ECE606: Solid State Devices
Lecture 22
MOScap Frequency Response
MOSFET I-V Characteristics

Gerhard Klimeck
gekco@purdue.edu

Outline

1. Background
2. Small signal capacitances
3. Large signal capacitance
4. Intermediate Summary
5. Sub-threshold (depletion) current
6. Super-threshold, inversion current
7. Conclusion

Ref: Sec. 16.4 of SDF
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**Small Signal Equivalent Circuit**

- **Gate**
- **semiconductor**
- **G₀** is small (only tunelling current)
- **C₀**
- **C_J**

For insulated devices, consider only majority carrier junction capacitance **C_J**
For insulated devices, consider only majority carrier junction capacitance $C_J$.
Junction Capacitance

\[ \frac{1}{C_G} = \frac{1}{C_S} + \frac{1}{C_o} \]

\[ C_S = \frac{d(-Q_s)}{d\psi_s} \]

\[ Q_s(\psi_s) \]

which we already understand!

Remember the Qs vs phi_s figure we mentioned in the previous lecture

Definition of \( m \) for later use

\[ m = \left(1 + \frac{C_S}{C_O}\right) \]

‘body effect coefficient’

\[ m = \left(1 + \kappa_s x_O / \kappa_0 W_T\right) \]

\( W_T \) depends on the voltage

in practice:

\[ 1.1 \leq m \leq 1.4 \]
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\[ C_{j,\text{acc}} \approx \frac{K_n e_0}{x_0} \equiv C_0 \]
\[ C_{j,\text{acc}} = \frac{C_s C_{s,\text{acc}}}{C_s + C_{s,\text{acc}}} \]
\[ C_{s,\text{acc}} \equiv \frac{K_s e_0}{W_{\text{acc}}} \]

Junction Capacitance in accumulation

Arrows is the charge induced by small signal
Two blue arrow \( C_0 \)
Two red arrow \( C_s \)
These two capacitors are in series
Junction Capacitance in depletion

\[ C_{j,\text{dep}} = \frac{C_s C_r}{C_s + C_r} = \frac{C_r}{1 + C_0/C_r} \]

\[ = \frac{C_0}{1 + \kappa_s \varepsilon_0 / \kappa_r \varepsilon_0 W} = \frac{C_r}{\sqrt{1 + \frac{V_G}{V_\delta}}} \]

\[ V_G = \frac{qN_A W}{\kappa_s \varepsilon_0} x_0 + \left( \frac{qN_A W^2}{2 \kappa_s \varepsilon_0} \right) \]

First term is the V drops on oxide

Second term is band bending

\[ \frac{\kappa_r W}{\kappa_s x_0} = \sqrt{1 + \frac{V_G}{V_\delta} - 1} \]

Purdue Klimek - ECE606 Fall 2012 - notes adopted from Alam

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Junction capacitance in inversion

\[ C_{j,\text{inv}} \approx \frac{\kappa_r \varepsilon_0}{x_0} \equiv C_0 \]

Time to generate inversion charge. ms to \( \mu \)s

\[ C_{j,\text{inv}} = \frac{C_r C_{\text{inv}}}{C_s + C_{\text{inv}}} \]

\[ C_{\text{inv}} = \frac{\kappa_r \varepsilon_0}{W_{\text{inv}}} \]

Purdue Klimek - ECE606 Fall 2012 - notes adopted from Alam
\[ Q_i = -C_G(V_G - V_I) \]

\[ C_G = C_{j,\text{inv}} = \frac{C_\alpha C_{\text{inv}}}{C_{\text{inv}} + C_\alpha} < C_o \]

\[ C_o = \frac{K_o \varepsilon_0}{x_0} \quad C_{\text{inv}} \equiv \frac{K_s \varepsilon_o}{W_{\text{inv}}} \]

\[ C_G = \frac{K_\alpha \varepsilon_0}{EOT_{\text{elec}}} \quad EOT_{\text{elec}} = x_0 + \left( \frac{K_\alpha \varepsilon_0}{K_s \varepsilon_0} \right) W_{\text{inv}} > x_0 \]

‘Equivalent oxide thickness - electrical’

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**High frequency curve at inversion**

\[ C_{j,\text{inv}} \approx \frac{K_s \varepsilon_0}{x_0} \equiv C_0 \]

The red region contribute to the \( C \), as if it is still in depletion

What about high frequency part of the curve?
High frequency response in MOS-C

Dielectric Relaxation

$$\tau = \frac{\sigma}{\kappa_3 \varepsilon_0}$$

SRH Recombination-Generation

$$R = \frac{np - n_i^2}{\tau_n (p + p_i) + \tau_p (n + n_i)} \to \frac{-n_i}{\tau_n + \tau_p}$$

Ref. Lecture no. 15
Ideal vs. Real C-V Characteristics

Blue dot: Flat band voltage ...

C/C₀

Red dot: Threshold voltage ...

C/C₀

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Equilibrium | DC | Small signal | Large Signal | Circuits
---|---|---|---|---
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Schottky
BJT/HBT
MOS

For large signal, the green do not have time to response; continue to deplete.
Small signal there is green because of the DC bias builds it.
Ideal vs. Real C-V Characteristics

**Relaxation from Deep Depletion**

- **Low frequency**
  - Depending on the measurement frequency, it will either merge with low-freq. or high-freq. curve.

- **High frequency**
  - Deep depletion

**Flat band voltage ...**

- **Ideal**
- **Real case**

**Threshold voltage ...**

- If the signal rise slower, it will be closer to ideal case
Low or High frequency?

$G = \frac{n_i}{2\tau}$

typically observe high-frequency CV

typically observe low-frequency CV
No deep-depletion as well

What happens if I shine light on a MOS capacitor?

