ECE606: Solid State Devices
Lecture 22
MOScap Frequency Response
MOSFET I-V Characteristics

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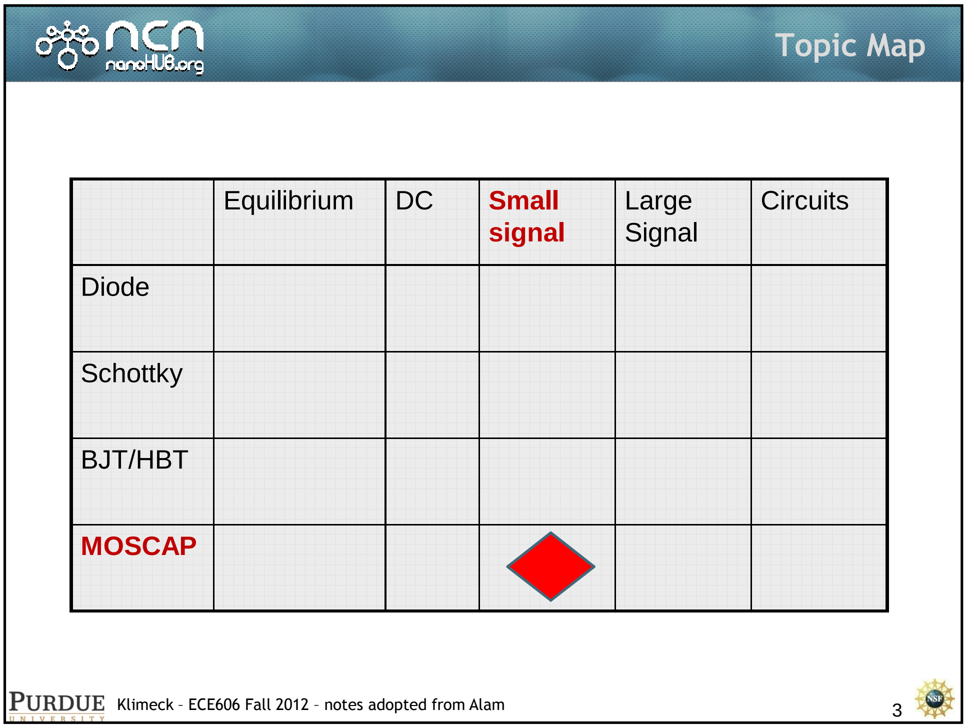
Outline

1. Background
2. Small signal capacitances
3. Large signal capacitance
4. Intermediate Summary

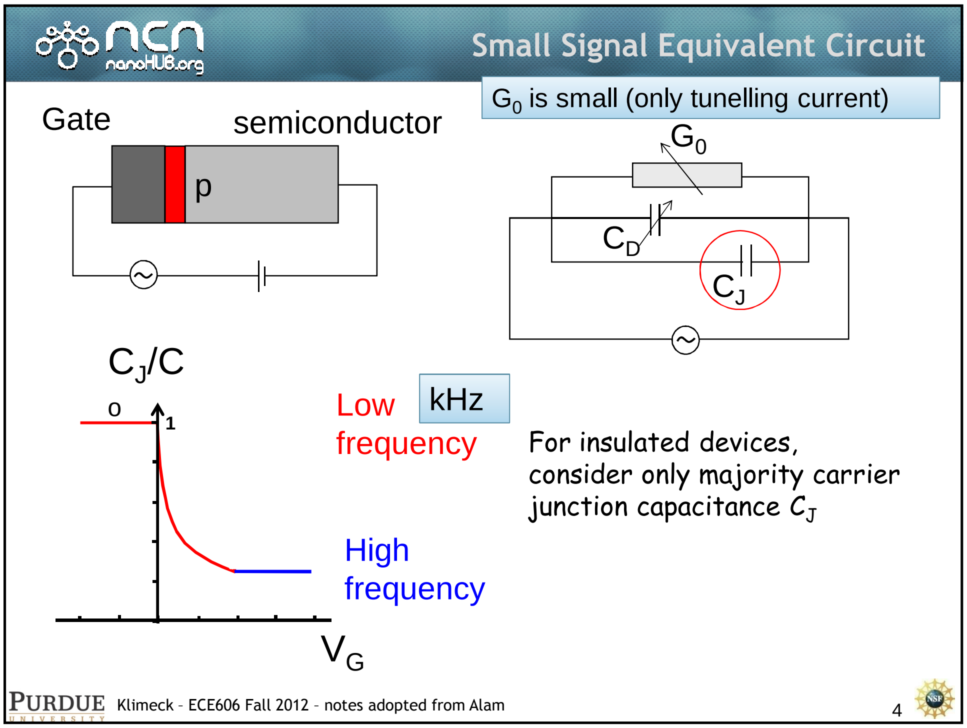
5. Sub-threshold (depletion) current
6. Super-threshold, inversion current
7. Conclusion

Ref: Sec. 16.4 of SDF



Topic Map

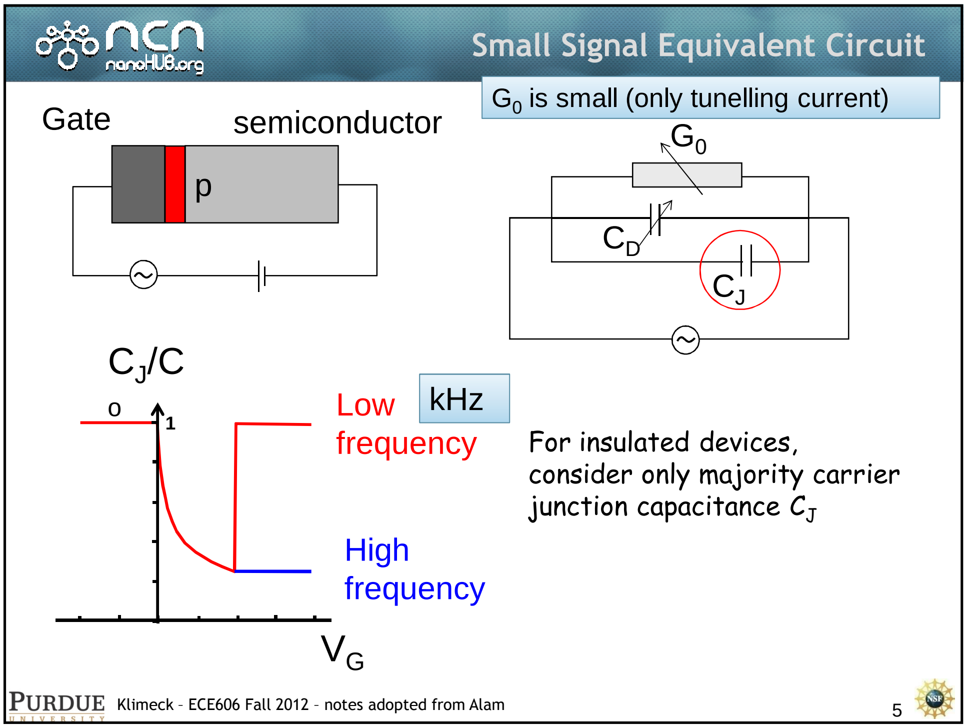
	Equilibrium	DC	**Small signal**	Large Signal	Circuits
Diode					
Schottky					
BJT/HBT					
MOSCAP			◆		



