

ECE606: Solid State Devices

Lecture 23

MOSFET I-V Characteristics

MOSFET non-idealities

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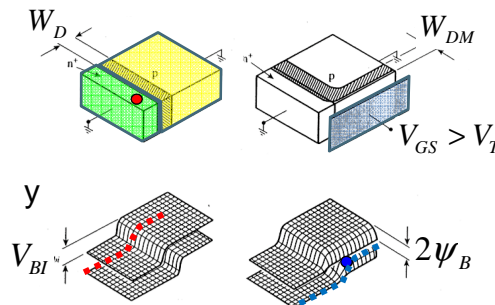
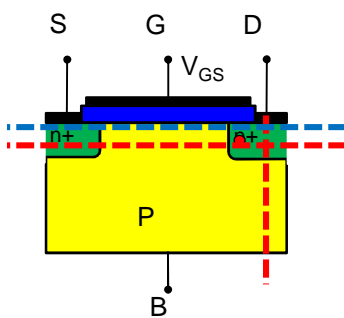
- 1) Square law/ simplified bulk charge theory
- 2) Velocity saturation in simplified theory
- 3) Few comments about bulk charge theory, small transistors
- 4) Flat band voltage - What is it and how to measure it?
- 5) Threshold voltage shift due to trapped charges
- 6) Conclusion

Ref: Sec. 16.4 of SDF Chapter 18, SDF

$$I_D = -\frac{W}{L_{ch}} \mu_{eff} \int_0^{V_{DS}} Q_i(V) dV$$

Formula overview –
derivation to follow

- 1) Square Law $Q_i(V) = -C_G [V_G - V_T - V]$
- 2) Bulk Charge $Q_i(V) = -C_G \left(V_G - V_{FB} - 2\psi_B - V - \frac{\sqrt{2q\epsilon_{Si} N_A (2\phi_B + V)}}{C_o} \right)$
- 3) Simplified Bulk Charge $Q_i(V) = -C_G [V_G - V_T - mV]$
- 4) "Exact" (Pao-Sah or Pierret-Shields)



No source-drain bias

Gated doped or p-MOS with adjacent n⁺ region

- a) gate biased at flat-band
- b) gate biased in inversion

A. Grove, *Physics of Semiconductor Devices*,
1967.

2D band diagram for an n-MOSFET

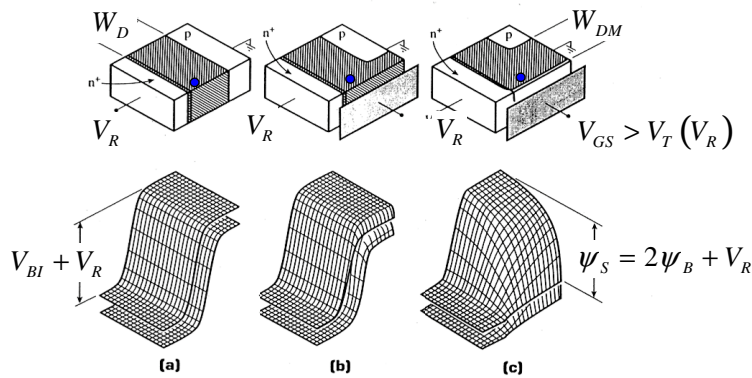
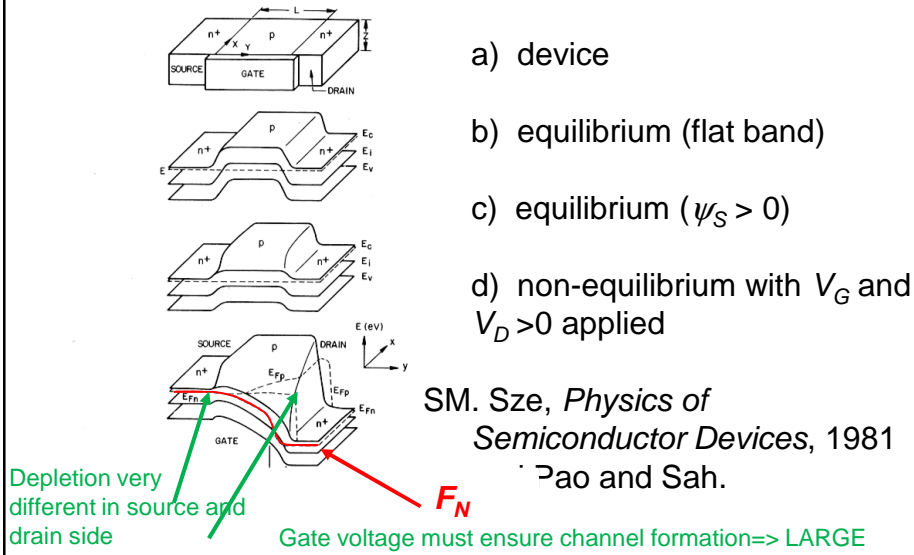


FIGURE 2.34. Gated diode or p-type MOS with adjacent n^+ region under nonequilibrium (reverse)

Gated doped or p-MOS with adjacent, reverse-biased n^+ region

- a) gate biased at flat-band
- b) gate biased in depletion
- c) gate biased in inversion

A. Grove, *Physics of Semiconductor Devices*, 1967.

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Inversion Charge in the Channel

Band diagrams (Gate - Channel - Drain)

$$Q_i = -C_{ox}(V_G - V_{th} - V) + qN_A(W_T(V) - W_T(V = 0))$$

Due to drain bias, additional gate voltage (as compared to threshold in MOSCAP) is now needed to invert the channel throughout its length

Body potential = constant, n+ region potential lowered

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Inversion Charge at one point in Channel

$$V_{th} = 2\phi_F - \frac{qN_A W_T(V = 0)}{C_{ox}}$$

$$V_{th}^* = (2\phi_F + V) - \frac{qN_A W_T(V)}{C_{ox}}$$

$$V_{th}^* = V_{th} + V - \frac{qN_A (W_T(V) - W_T(V = 0))}{C_{ox}}$$

$$Q_i = -C_{ox}(V_G - V_{th}^*)$$

Threshold voltage in the presence of drain bias

PURDUE Klimeck - ECE606 Fall 2012 - notes adopted from Alam

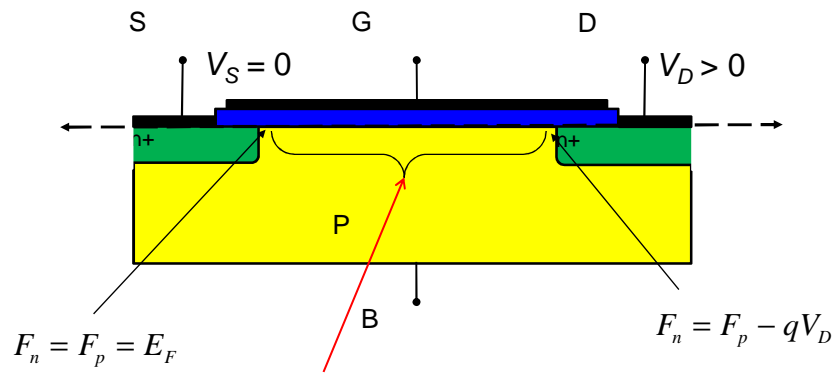
$$Q_i = -C_o(V_G - V_{th} - V) + qN_A(W_T(V) - W_T(V=0))$$

$$= -C_o(V_G - V_{th} - V) + \left[\sqrt{2q\kappa_s\epsilon_o N_A(2\phi_B + V)} - \sqrt{2q\kappa_s\epsilon_o N_A(2\phi_B)} \right]$$