Intermediate Summary

1) Since current flow through the oxide is small, we are primarily interested in the junction capacitance of the MOS-capacitor.
2) High frequency of MOS-C is very different than low-frequency C-V.
3) In MOSFET, we only see low frequency response.
4) Deep depletion is an important consideration for MOS-capacitor that does not happen in MOSFETs.
### Topic Map

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Subthreshold Region ($V_G < V_{th}$)

No voltages

What happens with a gate bias?
Remember this is a 2D device!

MOScap as discussed before with surface $\psi_s$

High injection

$\log I_D$ vs $V_G$

Subthreshold Slope $\Rightarrow 60$ mV/dec

$\Delta n$

$Q_1$

$log I_D$

$Q_2$

$\Psi_s$ or $V_G/m$

$m = \text{body coefficient typically 1.1~1.4}$

Back-gate grounded $\Rightarrow$ fixed potential

No voltages

Looks like 2 pn junctions in 2D - not 1D

Remember this is a 2D device!
Recall the definition of body coefficient \((m)\)

\[
m = \left(1 + \frac{C_S}{C_O}\right)
\]

‘Body Effect Coefficient’

\[
m = \left(1 + \kappa_S \nu \frac{1}{k_0 W_T}\right)
\]

in practice:

\[1.1 \leq m \leq 1.4\]
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Post-Threshold MOS Current ($V_G > V_{th}$)

$$I_D = \frac{W}{L_{ch}} \mu_{df} \int_{0}^{V_{so}} Q_s(V) dV$$

Formula overview – derivation to follow

1) **Square Law**
   $$Q_s(V) = -C_G \left[V_G - V_T - V\right]$$

2) **Bulk Charge**
   $$Q_s(V) = -C_G \left[V_G - V_{fb} - 2\phi_n - V - \frac{\sqrt{2q} \epsilon_s N_A (2\phi_n + V)}{C_o}\right]$$

3) **Simplified Bulk Charge**
   $$Q_s(V) = -C_G \left[V_G - V_T - mV\right]$$

4) “Exact” (Pao-Sah or Pierret-Shields)
Effect of Gate Bias

Gated doped p-MOS with adjacent n+ region
- a) gate biased at flat-band
- b) gate biased in inversion


The Effect of Drain Bias

2D band diagram for an n-MOSFET

- a) device
- b) equilibrium (flat band)
- c) equilibrium ($\psi_s > 0$)
- d) non-equilibrium with $V_G$ and $V_D > 0$ applied

Gated doped or p-MOS with adjacent, reverse-biased n+ region
a) gate biased at flat-band
b) gate biased in depletion
c) gate biased in inversion

Approximations for Inversion Charge

\[ Q_i = -C_o (V_G - V_{th} - V) + qN_A (W_T(V) - W_T(V = 0)) \]

\[ = -C_o (V_G - V_{th} - V) + \sqrt{2q\kappa_s\varepsilon_r N_A (2\phi_b + V)} - \sqrt{2q\kappa_s\varepsilon_r N_A (2\phi_b)} \]

Approximations:

\[ Q_i = -C_o (V_G - V_{th} - V) \quad \text{Square law approximation} \ldots \]

\[ Q_i = -C_o (V_G - V_{th} - mV) \quad \text{Simplified bulk charge approximation} \ldots \]
Elements of Square-law Theory

\[ Q_t(y) = -C_{ox} \left[ V_G - V_m - mV(y) \right] \]
Square Law Theory

\[ J_1 = Q_1 \mu \mathcal{E}_1 = Q_1 \mu \frac{dV}{dy} \]

\[ J_2 = Q_2 \mu \mathcal{E}_2 = Q_2 \mu \frac{dV}{dy} \]

\[ J_3 = Q_3 \mu \mathcal{E}_3 = Q_3 \mu \frac{dV}{dy} \]

\[ J_4 = Q_4 \mu \mathcal{E}_4 = Q_4 \mu \frac{dV}{dy} \]

\[ \sum_{i=1,N} J_i \frac{dy}{\mu} = \sum_{i=1,N} Q_i \frac{dV}{dy} \]

\[ J_D = \frac{\mu C_{ex}}{L_{ch}} \left[ (V_G - V_m) V_D - m \frac{V_D^2}{2} \right] \]
Square Law or Simplified Bulk Charge Theory

\[ I_D = \frac{W \mu C_m}{L_{ch}} \left[ (V_G - V_{th})V_D - mV_D^2 \right] \]

\[ \frac{dI_D}{dV} = 0 = (V_G - V_{th}) - mV_D \Rightarrow V_{D,sat} = \left( V_G^* - V_{th} \right)/m \]

\[ I_D = \frac{W \mu C_m}{2mL_{ch}} (V_G - V_T)^2 \]

\[ J_D = \frac{\mu C_m}{L_{ch}} \left[ (V_G - V_{th})V_D - mV_D^2 \right] \]

**Why does the curve roll over?**

\[ I_D = \frac{W \mu C_m}{2mL_{ch}} (V_G - V_T)^2 \]

\[ V_{DSAT} = \left( V_{GS} - V_T \right)/m \]

\[ Q_i \approx -C_{ox} (V_G - V_{th} - mV) \]

Expression is only valid for voltages up to pinch-off
Summary

1) MOSFET differs from MOSCAP in that the field from the S/D contacts now causes a current to flow.

2) Two regimes, diffusion-dominated Subthreshold and drift-dominated super-threshold characteristics, define the $I_D$-$V_D$-$V_G$ characteristics of a MOSFET.

3) The simple bulk charge theory allows calculation of drain currents and provide many insights, but there are important limitations of the theory as well.