Small Signal Equivalent Circuit

G_0 is small (only tunelling current)

For insulated devices, consider only majority carrier junction capacitance C_J



Small Signal Equivalent Circuit

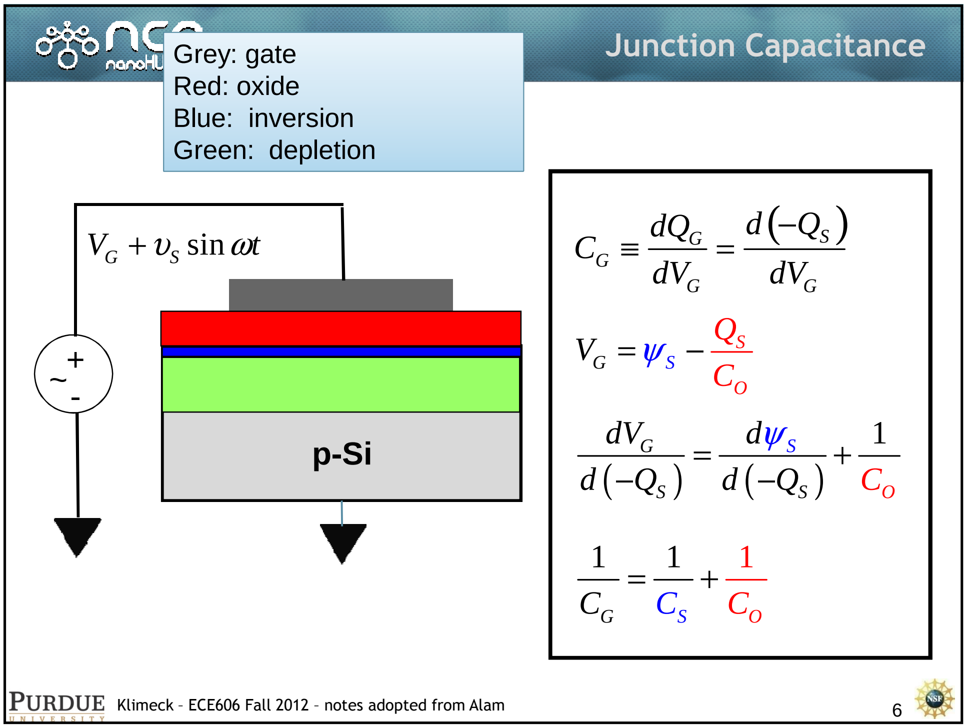
G_0 is small (only tunelling current)

Low frequency

kHz

High frequency

For insulated devices, consider only majority carrier junction capacitance C_J



Junction Capacitance

Grey: gate
Red: oxide
Blue: inversion
Green: depletion

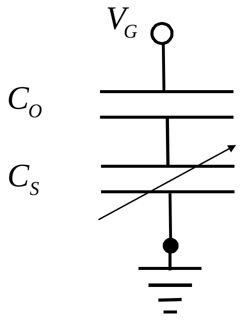
$$C_G \equiv \frac{dQ_G}{dV_G} = \frac{d(-Q_S)}{dV_G}$$

$$V_G = \psi_S - \frac{Q_S}{C_O}$$

$$\frac{dV_G}{d(-Q_S)} = \frac{d\psi_S}{d(-Q_S)} + \frac{1}{C_O}$$

$$\frac{1}{C_G} = \frac{1}{C_S} + \frac{1}{C_O}$$

Junction Capacitance

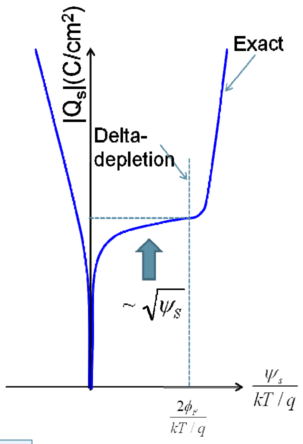


$$\frac{1}{C_G} = \frac{1}{C_S} + \frac{1}{C_O}$$

$$C_S \equiv \frac{d(-Q_S)}{d\psi_S}$$

C_s is not fixed

which we already understand! → $Q_S(\psi_S)$



Remember the Qs vs phi_s figure we mentioned in the previous lecture

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Definition of *m* for later use

$$m = (1 + C_S/C_O)$$

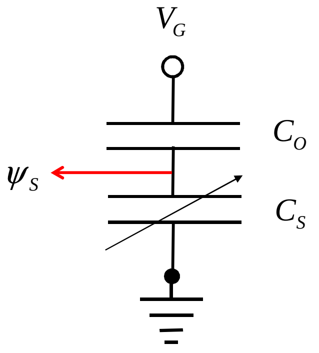
'body effect coefficient'

$$m = (1 + \kappa_S x_O/\kappa_0 W_T)$$

W_T depends on the voltage

in practice:

$$1.1 \le m \le 1.4$$



$$\psi_S = \frac{C_O}{C_O + C_S} V_G \equiv \frac{V_G}{m}$$

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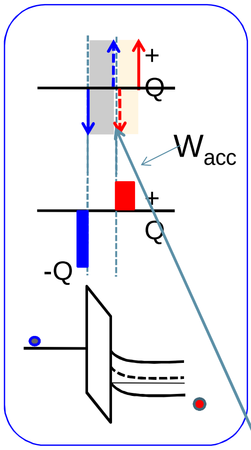
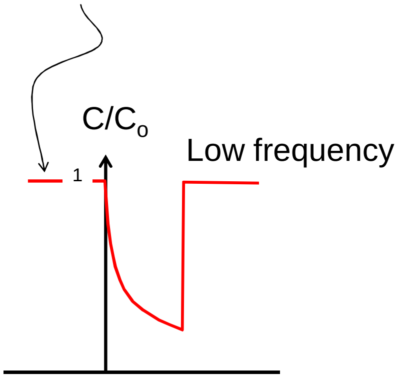
Junction Capacitance *in accumulation*

$$C_{j,acc} \approx \frac{\kappa_{ox}\varepsilon_0}{x_0} \equiv C_0$$

$$C_{j,acc} = \frac{C_o C_{s,acc}}{C_o + C_{s,acc}}$$

$$C_{s,acc} \equiv \frac{\kappa_s \varepsilon_0}{W_{acc}}$$





Arrows is the charge induced by small signal
Two blue arrow→ C_0
Two red arrow → C_s
These two capacitors are in series

Junction Capacitance *in depletion*

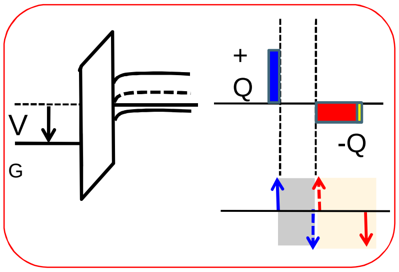
$$C_{j,dep} = \frac{C_0 C_s}{C_0 + C_s} = \frac{C_0}{1 + C_0/C_s}$$