Approximations:

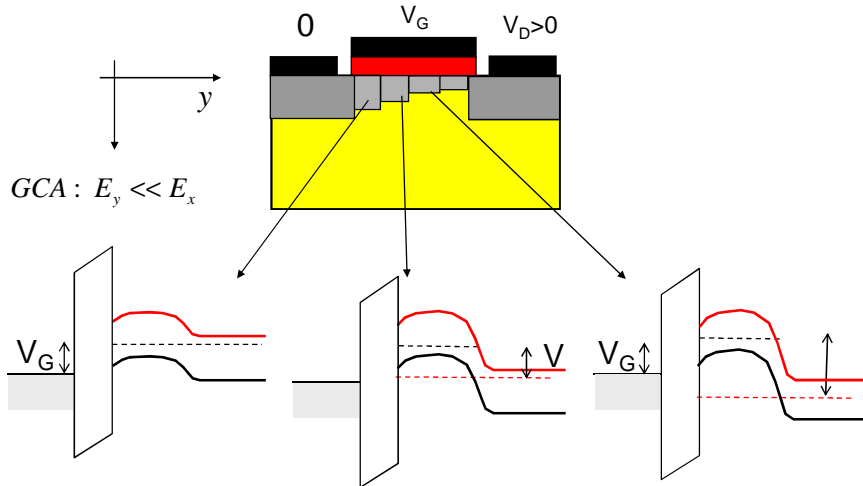
$$Q_i \approx -C_{ox}(V_G - V_{th} - V) \quad \text{Square law approximation ...}$$

$$Q_i \approx -C_{ox}(V_G - V_{th} - mV) \quad \text{Simplified bulk charge approximation ...}$$



F_n increasingly negative from source to drain
(reverse bias increases from source to drain)

Voltage and hence inversion charge vary spatially

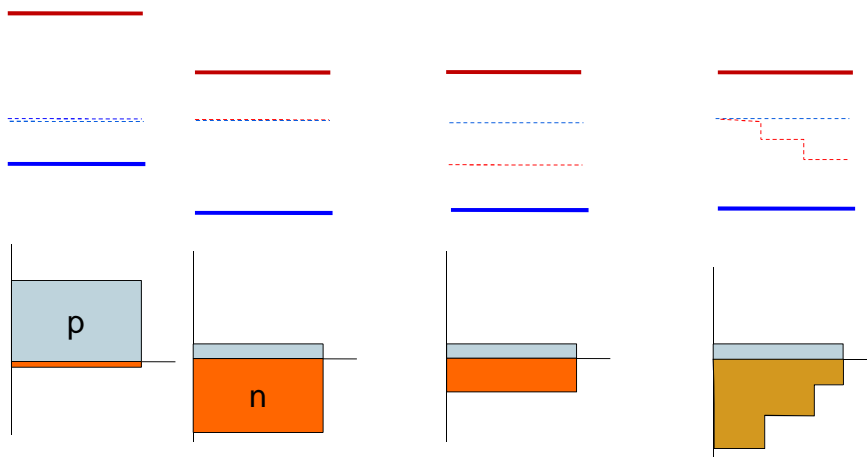


GCA: $E_y \ll E_x$

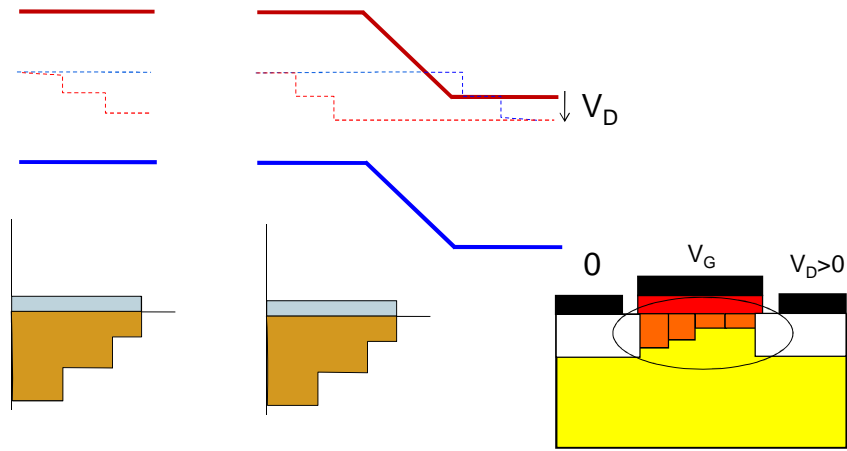
$$Q_i(y) = -C_{ox} [V_G - V_{th} - mV(y)]$$

