarrow \sum_{i=1,N} \frac{J_i dy}{\mu(y)} = \sum_{i=1,N} Q_i dV \]

Velocity Saturation

\[ J_D = \int_0^{L_D} \frac{dy}{\mu_0} \left[ 1 + \frac{1}{\xi} \frac{dV}{dy} \right] C_m \left( V_G - V_{th} \right) V_D - \frac{mV_D}{2} \]
\[ \frac{J_D}{\mu_0} \int_0^{L_D} dy \left[ 1 + \frac{1}{\xi} \frac{dV}{dy} \right] = C_m \left( V_G - V_{th} \right) V_D - \frac{mV_D}{2} \]
\[ \int_0^{L_D} J_D dy + \int_0^{V_D} V_D dV_D = C_m \left( V_G - V_{th} \right) V_D - \frac{mV_D}{2} \]
\[ J_D = \frac{\mu_0 C_m}{L_\alpha + \frac{V_D}{\xi}} \left( V_G - V_{th} \right) V_D - \frac{mV_D}{2} \]
Calculating $V_{DSAT}$

\[
\frac{dI_D}{dV_{DS}} = 0
\]
\[
I_D = \frac{\mu_C D}{L_{ch} + \frac{V_D}{E_c}} \left( V_G - V_{th} \right) V_D - \frac{m V_D^2}{2}
\]

Take log on both sides and then set the derivative to zero ....

\[
V_{DSAT} = \frac{2(V_G - V_{th}) / m}{1 + \sqrt{1 + 2\mu_c (V_G - V_{th}) / m V_{sat} L_{ch}}} < \frac{(V_{GS} - V_T)}{m}
\]
Velocity Saturation in short channel devices

\[ J_{D,sat} = \frac{\mu_{e} C_{ox}}{L_{th} + V_{D,sat}} \left[ (V_{G} - V_{th}) V_{D,sat} - \frac{mV_{D,sat}^2}{2} \right] \]

\[ - \frac{\mu_{e} E_{C,ox}}{V_{D,sat}} \left[ (V_{G} - V_{th}) V_{D,sat} - \frac{mV_{D,sat}^2}{2} \right] v_{sat} C_{ox} (V_{G} - V_{th}) \]

This expression can be derived by plugging in the value of \( V_{D,sat} \) for the short channel regime.

\[ I_{D} = W C_{ox} v_{sat} (V_{G} - V_{th}) \]

\[ V_{DSAT} = \frac{2 (V_{G} - V_{th}) / m}{1 + \sqrt{1 + 2 \mu_{0} (V_{G} - V_{th}) / m v_{sat} L_{th}}} \]

\[ V_{DSAT} \rightarrow \sqrt{2 v_{sat} L_{th} (V_{G} - V_{th}) / m \mu_{0}} \]

\[ I_{DSAT} = W C_{ox} v_{sat} (V_{G} - V_{th}) \frac{\sqrt{1 + 2 \mu_{0} (V_{G} - V_{th}) / m v_{sat} L_{th}} - 1}{\sqrt{1 + 2 \mu_{0} (V_{G} - V_{th}) / m v_{sat} L_{th}} + 1} \]

\[ I_{DSAT} = W C_{ox} v_{sat} (V_{G} - V_{th}) \]

Complete velocity saturation, 
Current independent of \( L \)
‘Signature’ of Velocity Saturation

\[ I_D = \frac{W}{2L_{th}} \mu_0 C_{ox} (V_G - V_{th})^2 \]

\[ I_D = W \nu_{sat} C_{ox} (V_G - V_{th}) \]

Can pull out oxide thickness from experimental curves... How?

\[ I_D \text{ and } (V_{GS} - V_T): \text{ In practice .....} \]

\[ I_D(V_D = V_{DD}) \sim (V_G - V_{th})^\alpha \]

Complete velocity saturation

Long channel

1 < \alpha < 2
Approximations for Inversion Charge

\[ Q_i = -C_{ox}(V_G - V_{th} - V) + qN_A(W_f(V) - W_f(V = 0)) \]

\[ = -C_{ox}(V_G - V_{th} - V) + \sqrt{2q\kappa_s\varepsilon_r N_A(2\phi_b + V)} - \sqrt{2q\kappa_s\varepsilon_r N_A(2\phi_b)} \]

Approximations:

\[ Q_i = -C_{ox}(V_G - V_{th} - V) \quad \text{Square law approximation …} \]

\[ Q_i = -C_{ox}(V_G - V_{th} - mV) \quad \text{Simplified bulk charge approximation …} \]

One could substitute the expression for \( Q_i \) above explicitly instead of using \( m \) to simplify the equation, resulting in a more complete bulk charge expression.
Complete Bulk-charge Theory