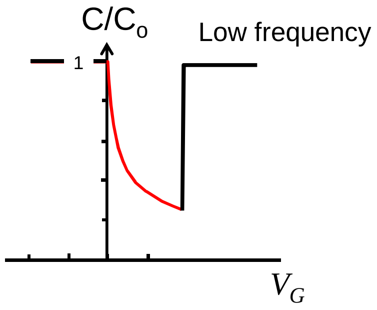
$$= \frac{C_0}{1 + \frac{\kappa_o \varepsilon_0}{x_0} \Big/ \frac{\kappa_s \varepsilon_0}{W}} = \frac{C_o}{\sqrt{1 + \frac{V_G}{V_\delta}}}$$

$$V_G = \frac{qN_A W}{\kappa_o \varepsilon_0} x_0 + \left(\frac{qN_A W^2}{2\kappa_s \varepsilon_0} \right)$$

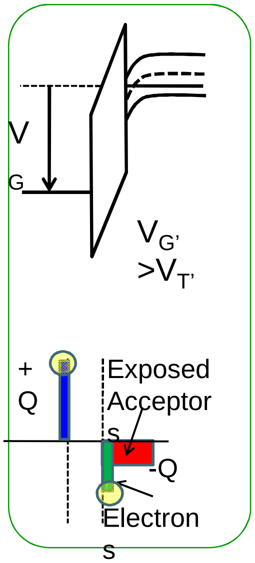
First term is the V drops on oxide
Second term is band bending

$$\frac{\kappa_o}{\kappa_s} \frac{W}{x_0} \approx \sqrt{1 + \frac{V_G}{V_\delta}} - 1$$





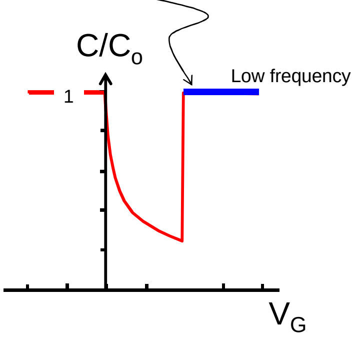
Junction capacitance *in inversion*



$$C_{j,inv} \approx \frac{\kappa_s \varepsilon_0}{x_0} \equiv C_0$$

Time to generate inversion charge. ms to μs

$$C_{j,inv} = \frac{C_o C_{inv}}{C_o + C_{inv}} \qquad C_{inv} \equiv \frac{\kappa_s \varepsilon_0}{W_{inv}}$$

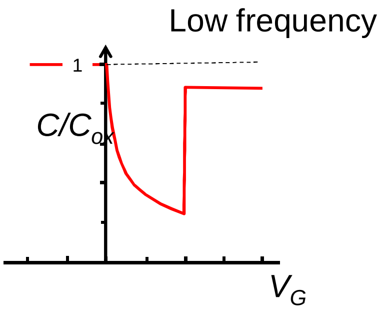


Equivalent Oxide Thickness

$$Q_i = -C_G\left(V_G - V_T\right)$$

$$C_G = C_{j,inv} = \frac{C_o C_{inv}}{C_{inv} + C_o} < C_o$$

$$C_o = \frac{\kappa_o \varepsilon_o}{x_0} \qquad C_{inv} \equiv \frac{\kappa_s \varepsilon_o}{W_{inv}}$$



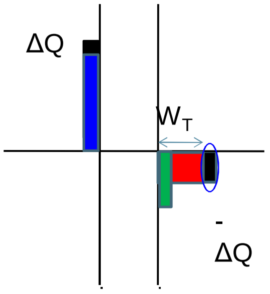
$$C_G = \frac{\kappa_{ox}\varepsilon_O}{EOT_{elec}} \qquad EOT_{elec} = x_O + \left(\frac{\kappa_{ox}\varepsilon_O}{\kappa_s\varepsilon_O}\right)W_{inv} > x_O$$

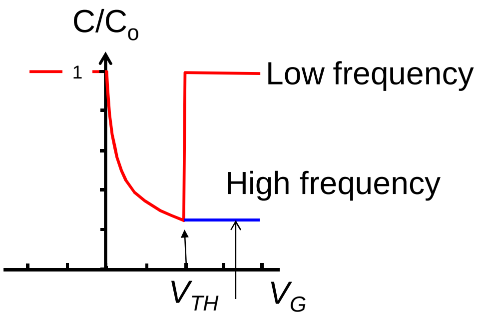
'Equivalent oxide thickness - electrical '

High frequency curve *at inversion*

$$C_{j,inv} \approx \frac{\kappa_s \varepsilon_0}{x_0} \equiv C_0$$

High Frequency

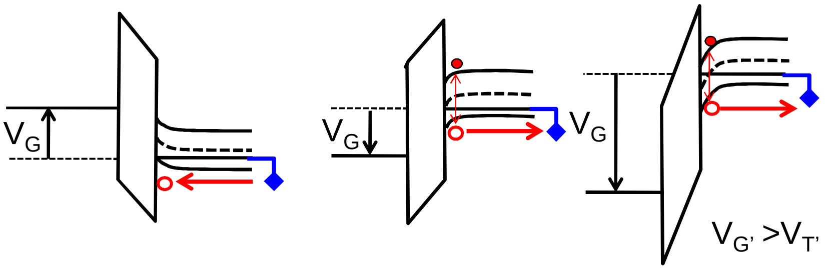




The red region contribute to the C, as if it is still in depletion

What about high frequency part of the curve?

Response Time



Dielectric Relaxation

$$\tau = \frac{\sigma}{\kappa_s \varepsilon_0}$$

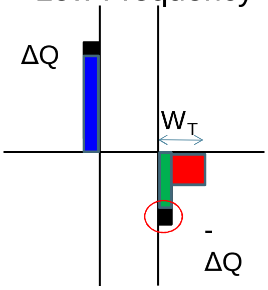
SRH Recombination-Generation

$$R = \frac{np - n_i^2}{\tau_n (p + p_1) + \tau_p (n + n_1)} \rightarrow \frac{-n_i}{\tau_n + \tau_p}$$

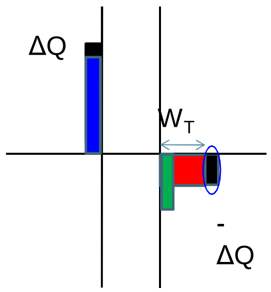
Ref. Lecture no. 15

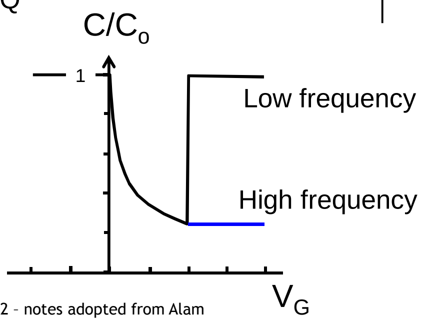
High frequency response in MOS-C

Low Frequency



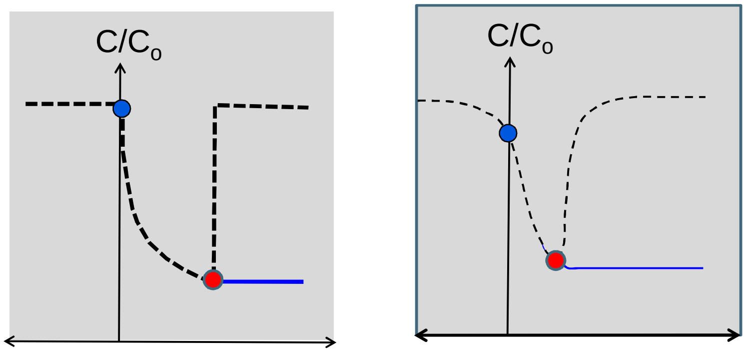
High Frequency





Ideal vs. Real C-V Characteristics

Blue dot: Flat band voltage …



Red dot: Threshold voltage …

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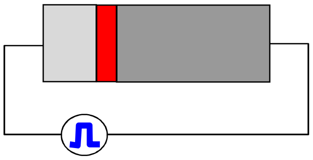
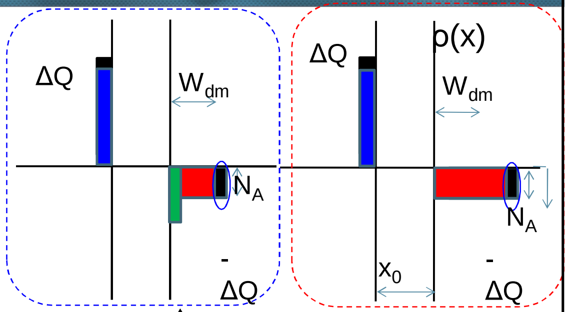
Schottky					
BJT/HBT					
MOS				◆	