$$n = N_c e^{-(E_c - F_n)/\beta}$$

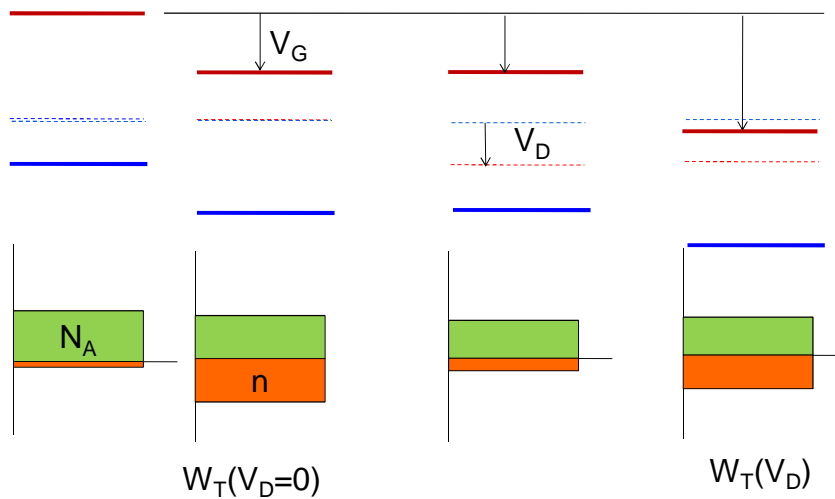
$$p = N_c e^{(E_v - F_p)/\beta}$$

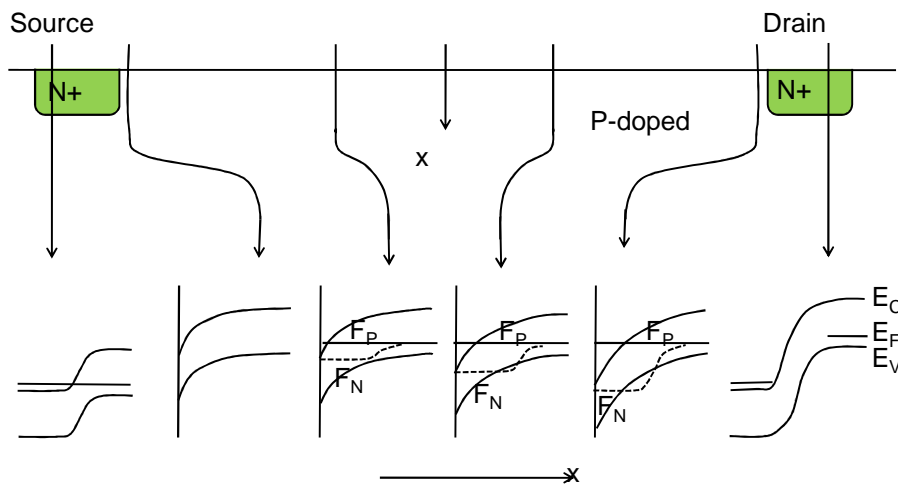
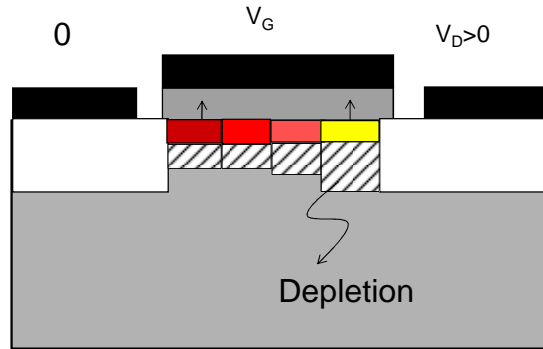


$$n = N_c e^{-(E_c - F_n) \beta} \quad p = N_c e^{(E_v - F_p) \beta}$$



$$n = N_c e^{-(E_c - F_n) \beta} \quad p = N_c e^{(E_v - F_p) \beta}$$





$$J_1 = Q_1 \mu \mathcal{E}_1 = Q_1 \mu \left. \frac{dV}{dy} \right|_1$$

$$J_2 = Q_2 \mu \mathcal{E}_2 = Q_2 \mu \left. \frac{dV}{dy} \right|_2$$

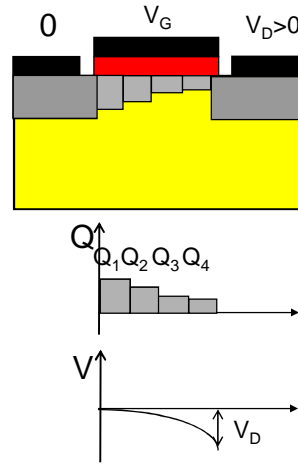
$$J_3 = Q_3 \mu \mathcal{E}_3 = Q_3 \mu \left. \frac{dV}{dy} \right|_3$$

$$J_4 = Q_4 \mu \mathcal{E}_4 = Q_4 \mu \left. \frac{dV}{dy} \right|_4$$

$$\sum_{i=1,N} \frac{J_i dy}{\mu} = \sum_{i=1,N} Q_i dV$$

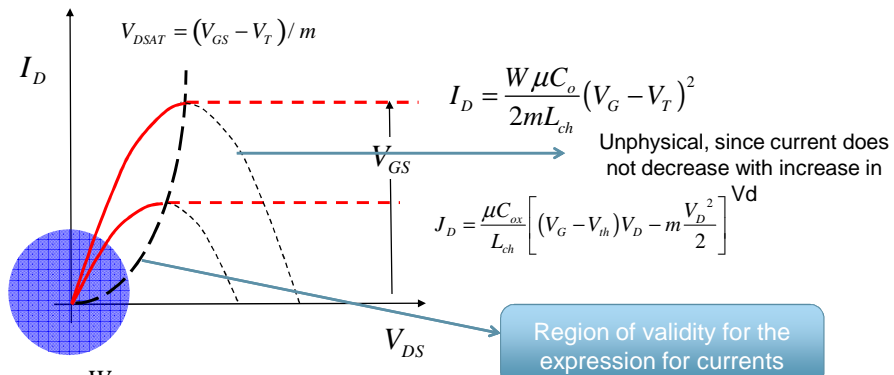
$$\frac{J_D}{\mu} \sum_{i=1,N} dy = \int_0^{V_D} C_{ox} (V_G - V_{th} - mV) dV$$

$$J_D = \frac{\mu C_{ox}}{L_{ch}} \left[(V_G - V_{th}) V_D - m \frac{V_D^2}{2} \right]$$



$$I_D = W \frac{\mu C_{ox}}{L_{ch}} \left[(V_G - V_{th}) V_D - m \frac{V_D^2}{2} \right]$$

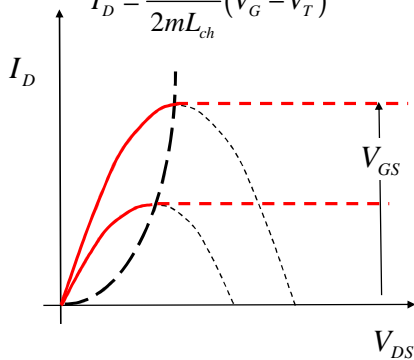
$$\frac{dI_D}{dV} = 0 = (V_G - V_{th}) - mV_D \Rightarrow V_{D,sat} = (V_G^* - V_{th})/m$$



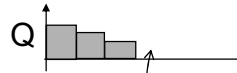
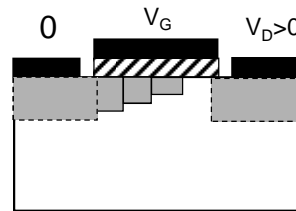
$$I_D = \mu C_o \frac{W}{L} (V_G - V_T) V_D$$

$$V_{DSAT} = (V_{GS} - V_T) / m$$

$$I_D = \frac{W \mu C_o}{2m L_{ch}} (V_G - V_T)^2$$

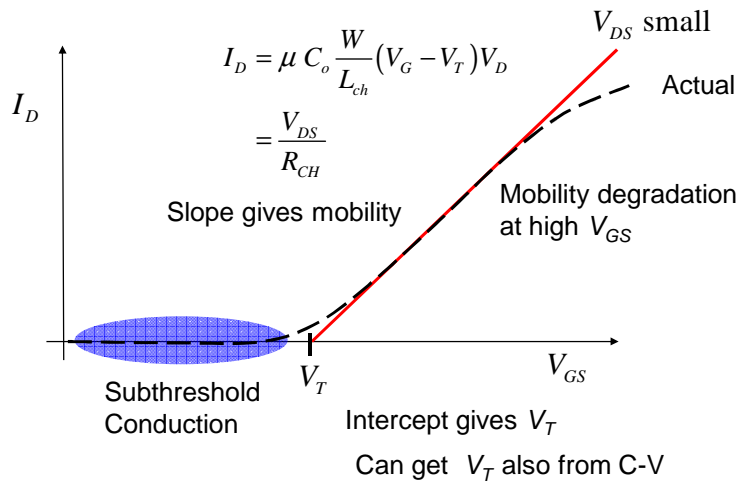


$$Q_i \approx -C_{ox} (V_G - V_{th} - mV)$$



This situation doesn't arise since electrons travelling from left to right are swept into the drain under the effect of the reverse bias applied

