ECE606: Solid State Devices  
Lecture 16  
p-n diode AC Response  

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*Diode in Non-Equilibrium (External DC+AC voltage applied)*
Why should we study AC Response?

Motivation

Radio

www.sci-toy.com

Outline

1) Conductance and series resistance
2) Majority carrier junction capacitance
3) Minority carrier diffusion capacitance
4) Conclusion

Ref. SDF, Chapter 7
Forward Bias Conductance

\[ I = I_o \left( e^{q(V_A - R_s I) \beta / m} - 1 \right) \]

\[ m = RG \text{ (2), } \text{diff (1), Ambipolar (2)} \]

\[ \ln \frac{I + I_o}{I_0} = q(V_A - R_s I) \frac{\beta}{m} \]

\[ \frac{m}{q \beta (I + I_o)} = \frac{dV_A}{dI} - R_s \]

\[ \frac{1}{g_{FB}} = R_s + \frac{m}{q \beta (I + I_o)} \]

Reverse Bias Conductance

\[ I = I_o \left( e^{q(V_A - R_s I) \beta / m} - 1 \right) - \frac{qn_i}{2\tau} B_0 \sqrt{V_{bi} - V_A} \]

\[ \approx -I_0 - \frac{qn_i}{2\tau} B_0 \sqrt{V_{bi} - V_A} \]

\[ \frac{1}{g_{RB}} = \frac{qn_i B_0}{2\tau \sqrt{V_{bi} - V_A}} \]
1) Conductance and series resistance

2) **Majority carrier junction capacitance**

3) Minority carrier diffusion capacitance

4) Conclusion
**Measurement of Built-in Potential**

\[
\frac{1}{C_j^2} = 2 \frac{2}{qN_D(x)k_eA^2} (V_{bi} - V_A)
\]

(Assume single sided p⁺-n junction)

Measure \( C_j \) and plot \( C_j^{-2} \) vs. \( V_A \) to derive \( V_{bi} \) from \( C_j \) measurements.
\[
\frac{1}{C_j^2} = \frac{2}{qN_D(x)K_e \varepsilon_0 A^2} (V_{bi} - V_A)
\]

\[
N_D(x) = \frac{2}{qK_e \varepsilon_0 A^2} \frac{1}{d(1/C_j^2)/dV_A}
\]

Measure doping concentration as a function of position

**Dielectric Relaxation Time (majority side)**

\[ J_n = qn \mu_N E + q \mathbf{D}_n \nabla n \]

\[
\frac{dn}{dt} = \frac{1}{q} \frac{dJ_n}{dx} - \mathbf{D}_n + \mathbf{D}_n
\]

Neglect

\[
\frac{d(\Delta n)}{dt} = \frac{1}{q} \frac{d(qn \mu_N E)}{dx} = N_D \mu_N \frac{dE}{dx}
\]

\[
\frac{dE}{dx} = \frac{q}{k_s \varepsilon_0} (\mathbf{D}_n - n_0 - \Delta n + N_D - \mathbf{D}_n)
\]

How long does it take for the signal to cross the junction?

\[
\tau_d = \frac{K_e \varepsilon_0}{\sigma} \approx 0.1 \text{ ps}
\]

Very fast

\[
\Delta n(t) = n_0 e^{-\frac{\sigma_s \tau_d}{k_s \varepsilon_0}} = n_0 e^{-\frac{1}{\tau_s}}
\]
1) Conductance and series resistance
2) Majority carrier junction capacitance
3) Minority carrier diffusion capacitance
4) Conclusion
Diffusion Capacitance for Minority Carriers

\[
\frac{\partial (n_0 + \Delta n_{dc} + \Delta n_{ac} e^{i\omega t})}{\partial t} = D_N \frac{d^2 (n_0 + \Delta n_{dc} + \Delta n_{ac} e^{i\omega t})}{dx^2} - \frac{\Delta n_{dc} + \Delta n_{ac} e^{i\omega t}}{\tau_n}
\]

\[
ja\Delta n_{ac} e^{i\omega t} = D_N \frac{d^2 \Delta n_{dc}}{dx^2} + e^{i\omega t} \frac{d^2 \Delta n_{ac}}{dx^2} - \frac{\Delta n_{dc}}{\tau_n} - e^{i\omega t} \frac{\Delta n_{ac}}{\tau_n}
\]