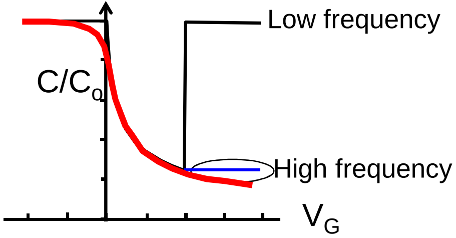
Large Signal Deep Depletion





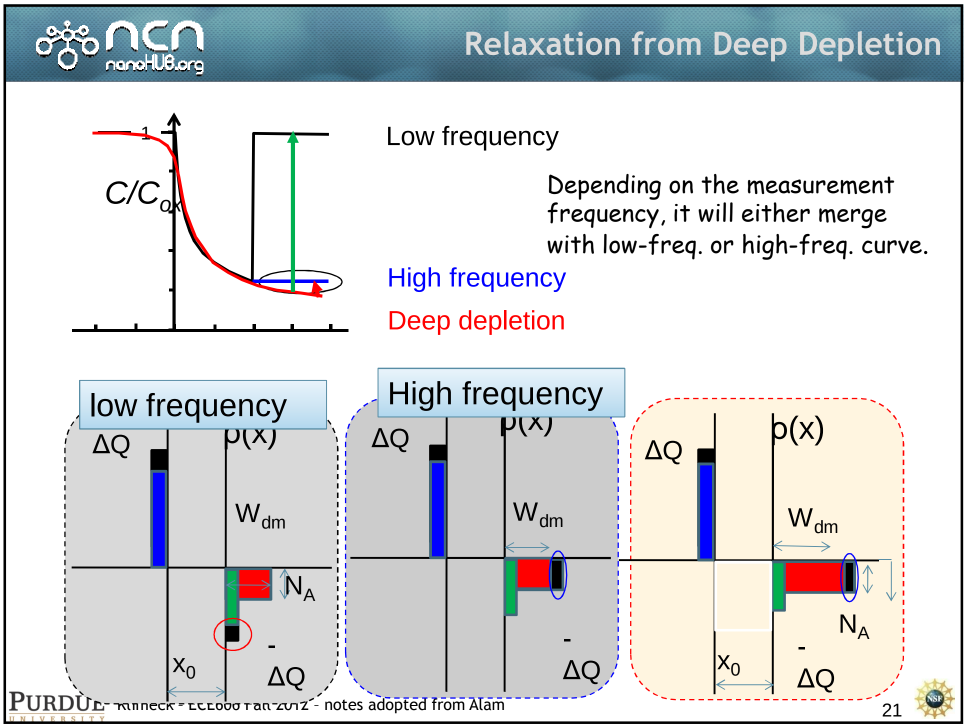
$$C_{j,dep} = \frac{C_0 C_s}{C_0 + C_s} = \frac{C_0}{1+\frac{\kappa_{ox}}{\kappa_s}\frac{W}{x_0}}$$

$$= \frac{C_o}{\sqrt{1+\frac{V_G}{V_\delta}}}$$



(even beyond threshold)

For large signal, the green do not have time to response;
→ continue to deplete.
Small signal there is green because of the DC bias builds it.



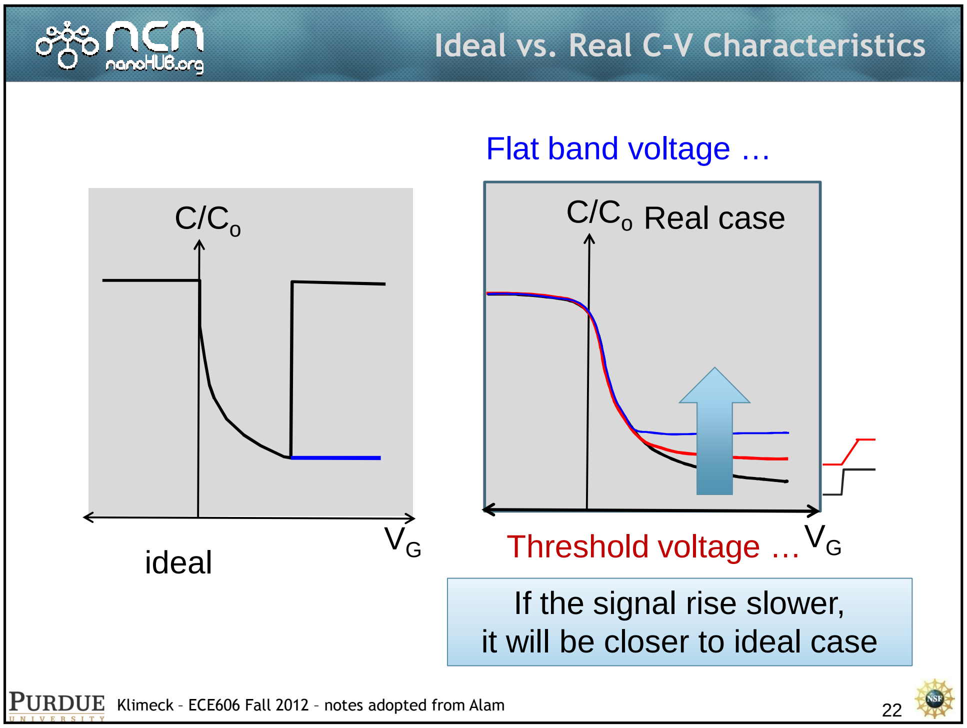
Relaxation from Deep Depletion

Low frequency

Depending on the measurement frequency, it will either merge with low-freq. or high-freq. curve.

High frequency

Deep depletion



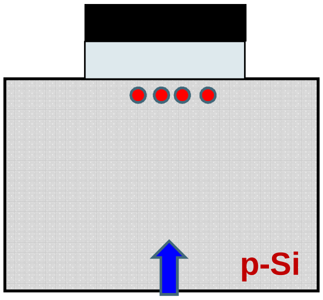
Ideal vs. Real C-V Characteristics

Flat band voltage …

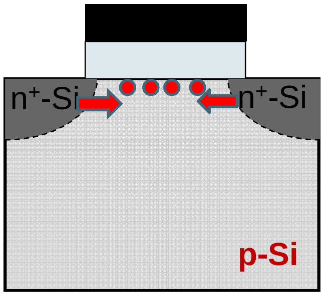
Threshold voltage …

If the signal rise slower, it will be closer to ideal case

Low or High frequency?



$$G = \frac{n_i}{2\tau}$$



typically observe hi-frequency CV

typically observe low-frequency CV
No deep-depletion as well

What happens if I shine light on a MOS capacitor?