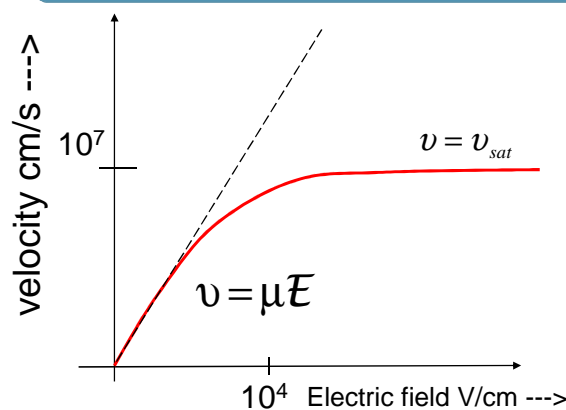
$$I_D = W \frac{\mu C_{ox}}{L_{ch}} \left[(V_G - V_{th}) V_D - m \frac{V_D^2}{2} \right]$$



- 1) Square law/ simplified bulk charge theory
- 2) Velocity saturation in simplified theory**
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Ref: Sec. 16.4 of SDF Chapter 18, SDF

Velocity saturates at high fields because of scattering



$$v_d = \frac{-\mu E}{[1 + (E/E_c)^2]^{1/2}}$$

$$v_d = \frac{-\mu E}{1 + (|E|/E_c)}$$

$$v_{d,sat} = \mu E_c$$

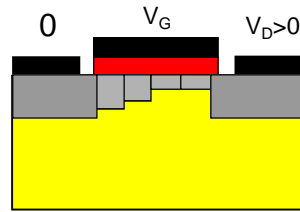
This expression can be used to re-derive the expression for current since since mobility is now, in principle, a function of distance

$$J_1 = Q_1 \mu_1 \mathcal{E}_1 = Q_1 \mu_1 \left. \frac{dV}{dy} \right|_1$$

$$J_2 = Q_2 \mu_2 \mathcal{E}_2 = Q_2 \mu_2 \left. \frac{dV}{dy} \right|_2$$

$$J_3 = Q_3 \mu_3 \mathcal{E}_3 = Q_3 \mu_3 \left. \frac{dV}{dy} \right|_3$$

$$J_4 = Q_4 \mu_4 \mathcal{E}_4 = Q_4 \mu_4 \left. \frac{dV}{dy} \right|_4$$



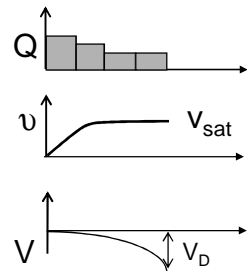
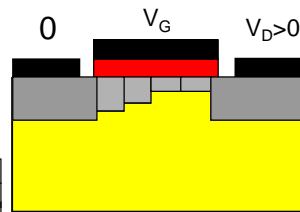
$$\Rightarrow \sum_{i=1,N} \frac{J_i dy}{\mu(y)} = \sum_{i=1,N} Q_i dV$$

$$J_D \sum_{i=1,N} \frac{dy}{\mu_0 \left[1 + \frac{|\mathcal{E}|}{\mathcal{E}_c} \right]} = \int_0^{V_D} C_{ox} (V_G - V_{th} - mV) dV$$

$$\frac{J_D}{\mu_0} \int_0^{L_{ch}} dy \left[1 + \frac{1}{\mathcal{E}_c} \frac{dV}{dy} \right] = C_{ox} \left[(V_G - V_{th}) V_D - \frac{m V_D^2}{2} \right]$$

$$\int_0^{L_{ch}} J_D dy + \int_0^{V_{DS}} \frac{J_D}{\mathcal{E}_c} dV = C_{ox} \left[(V_G - V_{th}) V_D - \frac{m V_D^2}{2} \right]$$

$$J_D = \frac{\mu_0 C_{ox}}{L_{ch} + \frac{V_D}{\mathcal{E}_c}} \left[(V_G - V_{th}) V_D - \frac{m V_D^2}{2} \right]$$



$$J_D = \frac{\mu_0 C_{ox}}{L_{ch} + \frac{V_D}{\mathcal{E}_c}} \left[(V_G - V_{th}) V_D - \frac{m V_D^2}{2} \right]$$

- At very small channel lengths and high drain biases, the current expression becomes independent of the channel length
- In the linear region in the I - V_d characteristics, you have a resistance that doesn't depend on the length of the channel



$$\frac{dI_D}{dV_{DS}} = 0$$

$$\frac{I_D}{W} = \frac{\mu_o C_{ox}}{L_{ch} + \frac{V_D}{\mathcal{E}_c}} \left[(V_G - V_{th}) V_D - m \frac{V_D^2}{2} \right]$$

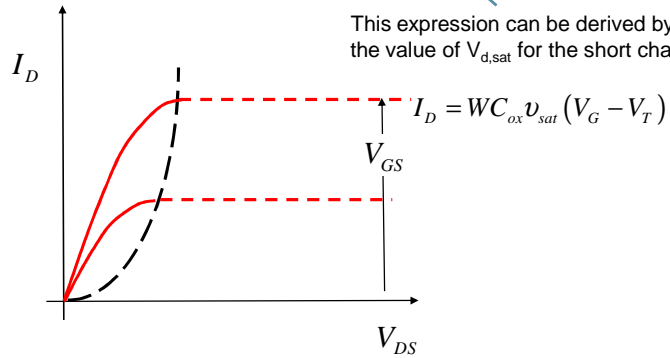
Take log on both sides and then set the derivative to zero

$$V_{DSAT} = \frac{2(V_G - V_{th}) / m}{1 + \sqrt{1 + 2\mu_o (V_G - V_{th}) / m v_{sat} L_{ch}}} < \frac{(V_{GS} - V_T)}{m}$$



$$J_{D,sat} = \frac{\mu_0 C_{ox}}{L_{ch} + \frac{V_{D,sat}}{\epsilon_c}} \left[(V_G - V_{th}) V_{D,sat} - \frac{m V_{D,sat}^2}{2} \right]$$

$$\sim \frac{\mu_0 \epsilon_c C_{ox}}{V_{D,sat}} \left[(V_G - V_{th}) V_{D,sat} - \frac{m V_{D,sat}^2}{2} \right] \sim v_{sat} C_{ox} (V_G - V_{th})$$