Additional V dependent terms abstracted into \( m \) previously

\[
\frac{J_D}{\mu_0} \sum_{i=1,N} dy = \int_0^{V_D} C_o (V_G - V_{th} - V) dV + \int_0^{V_D} \left[ \ldots \right] dV
\]

\[
\frac{J_D}{\mu_0} \int_0^{L_{ch}} dy = \int_0^{V_D} C_o (V_G - V_{th} - V) dV + \int_0^{V_D} \left[ \ldots \right] dV
\]

\[
J_D = \frac{\mu_0 C_{ox}}{L_{ch}} \left[ (V_G - V_{th}) V_D - \frac{V_D^2}{2} - \frac{4 q N_s W_T}{3 C_o} \phi_f \left( 1 + \frac{V_D}{2 \phi_f} \right)^{3/2} - \left( 1 + \frac{3V_D}{4 \phi_f} \right) \right]
\]

(Eq. 17.28 in SDF) --- Explicit dependence on bulk doping

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Velocity Overshoot

\[ \nu = \mu_n (E) E \] Valid for bulk semiconductors, not valid at top of the barrier
Intermediate Summary

1) Velocity saturation is an important consideration for short channel transistors (e.g., $V_D=1\text{V}$, $L_{ch}=20\text{nm}$). Therefore, $\alpha \sim 1$ for most modern transistors.

2) Bulk charge theory explains why MOSFET current depends on substrate (bulk) doping. In the simplified bulk charge theory, doping dependence is encapsulated in $m$.

3) Additional considerations of velocity overshoot could complicate calculation of current.

4) Good news is that for very short channel transistors, electrons travel from source to drain without scattering. A considerably simpler ‘Lundstrom theory of MOSFET’ applies.

Frank, Laux, and Fischetti, IEDM Tech. Dig., p. 553, 1992
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\[ I_D(V_D = V_{DD}) \sim (V_G - V_{th})^\alpha \]
1 < \alpha < 2

Ref: Sec. 16.4 of SDF Chapter 18, SDF

\[ V_{th} = V_{th,\text{ideal}} + \phi_{MS} - \frac{\gamma M Q_M}{C_O} - \frac{Q_F}{C_O} - \frac{Q_{II}(\phi_s)}{C_O} \]

In the idealized MOS capacitor, the Fermi Levels in metal and semiconductor align perfectly so that at zero applied bias, the energy bands are flat.

Recall that

\[ Q_i = C_{ox}(V_G - V_{th,\text{ideal}}) \]

\[ V_{th,\text{ideal}} = \psi_s - \frac{Q_0}{C_{ox}} \bigg|_{V_F = 2\phi_F} \]
Potential, Field, Charges

No built in potential, fields or charges at zero applied bias in the idealized MOS structure

Real MOS Capacitor with $\Phi_M < \Phi_S$

In reality, the metal and semiconductor Fermi Levels are never aligned perfectly \( \Rightarrow \) when you bring them together there is charge transfer from the bulk of the semiconductor to the surface so that we have alignment

Do we need to apply less or more $V_G$ to invert the channel?
How to Calculate Built-in or Flat-band Voltage

The presence of a flatband voltage lowers or raises the threshold voltage of a MOS structure. Engineering question: Is it desirable to have a metal having a work function greater or less than the electron affinity $(\chi_s - E_F)$ in the semiconductor?

$$q V_{bi} = (\chi_s + E_F - \Delta_F) - \Phi_M$$

Therefore,

$$Q_f = C_{ox} (V_G - V_{th})$$

$$V_{th} = \left( 2 \phi_f - \frac{Q_f}{C_{ox}} \right) - V_{FB}$$
Measure of Flat-band shift from C-V Characteristics

The transition point between accumulation and depletion in a non-ideal MOS structure is shifted to the left when the metal work function is smaller than the electron affinity \( \gamma (E_c - E_F) \). At zero applied bias the semiconductor is already depleted so that a very small positive bias inverts the channel. The flatband voltage is the amount of voltage required to shift the curve such that the transition point is at zero bias.

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Ref: Sec. 16.4 of SDF Chapter 18, SDF

\[
V_{th} = V_{th,ideal} + \phi_{MS} - \frac{\gamma M Q_M}{C_{ox}} - \frac{Q_F}{C_{ox}} - \frac{Q_{IT}(\phi_t)}{C_{ox}}
\]
Distributed Trapped charge in the Oxide

In the absence of charges in the oxide, the field is constant (dV/dx = constant). The presence of a charge distribution inside the oxide changes the field inside the oxide and effectively traps field lines coming from the gate. As a result, depending on the polarity of charges in the oxide, the threshold voltage is modified.

\[
V_{\text{th}} = \psi^s - \frac{Q_F}{C_{\text{ox}}} - \gamma \frac{Q_M}{C_{\text{ox}}}
\]

\[
Q_M = \int_{x_0}^{x} \rho_{\text{ox}}(x) dx
\]

\[
Q_M = \int_{0}^{x_0} \rho_{\text{ox}}(x) dx
\]

\[
\rho_{\text{ox}}(x) = \frac{Q_M}{x_0 - x}
\]
Kirchoff’s Law – balancing voltages

\[ V^*_G = V^*_{ox} + \psi_s \]