Intermediate Summary

1) Since current flow through the oxide is small, we are primarily interested in the junction capacitance of the MOS-capacitor.
2) High frequency of MOS-C is very different than low-frequency C-V.
3) In MOSFET, we only see low frequency response.
4) Deep depletion is an important consideration for MOS-capacitor that does not happen in MOSFETs.

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MOSCAP **MOSFET**		◆			

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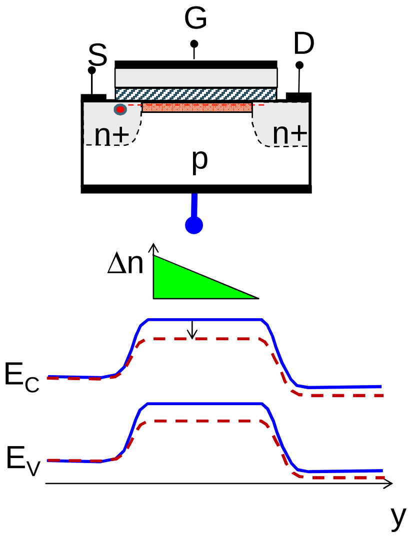
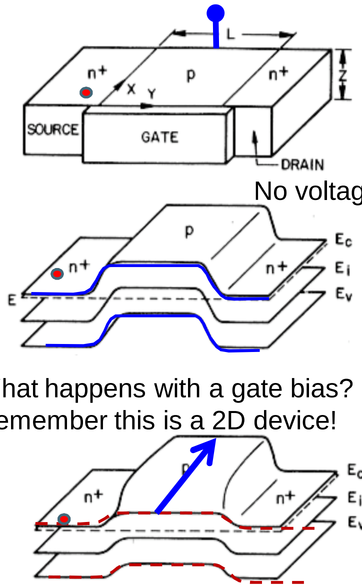
Subthreshold Region ($V_G < V_{th}$)





No voltages

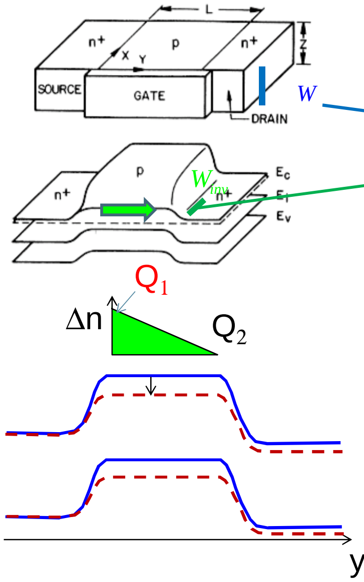
Looks like
2 pn junctions
In 2D – not 1D

What happens with a gate bias?
Remember this is a 2D device!

**MOScap as
discussed before
with surface psi_s**

Back-gate grounded => fixed potential

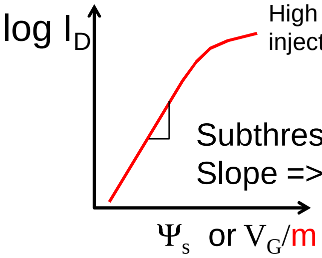
Subthreshold Region ($V_G < V_{th}$)



$$I_D = qD_n \frac{Q_1 - Q_2}{L_{ch}}$$

$$= q\frac{D}{L_{ch}}\left[W \times W_{inv} \times \frac{n_i^2}{N_A}\left(e^{q\psi_s\beta} - 1\right)\right]$$

$$\approx qWW_{inv}\frac{D}{L_{ch}}\frac{n_i^2}{N_A}\left(e^{qV_G\beta/m} - 1\right)$$

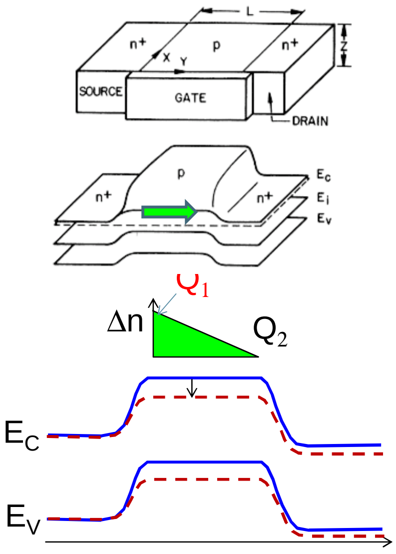


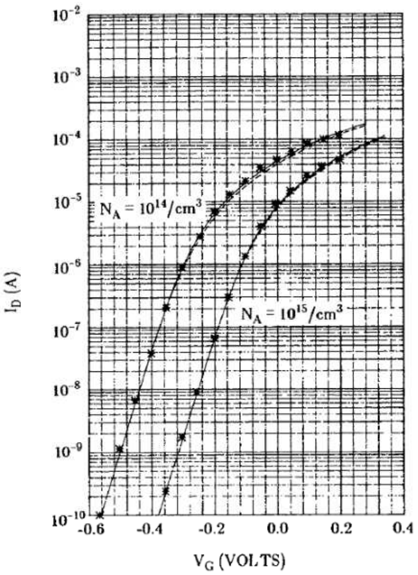
High
injection

Subthreshold
Slope =>60mV/dec

m = body coefficient
typically 1.1~1.4

Subthreshold Region ($V_G < V_{th}$)





Recall the definition of body coefficient (m)

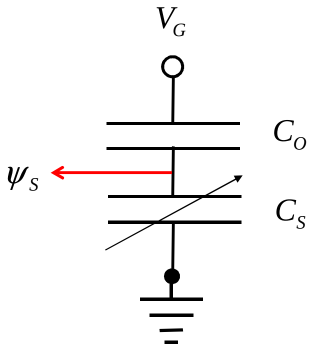
$$m = \left(1 + C_S / C_O\right)$$

'Body Effect Coefficient'

$$m = \left(1 + \kappa_S x_O / \kappa_0 W_T\right)$$

in practice:

$$1.1 \le m \le 1.4$$



$$\psi_S = \frac{C_O}{C_O + C_S} V_G \equiv \frac{V_G}{m}$$

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Post-Threshold MOS Current (V_G>V_{th})

$$\boxed{I_D = -\frac{W}{L_{ch}}\mu_{eff}\int_0^{V_{DS}} Q_i(V)\,dV}$$

Formula overview – derivation to follow

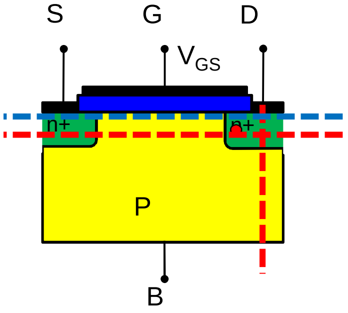
1) Square Law $Q_i(V) = -C_G\left[V_G - V_T - V\right]$