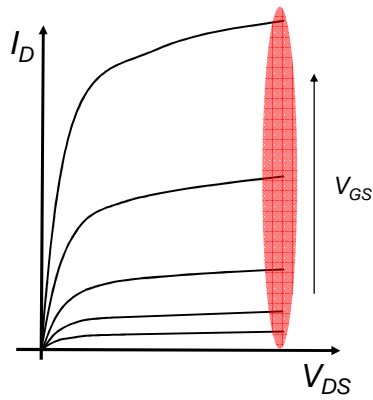
$$V_{DSAT} = \frac{2(V_G - V_{th})/m}{1 + \sqrt{1 + 2\mu_0(V_G - V_{th})/m v_{sat} L_{ch}}}$$

$$V_{DSAT} \rightarrow \sqrt{2v_{sat} L_{ch} (V_G - V_{th})/m\mu_0}$$

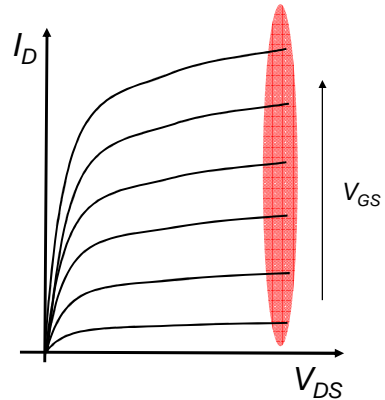
$$I_{DSAT} = W C_{ox} v_{sat} (V_G - V_{th}) \frac{\sqrt{1 + 2\mu_0(V_G - V_{th})/m v_{sat} L_{ch}} - 1}{\sqrt{1 + 2\mu_0(V_G - V_{th})/m v_{sat} L_{ch}} + 1}$$

$$I_{DSAT} = W C_{ox} v_{sat} (V_G - V_{th})$$

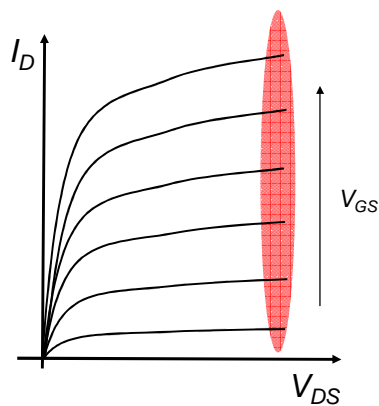
Complete velocity saturation
Current independent of L



$$I_D = \frac{W}{2L_{ch}} \mu_0 C_{ox} \frac{(V_G - V_{th})^2}{m}$$



$$I_D = W v_{sat} C_{ox} (V_G - V_{th})$$



$$I_D(V_D = V_{DD}) \sim (V_G - V_{th})^\alpha$$

$$1 < \alpha < 2$$

Complete velocity saturation

Long channel

- 1) Square law/ simplified bulk charge theory
- 2) Velocity saturation in simplified theory
- 3) Few comments about bulk charge theory, small transistors**
- 4) Flat band voltage - What is it and how to measure it?
- 5) Threshold voltage shift due to trapped charges
- 6) Conclusion

Ref: Sec. 16.4 of SDF Chapter 18, SDF

$$\begin{aligned}
 Q_i &= -C_o(V_G - V_{th} - V) + qN_A(W_T(V) - W_T(V=0)) \\
 &= -C_o(V_G - V_{th} - V) + \sqrt{2q\kappa_s\epsilon_o N_A(2\phi_B + V)} - \sqrt{2q\kappa_s\epsilon_o N_A(2\phi_B)}
 \end{aligned}$$

Approximations:

$$Q_i \approx -C_{ox}(V_G - V_{th} - V) \quad \text{Square law approximation ...}$$

$$Q_i \approx -C_{ox}(V_G - V_{th} - mV) \quad \text{Simplified bulk charge approximation ...}$$

One could substitute the expression for Q_i above explicitly instead of using m to simplify the equation, resulting in a more complete bulk charge expression

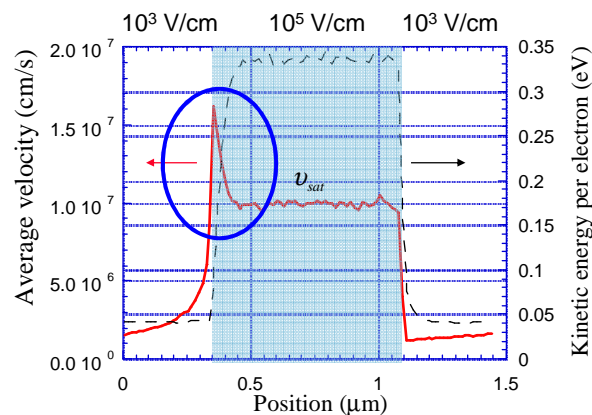
Additional V dependent terms
abstracted into m previously

$$\frac{J_D}{\mu_0} \sum_{i=1,N} dy = \int_0^{V_D} C_O (V_G - V_{th} - V) dV + \int_0^{V_D} [\dots\dots\dots] dV$$

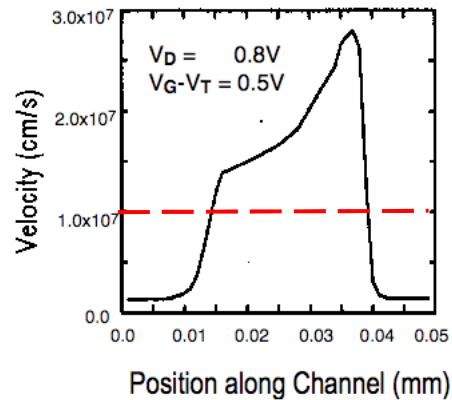
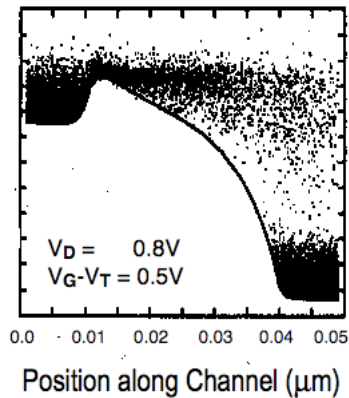
$$\frac{J_D}{\mu_0} \int_0^{L_{ch}} dy = \int_0^{V_D} C_O (V_G - V_{th} - V) dV + \int_0^{V_D} [\dots\dots\dots] dV$$

$$J_D = \frac{\mu_0 C_{ox}}{L_{ch}} \left[(V_G - V_{th}) V_D - \frac{V_D^2}{2} - \frac{4 q N_A W_T}{3 C_O} \phi_F \left\{ \left(1 + \frac{V_D}{2 \phi_F} \right)^{3/2} - \left(1 + \frac{3 V_D}{4 \phi_F} \right) \right\} \right]$$

(Eq. 17.28 in SDF) ... Explicit dependence on bulk doping



$v \neq \mu_n (E) E \longrightarrow$ Valid for bulk semiconductors,
not valid at top of the barrier



Frank, Laux, and Fischetti, IEDM Tech. Dig., p. 553, 1992

- 1) Velocity saturation is an important consideration for short channel transistors (e.g., $V_D=1V$, $L_{ch}=20\text{nm}$). Therefore, $\alpha \sim 1$ for most modern transistors.
- 2) Bulk charge theory explains why MOSFET current depends on substrate (bulk) doping. In the simplified bulk charge theory, doping dependence is encapsulated in m .
- 3) Additional considerations of velocity overshoot could complicate calculation of current.
- 4) Good news is that for very short channel transistors, electrons travel from source to drain without scattering. A considerably simpler 'Lundstrom theory of MOSFET' applies.