DC: \( 0 = D_N \frac{d^2 \Delta n_{dc}}{dx^2} - \frac{\Delta n_{dc}}{\tau_n} \Rightarrow \Delta n_{dc} = Ae^{\frac{x}{\tau_n}} + Be^{-\frac{x}{\tau_n}} \)

AC: \( 0 = D_N \frac{d^2 \Delta n_{ac}}{dx^2} - (j\omega \tau_n + 1) \frac{\Delta n_{ac}}{\tau_n} \Rightarrow \Delta n_{ac} = Ce^{\frac{x}{\tau_n}} + De^{-\frac{x}{\tau_n}} \rightarrow Ce^{\frac{x}{\tau_n}} \)

\[
L_n^* = \sqrt{D_n \tau_n / (1 + j\omega \tau_n)} \quad \tau_n^* = \tau_n / (1 + j\omega \tau_n)
\]

AC Boundary Conditions

\[
\Delta n_{dc} (x = 0) = \frac{n_0^2}{N_A} \left( \frac{qV_n}{kT} - 1 \right)
\]

\[
\left(\Delta n_{ac} + \Delta n_{ac} e^{i\omega t}\right) = \frac{n_0^2}{N_A + \Delta n_{ac} e^{i\omega t}} \left( e^{\frac{qV_n + qV_c e^{i\omega t}}{kT}} - 1 \right)
\]

\[
\left(\Delta n_{dc} + \Delta n_{dc} e^{i\omega t}\right) = \frac{n_0^2}{N_A} \left( e^{\frac{qV_n}{kT}} e^{\frac{qV_c e^{i\omega t}}{kT}} - 1 \right)
\]

Taylor expansion

\[
= \frac{n_0^2}{N_A} \left( e^{\frac{qV_n}{kT}} \left( 1 + \frac{qV_c e^{i\omega t}}{kT} - 1 \right) \right)
\]

\[
\Delta n_{ac} (x = 0) = \frac{qV_n}{kT} \frac{n_0^2}{N_A} e^{\frac{qV_n}{kT}} = C
\]
### AC Current and Impedance

\[
\Delta n_{ac}(x=0) = \frac{qV_{ac}}{kT} \frac{n_i^2}{N_A} e^{\frac{qV_{ac}}{kT}} = C
\]

\[
\Delta n_{ac}(x) = Ce^{\frac{-x}{\tau_n}} + De^{\frac{x}{\tau_n}} \rightarrow Ce^{\frac{-x}{\tau_n}}
\]

Finally...

\[
J_{ac} = -qD_n \left. \frac{d\Delta n_{ac}}{dx} \right|_{x=0} = \frac{qD_n}{L_n} qV_{ac} \frac{n_i^2}{kT} N_A e^{\frac{qV_{ac}}{kT}}
\]

**AC Current**

\[
Y_{ac} = \frac{J_{ac}}{V_{ac}} = \frac{q^2 D_n}{L_n^2} \frac{n_i^2}{kT} N_A e^{\frac{qV_{ac}}{kT}} \equiv G_0 \sqrt{1 + j\omega\tau_n}
\]

**AC Impedance**

---

### Diffusion Conductance and Capacitance

\[
G_D \propto \sqrt{\omega}
\]

\[
Y_{ac} = G_D + j\omega C_D \equiv G_0 \sqrt{1 + j\omega\tau_n}
\]

Separate in real & imaginary parts ...

\[
G_D = \frac{G_0}{\sqrt{2}} \left[ \sqrt{1 + \omega^2\tau_n^2} + 1 \right]^{1/2}
\]

\[
\omega C_D = \frac{G_0}{\sqrt{2}} \left[ \sqrt{1 + \omega^2\tau_n^2} - 1 \right]^{1/2}
\]