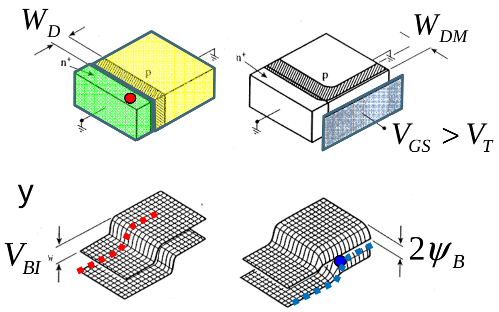
2) Bulk Charge $Q_i(V) = -C_G\left(V_G - V_{FB} - 2\psi_B - V - \frac{\sqrt{2q\varepsilon_{Si}N_A(2\phi_B+V)}}{C_O}\right)$

3) Simplified Bulk Charge $Q_i(V) = -C_G\left[V_G - V_T - mV\right]$

4) “Exact” (Pao-Sah or Pierret-Shields)

Effect of Gate Bias





No source-drain bias

Gated doped or p-MOS with adjacent n$^+$ region

a) gate biased at flat-band
b) gate biased in inversion

A. Grove, *Physics of Semiconductor Devices*, 1967.

The Effect of Drain Bias

2D band diagram for an n-MOSFET

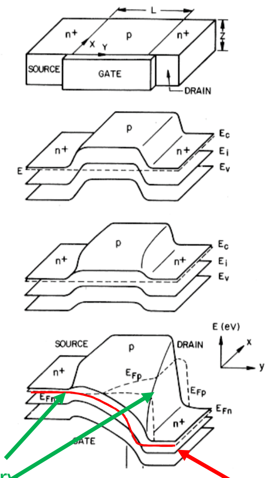
a) device

b) equilibrium (flat band)

c) equilibrium ($\psi_S > 0$)

d) non-equilibrium with V_G and $V_D > 0$ applied

SM. Sze, *Physics of Semiconductor Devices*, 1981
Pao and Sah.



Depletion very different in source and drain side

F_N

Gate voltage must ensure channel formation=> LARGE

Effect of a Reverse Bias at Drain

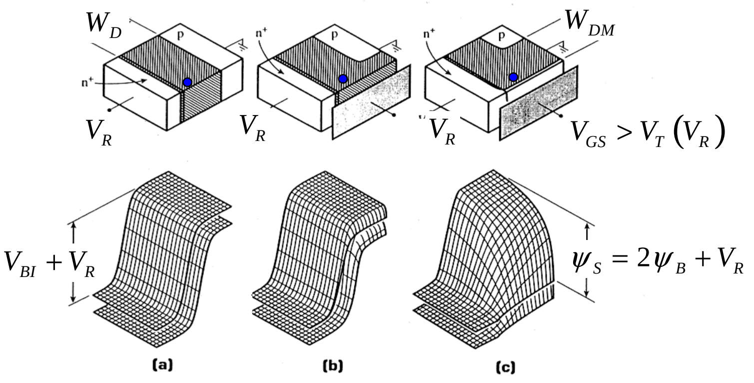


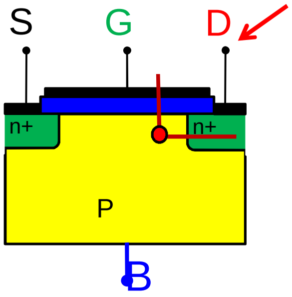
FIGURE 2.34. Gated diode or p-type MOS with adjacent n$^+$ region under nonequilibrium (reverse

Gated doped or p-MOS with adjacent, reverse-biased n$^+$ region

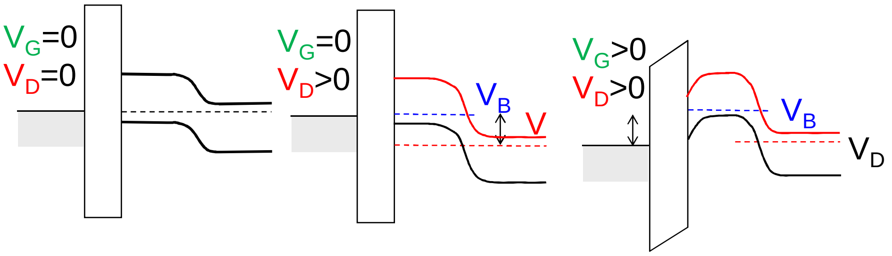
- a) gate biased at flat-band
- b) gate biased in depletion
- c) gate biased in inversion

A. Grove, *Physics of Semiconductor Devices*, 1967.

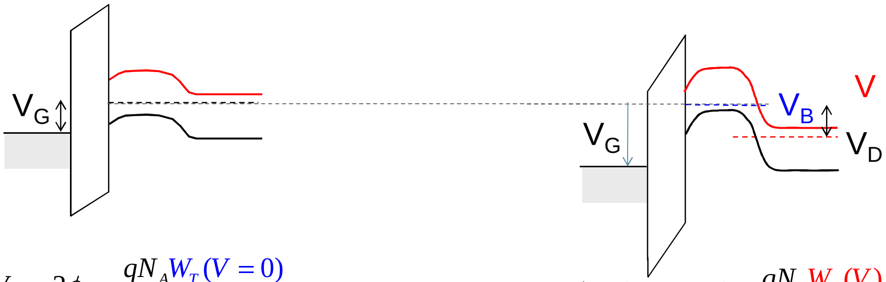
Inversion Charge in the Channel



$$Q_i = -C_{ox}(V_G - V_{th} - V) + qN_A(W_T(V) - W_T(V=0))$$



Inversion Charge at one point in Channel



$$V_{th} = 2\phi_F - \frac{qN_A W_T(V=0)}{C_{ox}}$$

$$V_{th}^* = (2\phi_F + V) - \frac{qN_A W_T(V)}{C_{ox}}$$

$$V_{th}^* = V_{th} + V - \frac{qN_A\left(W_T(V) - W_T(V=0)\right)}{C_{ox}}$$

$$Q_i = -C_{ox}(V_G - V_{th}^{\ *})$$

Approximations for Inversion Charge

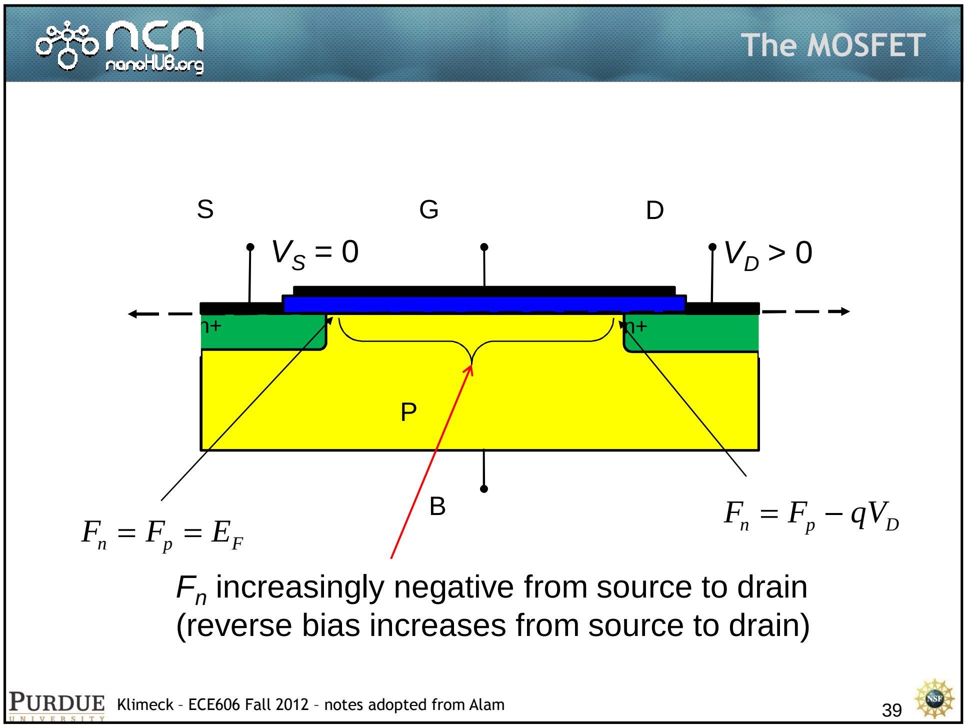
$$Q_i = -C_O(V_G - V_{th} - V) + qN_A\left(W_T(V) - W_T(V=0)\right)$$