Reduced gate charge

\[ \frac{d^2 V_{ox}}{dx^2} = \frac{dE_{ox}}{dx} = \frac{\rho_{ox}(x)}{\kappa_{ox}E_0} \]

\[ \int \frac{dE_{ox}}{E_{ox}} = \int_0^{\psi_s} \frac{\rho_{ox}(x')dx'}{\kappa_{ox}E_0} \]

Bulk charge

\[ \int \rho_{ox}(x')dx' = \int_0^{\psi_s} \frac{\rho_{ox}(x')dx'}{\kappa_{ox}E_0} \]

Interface charge

\[ \int \rho_{ox}(x')dx' = \int_0^{\psi_s} \frac{\rho_{ox}(x')dx'}{\kappa_{ox}E_0} \]

Ideal charge-free oxide

\[ V_{ox} = \frac{\kappa_{ox}}{\kappa_{ox}^*} \int_0^x \rho_{ox}(x')dx' \]

\[ V_{ox} = \frac{\kappa_{ox}}{\kappa_{ox}^*} \int_0^x \rho_{ox}(x')dx' \]

\[ V_{ox} = \frac{\kappa_{ox}}{\kappa_{ox}^*} \int_0^x \rho_{ox}(x')dx' \]

\[ V_{ox} = \frac{\kappa_{ox}}{\kappa_{ox}^*} \int_0^x \rho_{ox}(x')dx' \]

An Intuitive View

Gate Voltage and Oxide Charge

\[ V^*_G = V^*_{ox} + \psi_s \]

Kirchoff’s Law – balancing voltages

\[ \frac{d^2 V_{ox}}{dx^2} = \frac{dE_{ox}}{dx} = \frac{\rho_{ox}(x)}{\kappa_{ox}E_0} \]

\[ \int \frac{dE_{ox}}{E_{ox}} = \int_0^{\psi_s} \frac{\rho_{ox}(x')dx'}{\kappa_{ox}E_0} \]

Known from boundary conditions in semiconductor and continuity of E

\[ \int \rho_{ox}(x')dx' = \int_0^{\psi_s} \frac{\rho_{ox}(x')dx'}{\kappa_{ox}E_0} \]

\[ \int \rho_{ox}(x')dx' = \int_0^{\psi_s} \frac{\rho_{ox}(x')dx'}{\kappa_{ox}E_0} \]

\[ \int \rho_{ox}(x')dx' = \int_0^{\psi_s} \frac{\rho_{ox}(x')dx'}{\kappa_{ox}E_0} \]

\[ \int \rho_{ox}(x')dx' = \int_0^{\psi_s} \frac{\rho_{ox}(x')dx'}{\kappa_{ox}E_0} \]
Gate Voltage and Oxide Charge

\[
\Delta V_{ox} = \frac{K_s}{K_o} x_0 E_s(x_0) - \int_0^{x_0} x \rho_{ox}(x) \, dx
\]

\[
= \frac{K_s}{K_o} x_0 E_s(x_0) - \frac{1}{C_{ox} x_0} \int_0^{x_0} x \rho_{ox}(x) \, dx
\]

\[
V_{th} = \psi_s(= 2\phi_p) + \Delta V_{ox}
\]

\[
= \psi_s(= 2\phi_p) + \frac{K_s}{K_o} x_0 E_s(x_0) - \frac{1}{C_{ox} x_0} \int_0^{x_0} x \rho_{ox}(x) \, dx
\]

\[
= V_{th,ideal} - \frac{1}{C_{ox} x_0} \int_0^{x_0} x \rho_{ox}(x) \, dx
\]

\[
= V_{th,ideal} - \frac{Q_{st}}{C_{ox}} \gamma_M
\]

Interpretation for Bulk Charge

\[
V_{th} = V_{th,ideal} - \frac{1}{C_{ox} x_0} \int_0^{x_0} x \rho_{ox}(x) \delta(x - x_0) \, dx
\]

\[
= V_{th,ideal} - \frac{x_0 Q_{st}(x_0)}{C_{ox}}
\]
**Interpretation for Interface Charge**

\[ V_{th} = V_{th}^* - \frac{1}{C_{ox}x_0} \int_0^\infty x\rho_{ox}(x)\delta(x - x_c)dx \]

\[ = V_{th}^* \frac{Q_x}{C_p} \]

---

**Time-dependent shift of Trapped Charge**

\[ V_{th} = V_{th,ideal} - \frac{1}{C_{ox}x_0} \int_0^\infty xQ_{ox}(x)\times\delta(x - x_c(t))dx \]

\[ = V_{th,ideal} \frac{x_c(t)Q_{ox}(x)}{x_0} \frac{C_p}{C_{ox}} \]

Sodium related bias temperature instability (BTI) issue
1) Non-ideal threshold characteristics are important consideration of MOSFET design.
2) The non-idealities arise from differences in gate and substrate work function, trapped charges, interface states.
3) Although nonideal effects often arise from transistor degradation, there are many cases where these effects can be used to enhance desirable characteristics.