Product of G_D and C_D is frequency-independent

**C_D \propto \frac{1}{\sqrt{\omega}}**
1) Small signal response relevant for many analog applications.
2) Small signal parameters always refer to the DC operating conditions, as such the parameter changes with bias condition.
3) Important to distinguish between majority and minority carrier capacitance. Their relative importance depends on specific applications.
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**Equilibrium** | **DC** | **Small signal** | **Large Signal** | **Circuits**
--- | --- | --- | --- | ---
Diode | | | | 
Schottky | | | | 
BJT/HBT | | | | 
MOSFET | | | | 

**Outline**

1) **Large signal response and charge control model**
2) Turn-off characteristics
3) Turn-on characteristics
4) Other applications
5) Conclusion

Ref. SDF, Chapter 8
If transition is slow, every point is in quasi-equilibrium → treat them like DC

If transition is very fast

Before transition occur constant voltage, current change from $I_F$ to $I_R$
constant current, voltage change form 1V to 0V

A Closer Look to Fast Transition
Definitions

Charge control equations:
Approximation when you have large transient response

Continuity Equations

Full analytical solution impossible for large signal….

\[ \nabla \cdot D = q (p - n + N_D^+ - N_A^-) \]

\[ \frac{\partial n}{\partial t} = \frac{1}{q} \nabla \cdot J_N - r_N + g_N \]

\[ J_N = q n \mu_N \mathcal{E} + q D_N \nabla n \]

\[ \frac{\partial p}{\partial t} = \frac{1}{q} \nabla \cdot J_p - r_p + g_p \]

\[ J_p = q p \mu_p \mathcal{E} - q D_p \nabla p \]

Charge control equations: Approximation when you have large transient response
How Does Current Flip Without Voltage Flipping?

Where did the charge go?
1. Back to the left-hand side
2. Recombine with the trap and the majority carrier

When voltage starts to change, \( n(x,t) \) suddenly bended downwards, \( \frac{dn}{dx} \) change sign and current flip from \( p \rightarrow n \) to \( n \rightarrow p \)

Large Signal Charge Control Model

\[
\frac{\partial n}{\partial t} = \frac{1}{q} \frac{dJ_n}{dx} - r_n + \frac{\Delta n}{\tau_n}
\]

\[
\frac{\partial (\Delta n)}{\partial t} = D_n \frac{d^2 (\Delta n)}{dx^2} - \frac{\Delta n}{\tau_n}
\]

\[
J_n = qn \mu_n \mathcal{E} + qD_n \frac{dn}{dx}
\]

minority carrier
5) Conclusion

1) Large signal response and charge control model

2) **Turn-off characteristics**

3) Turn-on characteristics

4) Other applications

5) Conclusion
**Turn-off Characteristics: Determine \( t_s \)**

\[
\frac{\partial Q}{\partial t} = i_{diff} - \frac{Q}{\tau_n}
\]

Why does the current remain constant even with \( t > 0 \)?

**Boundary Condition**

\[
\frac{\partial Q}{\partial t} = i_{diff} - \frac{Q}{\tau_n}
\]

\( t < 0 \) \( \frac{\partial Q}{\partial t} = I_F - \frac{Q(0^-)}{\tau_n} \)

\( Q(0^-) = I_F \tau_n = Q(0^+) \)

Note. For a capacitor, voltage cannot change instantly. So charge cannot change instantly…. 
Since $V_j$ can’t be larger than the band gap, which is smaller than $V_R$, the diode will be forced to supply the negative current $I_R$.