$$= -C_O(V_G - V_{th} - V) + \left[\sqrt{2q\kappa_S\varepsilon_o N_A\left(2\phi_B + V\right)} - \sqrt{2q\kappa_S\varepsilon_o N_A\left(2\phi_B\right)}\right]$$

Approximations:

$Q_i \approx -C_{ox}(V_G - V_{th} - V)$ Square law approximation …

$Q_i \approx -C_{ox}(V_G - V_{th} - mV)$ Simplified bulk charge approximation …

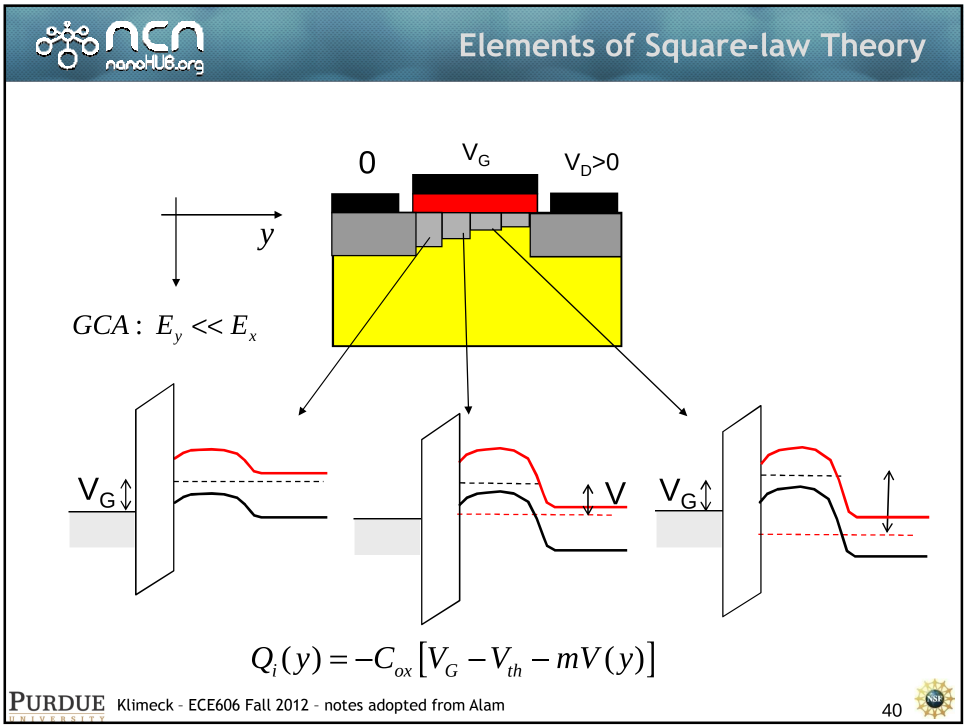


The MOSFET

$F_n = F_p = E_F$

$F_n = F_p - qV_D$

F_n increasingly negative from source to drain
(reverse bias increases from source to drain)

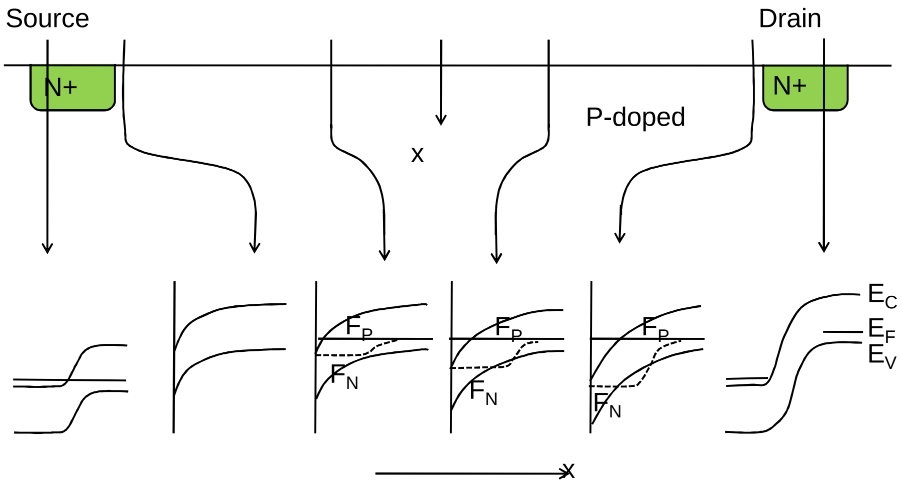


Elements of Square-law Theory

$GCA: \ E_y << E_x$

$$Q_i(y) = -C_{ox}\left[V_G - V_{th} - mV(y)\right]$$

Another view of Channel Potential



Square Law Theory

$$J_1 = Q_1 \mu \mathcal{E}_1 = Q_1 \mu \left.\frac{dV}{dy}\right|_1$$

$$J_2 = Q_2 \mu \mathcal{E}_2 = Q_2 \mu \left.\frac{dV}{dy}\right|_2$$

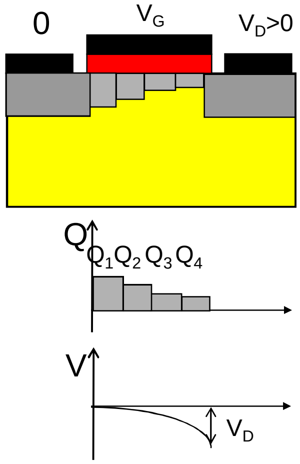
$$J_3 = Q_3 \mu \mathcal{E}_3 = Q_3 \mu \left.\frac{dV}{dy}\right|_3$$