- 1) Square law/ simplified bulk charge theory
- 2) Velocity saturation in simplified theory
- 3) Few comments about bulk charge theory, small transistors
- 4) Flat band voltage - What is it and how to measure it?**
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$$I_D(V_D = V_{DD}) \sim (V_G - V_{th})^\alpha$$

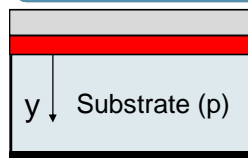
$$1 < \alpha < 2$$

Ref: Sec. 16.4 of SDF Chapter 18, SDF

$$V_{th} = V_{th,ideal} + \phi_{MS} - \frac{\gamma_M Q_M}{C_O} - \frac{Q_F}{C_O} - \frac{Q_{IT}(\phi_s)}{C_O}$$

(1) Idealized MOS Capacitor

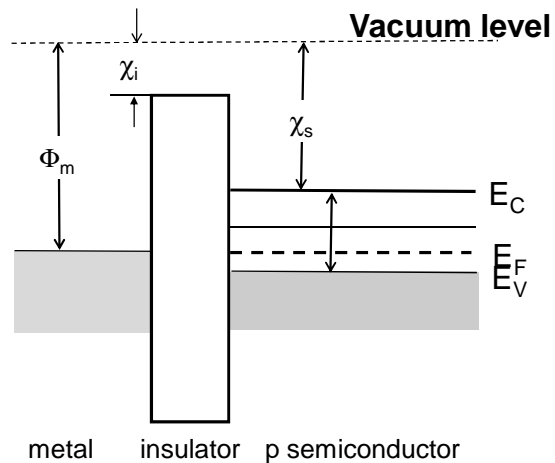
In the idealized MOS capacitor, the Fermi Levels in metal and semiconductor align perfectly so that at zero applied bias, the energy bands are flat

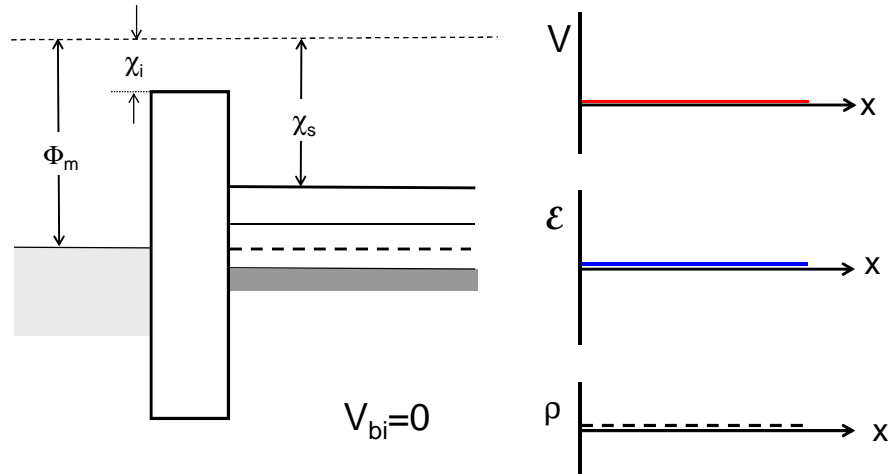


Recall that

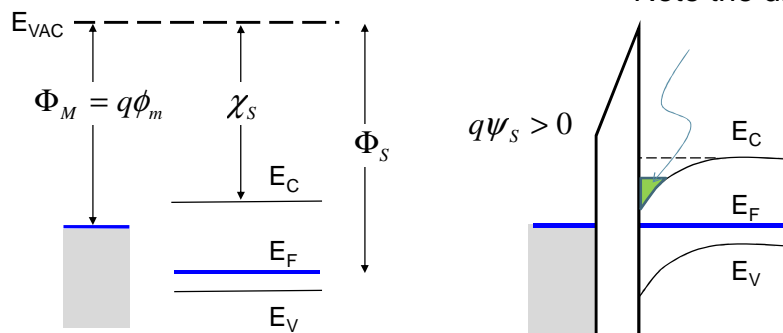
$$Q_i = C_{ox}(V_G - V_{th,ideal})$$

$$V_{th,ideal} = \psi_s - \frac{Q_B}{C_{ox}} \Big|_{\psi_s = 2\phi_F}$$





No built in potential, fields or charges at zero applied bias in the idealized MOS structure

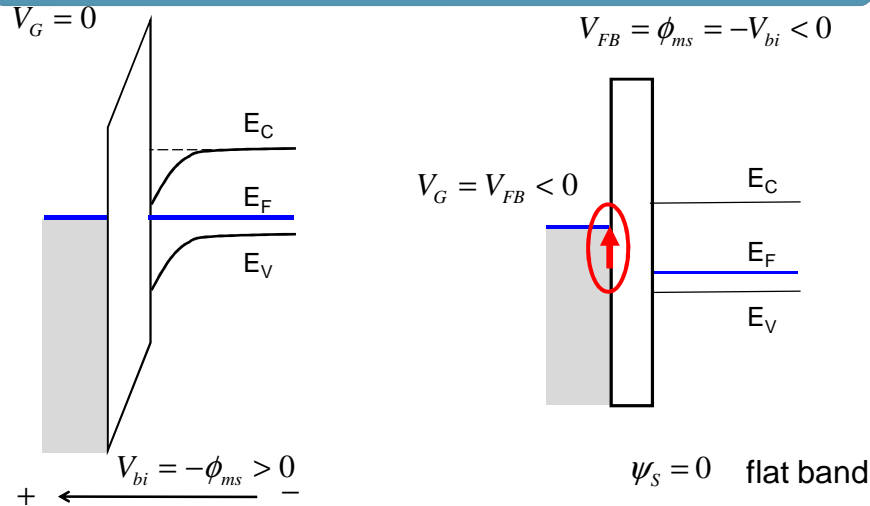


In reality, the metal and semiconductor Fermi Levels are never aligned perfectly → when you bring them together there is charge transfer from the bulk of the semiconductor to the surface so that we have alignment

Do we need to apply less or more V_G to invert the channel ?

Physical Interpretation of Flatband Voltage

The Flatband Voltage is the voltage applied to the gate that gives zero-band bending in the MOS structure. Applying this voltage nullifies the effect of the built-in potential. This voltage needs to be incorporated into the idealized MOS analysis while calculating threshold voltage



How to Calculate Built-in or Flat-band Voltage

The presence of a flatband voltage lowers or raises the threshold voltage of a MOS structure. Engineering question \rightarrow Is it desirable to have a metal having a work function greater or less than the electron affinity + (Ec-Ef) in the semiconductor?