\[
\frac{\partial Q}{\partial t} = \frac{Q_{\text{diff}}}{\tau_n} - \frac{Q}{\tau_n}
\]

\[
t > 0 \quad \frac{\partial Q}{\partial t} = -I_R - \frac{Q}{\tau_n}
\]

\[
t_s = \tau_n \ln \left( \frac{I_R + Q(t_s) \tau_n}{I_R + Q(0^+) \tau_n} \right)
\]

\[
\int_{Q(0^+)}^{Q(t)} \frac{dQ}{I_R + \frac{Q}{\tau_n}} = \int_0^{t_s} dt
\]
Storage Time

\[ t_s = \tau_n \ln \left( \frac{I_R + Q(0^-)}{I_R + Q(0^+)} \right) = \tau_n \ln \left( \frac{I_R + I_F}{I_R} \right) \]

Shorten it for fast response!

Turn-off Voltage Transient

\[ v_A(t) = \frac{kT}{q} \ln \frac{n_p(0,t)}{n_{po}} \]

\[ Q_n(t) = \tau_p \ln \left( -I_R + (I_R + I_F)e^{-t/\tau_p} \right) \]

Allows easy calculation of \( n_p(0,t) \).
Recovery Time

\[
erf \left( \frac{t_r}{\tau_p} \right) + \frac{e^{-t_r/\tau_p}}{\sqrt{\pi} \tau_p} = 1 + 0.1 \frac{I_F}{I_R}
\]

Useful formula ...

\[
erf (\sqrt{x}) = \left[ 1 - e^{-x^{1.27+0.15x}/(1+0.15x)} \right]^{0.5}
\]
1) Large signal response and charge control model
2) Turn-off characteristics
3) Turn-on characteristics
4) Other applications
5) Conclusion
3) turn-on characteristics

\[ \frac{\partial Q}{\partial t} = i_{\text{diff}} - \frac{Q}{\tau_n} \]

For \( t > 0 \)

\[ \frac{\partial Q}{\partial t} = I_F \frac{Q}{\tau_n} \]

Check:

\[ Q(t \to \infty) = I_F \tau_n \left( 1 - e^{-\frac{t}{\tau_n}} \right) \]

\[ Q(t=0) = 0 \]

\[ Q(t \to \infty) = I_F \tau_n \]

Outline

1) large signal response and charge control model
2) turn-off characteristics
3) turn-on characteristics
4) other applications
5) conclusion
Electrons get scattered to the other side, the average time is...

\[ \frac{\partial Q}{\partial t} = i_{\text{diff}} - \frac{Q}{\tau_n} \]

\[ \frac{Q(0^+) - Q(t = \infty)}{\tau_{\text{diff}}} = i_{\text{diff}} \]

\[ \tau_{\text{diff}} = \frac{Q(0^+)}{i_{\text{diff}}} = \frac{q}{2D_n} \frac{\Delta n_p(0)}{W_p} \]

\[ i_{\text{diff}} = \frac{Q}{\tau_{\text{diff}}} \]

\[ i_{\text{diff}} = \frac{q \times \frac{1}{2} n_i^2 (e^{\psi_s \varphi} - 1) \times W_p}{2D_n} = q \frac{D_n}{W_p} n_i^2 \frac{W_p}{\varphi_s} \]

\[ \Delta n(x, t) = C + Dx \]

No recombination

Only half of them goes to the right

Diffusion velocity

Steady State Diode Current

\[ \frac{\partial Q}{\partial t} = i_{\text{diff}} - \frac{Q}{\tau_n} \]

\[ i_{\text{diff}} = \frac{Q}{\tau_{\text{diff}}} \]

\[ i_{\text{diff}} = \frac{q \times \frac{1}{2} n_i^2 (e^{\psi_s \varphi} - 1) \times W_p}{2D_n} = q \frac{D_n}{W_p} n_i^2 \frac{W_p}{\varphi_s} \]

Exact Expression!
Large signal response of devices of great importance for digital applications.

Analytical solution of partial differential equation often difficult (if not impossible), therefore approximate methods like Charge-control approximation often help simplify the solution and still provide a great deal of insight into the dynamics of switching operation.

Be careful in using the boundary condition which is often dictated by external circuit conditions.