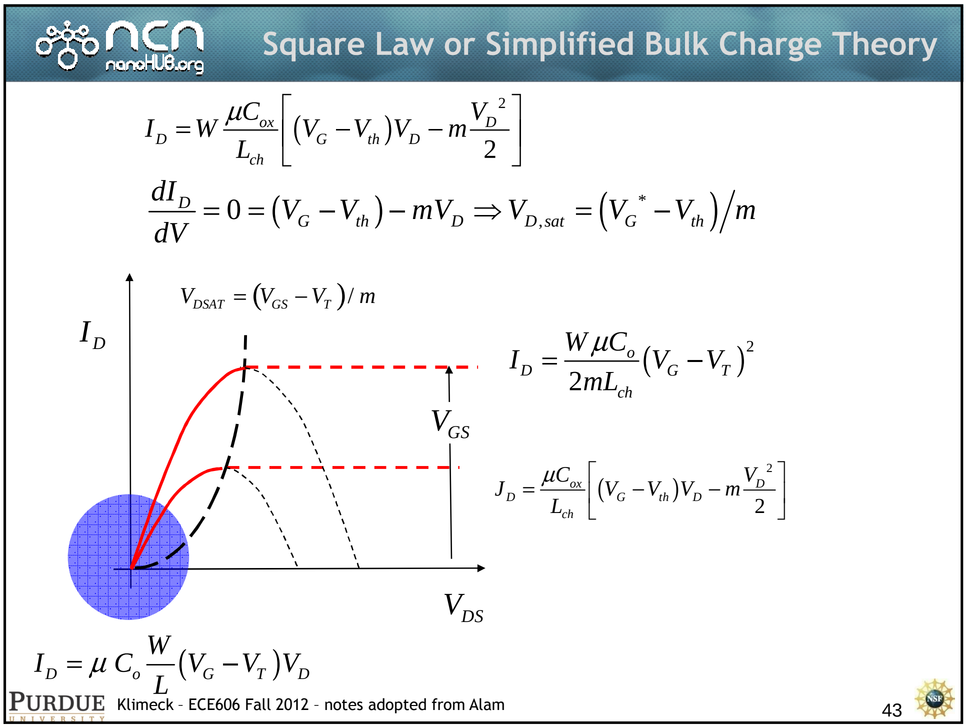
$$J_4 = Q_4 \mu \mathcal{E}_4 = Q_4 \mu \left.\frac{dV}{dy}\right|_4$$

$$\sum_{i=1,N} \frac{J_i dy}{\mu} = \sum_{i=1,N} Q_i \, dV$$

$$\frac{J_D}{\mu} \sum_{i=1,N} dy = \int_0^{V_D} C_{ox}(V_G - V_{th} - mV)dV$$

$$J_D = \frac{\mu C_{ox}}{L_{ch}}\left[(V_G - V_{th})V_D - m\frac{V_D^{\ 2}}{2}\right]$$





Square Law or Simplified Bulk Charge Theory

$$I_D = W\frac{\mu C_{ox}}{L_{ch}}\left[(V_G - V_{th})V_D - m\frac{V_D^{\ 2}}{2}\right]$$

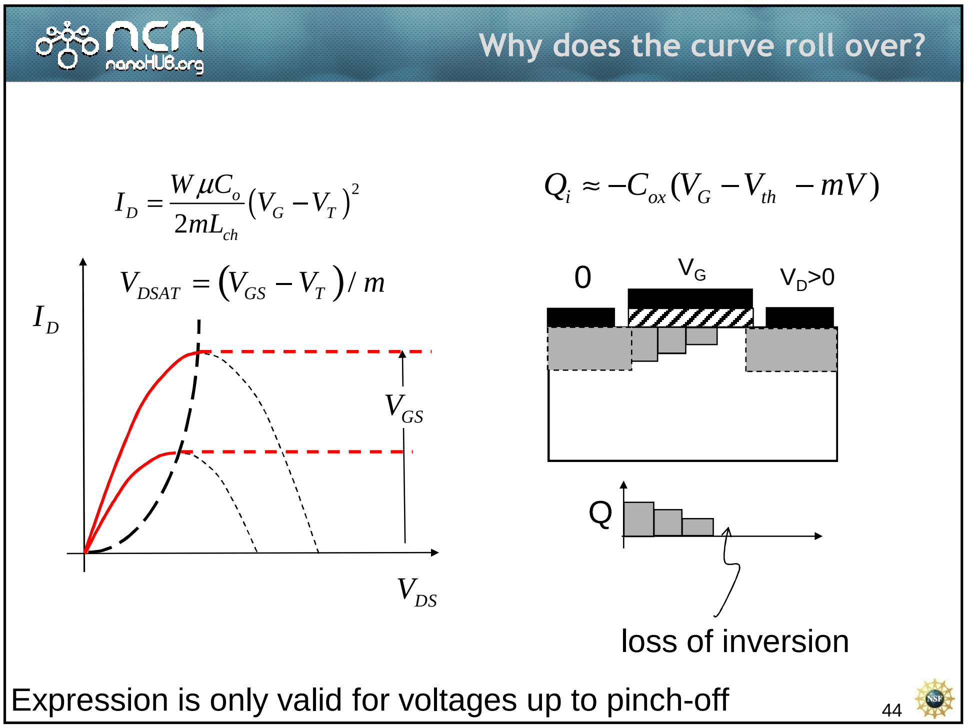
$$\frac{dI_D}{dV} = 0 = (V_G - V_{th}) - mV_D \Rightarrow V_{D,sat} = \left(V_G^{\ *} - V_{th}\right)/m$$

$$V_{DSAT} = (V_{GS} - V_T)/m$$

$$I_D = \frac{W\mu C_o}{2mL_{ch}}(V_G - V_T)^2$$

$$J_D = \frac{\mu C_{ox}}{L_{ch}}\left[(V_G - V_{th})V_D - m\frac{V_D^{\ 2}}{2}\right]$$

$$I_D = \mu\, C_o \frac{W}{L}(V_G - V_T)V_D$$



Why does the curve roll over?

$$I_D = \frac{W\mu C_o}{2mL_{ch}}(V_G - V_T)^2$$

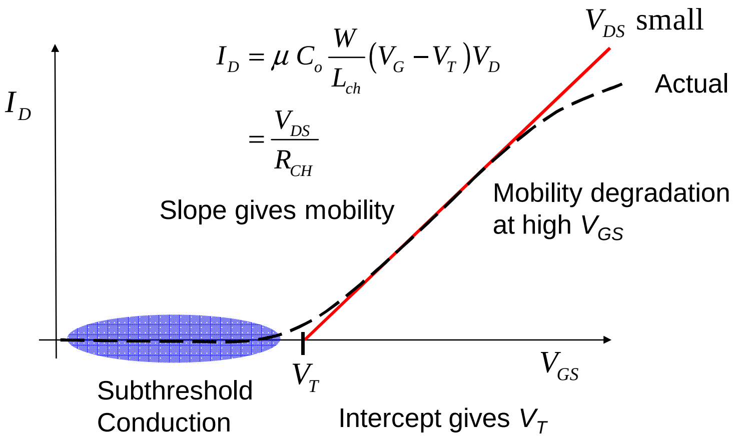
$$V_{DSAT} = \left(V_{GS} - V_T\right)/m$$

$$Q_i \approx -C_{ox}(V_G - V_{th} \ - mV)$$

loss of inversion

Expression is only valid for voltages up to pinch-off

Linear Region (Low V_{DS})



$$I_D = \mu C_o \frac{W}{L_{ch}} (V_G - V_T) V_D$$
$$= \frac{V_{DS}}{R_{CH}}$$

Slope gives mobility

Mobility degradation at high V_{GS}

Subthreshold Conduction

Intercept gives V_T

Can get V_T also from C-V

Summary

1) MOSFET differs from MOSCAP in that the field from the S/D contacts now causes a current to flow.
2) Two regimes, diffusion-dominated Subthreshold and drift-dominated super-threshold characteristics, define the I_D-V_D-V_G characteristics of a MOSFET.
3) The simple bulk charge theory allows calculation of drain currents and provide many insights, but there are important limitations of the theory as well.