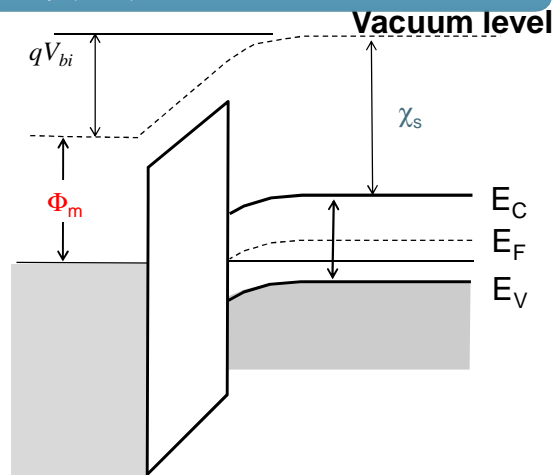
$$qV_{bi} = (\chi_s + E_g - \Delta_p) - \Phi_M$$

$$= qV_{FB} \equiv \phi_{MS}$$

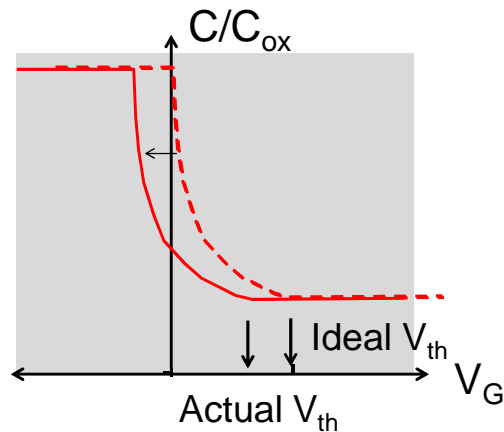
Therefore,

$$Q_i = C_{ox}(V_G - V_{th})$$

$$V_{th} = \left(2\phi_F - \frac{Q_B}{C_{ox}} \right) - V_{FB}$$



The transition point between accumulation and depletion in a non-ideal MOS structure is shifted to the left when the metal work function is smaller than the electron affinity $+(E_c - E_f)$. At **zero applied bias the semiconductor is already depleted** so that a very small positive bias inverts the channel. The flatband voltage is the amount of voltage required to shift the curve such that the transition point is at zero bias.

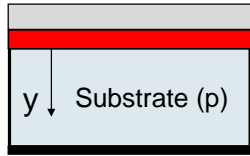


- 1) Square law/ simplified bulk charge theory
- 2) Velocity saturation in simplified theory
- 3) Few comments about bulk charge theory, small transistors
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Ref: Sec. 16.4 of SDF Chapter 18, SDF

$$V_{th} = V_{th,ideal} + \phi_{MS} - \frac{\gamma_M Q_M}{C_{ox}} - \frac{Q_F}{C_{ox}} - \frac{Q_{IT}(\phi_s)}{C_{ox}}$$

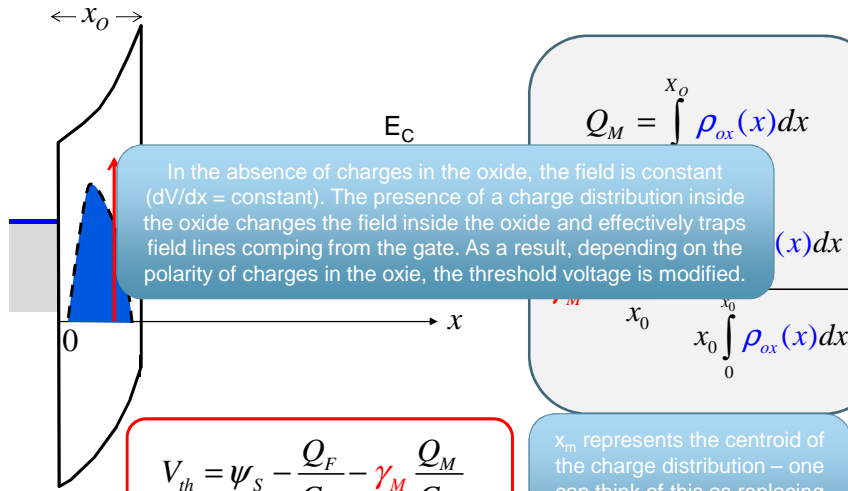
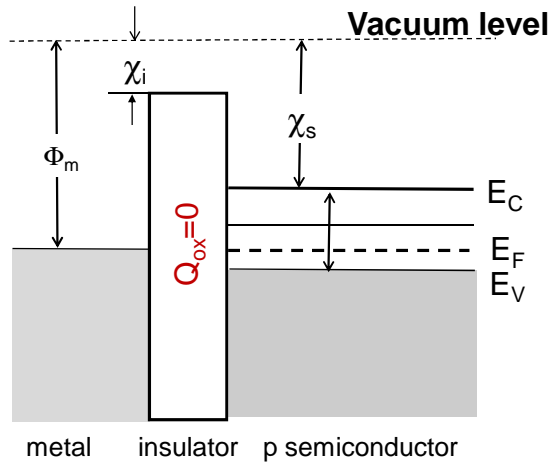




Recall that

$$Q_i = C_{ox} (V_G - V_{th,ideal})$$

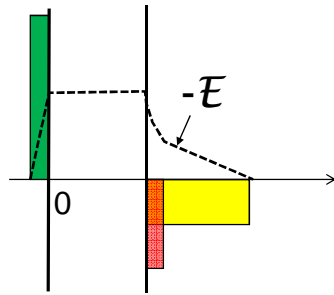
$$V_{th,ideal} = \psi_s - \frac{Q_B}{C_{ox}} \Big|_{\psi_s = 2\phi_F}$$



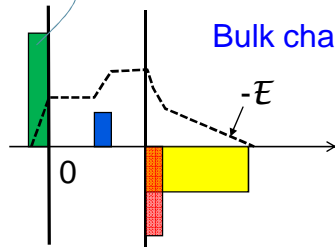
$$V_{th} = \psi_s - \frac{Q_F}{C_{ox}} - \gamma_M \frac{Q_M}{C_{ox}}$$

x_m represents the centroid of the charge distribution – one can think of this as replacing the entire distribution with a delta charge at this point

Ideal charge-free oxide

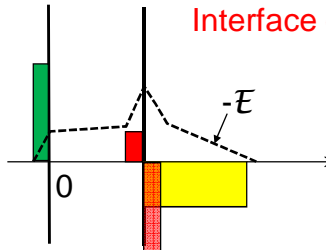


Reduced gate charge



Bulk charge

Interface charge



$$V_G = V_{ox} + \psi_s$$

Kirchoff's Law – balancing voltages

$$\frac{d^2 V_{ox}}{dx^2} = \frac{dE_{ox}}{dx} = \frac{\rho_{ox}(x)}{\kappa_{ox} \epsilon_0}$$

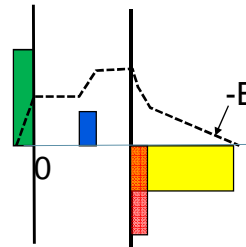
$$\int_{E(x)}^{E(x_0)} dE_{ox} = \int_x^{x_0} \frac{\rho_{ox}(x') dx'}{\kappa_{ox} \epsilon_0}$$

$$\frac{dV_{ox}}{dx} = E_{ox}(x_0) - E_{ox}(x) = E_{ox}(x_0) - \int_0^x \frac{\rho_{ox}(x') dx'}{\kappa_{ox} \epsilon_0}$$

Known from boundary conditions in semiconductor and continuity of E

$$V_{ox} = \frac{\kappa_s}{\kappa_{ox}} x_0 E_s(x_0) - \int_0^{x_0} dx \int_0^x \frac{\rho_{ox}(x') dx'}{\kappa_{ox} \epsilon_0}$$

$$= \frac{\kappa_s}{\kappa_{ox}} x_0 E_s(x_0) - \int_0^{x_0} \frac{x \rho_{ox}(x) dx}{\kappa_{ox} \epsilon_0}$$



$$\Delta V_{ox} = \frac{\kappa_s}{\kappa_{ox}} x_0 \mathcal{E}_s(x_0) - \int_0^{x_0} \frac{x \rho_{ox}(x) dx}{\left(\frac{\kappa_{ox} \epsilon_0}{x_0} \right) x_0}$$

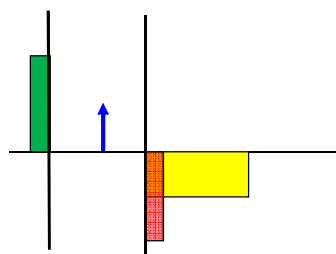
$$= \frac{\kappa_s}{\kappa_{ox}} x_0 \mathcal{E}_s(x_0) - \frac{1}{C_{ox} x_0} \int_0^{x_0} x \rho_{ox}(x) dx$$

$$V_{th} = \psi_s (= 2\phi_F) + \Delta V_{ox}$$

$$= \psi_s (= 2\phi_F) + \frac{\kappa_s}{\kappa_{ox}} x_0 \mathcal{E}_s(x_0) - \frac{1}{C_o x_0} \int_0^{x_0} x \rho_{ox}(x) dx$$

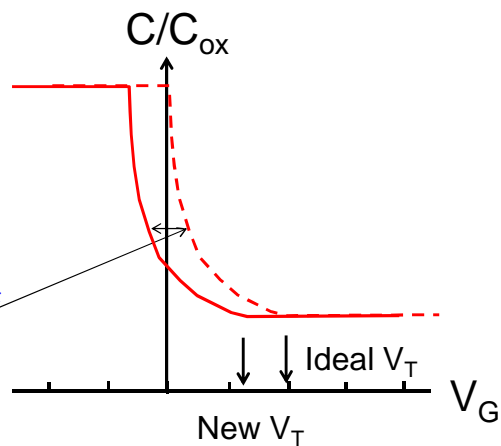
$$= V_{th,ideal} - \frac{1}{C_{ox} x_0} \int_0^{x_0} x \rho_{ox}(x) dx$$

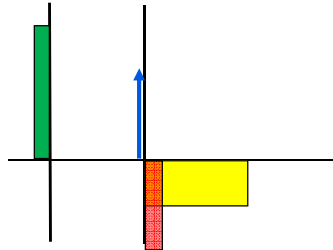
$$= V_{th,ideal} - \frac{Q_M}{C_{ox}} \gamma_M$$



$$V_{th} = V_{th,ideal} - \frac{1}{C_o x_0} \int_0^{x_0} x \rho_{ox}(x) \delta(x - x_1) dx$$

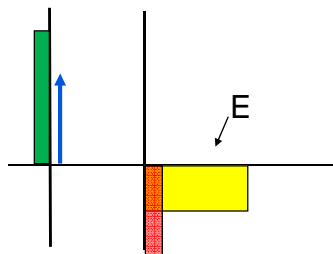
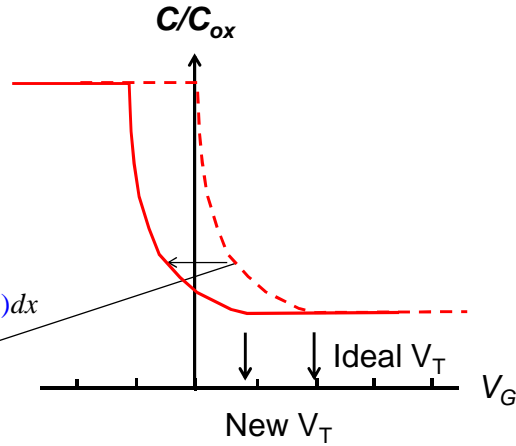
$$= V_{th,ideal} - \frac{x_1 Q_M(x_1)}{x_0 C_o}$$





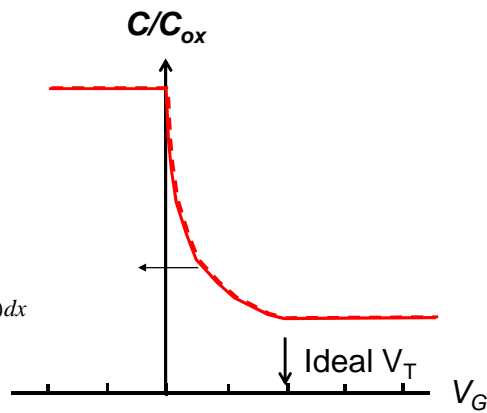
$$V_{th} = V_{th}^* - \frac{1}{C_o x_0} \int_0^{x_0} x \rho_{ox}(x) \delta(x - x_o) dx$$

$$= V_{th}^* - \frac{Q_F}{C_o}$$

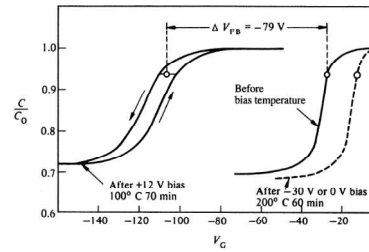
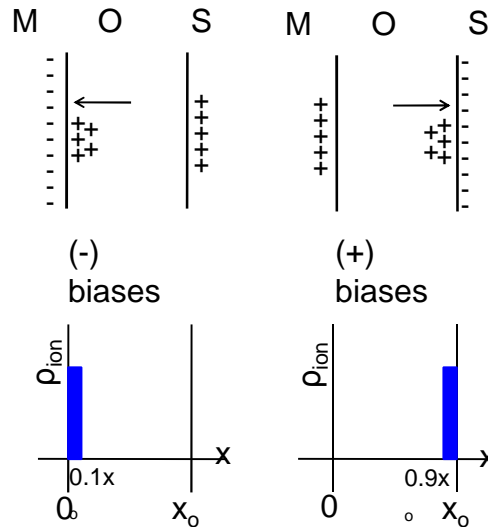


$$V_{th} = V_{th,ideal} - \frac{1}{C_{ox} x_0} \int_0^{x_0} x Q_{ox}(x) \times \delta(x - x_1(t)) dx$$

$$= V_{th,ideal} - \frac{x_1(t)}{x_0} \times \frac{Q_{ox}(x)}{C_{ox}}$$



Sodium related bias temperature instability (BTI) issue



- 1) Non-ideal threshold characteristics are important consideration of MOSFET design.
- 2) The non-idealities arise from differences in gate and substrate work function, trapped charges, interface states.
- 3) Although nonideal effects often arise from transistor degradation, there are many cases where these effects can be used to enhance desirable characteristics.

