Digital Signal Processing I **Exam 3** **Fall 1999**
Session 27 **Live: 30 Nov. 1999**

Cover Sheet

Test Duration: 75 minutes.
Open Book but Closed Notes.
Calculators not allowed.
This test contains **three** problems.
All work should be done in the blue books provided.
Do **not** return this test sheet, just return the blue books.

Prob. No.	Topic of Problem	Points
1.	DFT and Time-Domain Aliasing	30
2.	Windows	40
3.	Sum of Sinewaves Model	30

Problem 1. [30 points]

Consider a one-sided geometric sequence of the form

$$x[n] = 31 \left\{ \frac{1}{2} \right\}^n u[n]$$

where the constant 31 is just a multiplicative scalar that ultimately makes the numbers work out nice. The DTFT of $x[n]$ is $X(\omega) = \frac{31}{1-0.5e^{-j\omega}}$. Let $X_N[k]$ be obtained as N samples of $X(\omega)$ equi-spaced in frequency over $0 \leq \omega < 2\pi$. Further, let $x_N[n]$ be obtained as the N-point Inverse DFT of $X_N[k] = \frac{31}{1-0.5e^{-jk2\pi/N}}$. Determine $x_N[n]$ when $N = 5$. That is, determine and explicitly list the numerical values of $x_N[0]$, $x_N[1]$, $x_N[2]$, $x_N[3]$, $x_N[4]$, when $N = 5$. **Actually computing an Inverse DFT is NOT the way to solve this problem.**

Problem 2. [40 points] Consider the following window of length $M-1$, where M is an even number.

$$w[n] = e^{j\frac{2\pi}{M}n} \left\{ u[n] - u[n - \frac{M}{2}] \right\} * e^{-j\frac{2\pi}{M}n} \left\{ u[n] - u[n - \frac{M}{2}] \right\}$$

This "new" window is obtained as the convolution of one rectangular window of length $\frac{M}{2}$ modulated by $e^{j\frac{2\pi}{M}n}$ with another rectangular window of length $\frac{M}{2}$ modulated by $e^{-j\frac{2\pi}{M}n}$.

(a) Determine a closed-form expression for $w[n]$ (that is, determine a simple analytical expression for the result obtained from performing the convolution.) Sketch $w[n]$ for $n = 0, 1, ..., M-2$.

(b) Is $w[n]$ a symmetric or anti-symmetric window? Briefly justify your answer (that is, don't just guess.)

(c) Let $W(\omega)$ denote the DTFT of $w[n]$. Determine a closed-form expression for $W[\omega]$. Plot the magnitude $|W(\omega)|$ over $-\pi < \omega < \pi$ showing as much detail as possible. Explicitly point out the numerical values of the specific frequencies for which $|W(\omega)| = 0$.

(d) *Analysis of mainlobe width of* $W(\omega)$: What is the null-to-null mainlobe width of $W(\omega)$? Is the mainlobe width of $W(\omega)$ the same, larger, or smaller than the mainlobe width of the DTFT of a rectangular window of the same length, $M-1$? Briefly explain.

(e) *Analysis of peak sidelobe of* $W(\omega)$: Is the peak sidelobe of $W(\omega)$ the same, larger, or smaller than the peak sidelobe of the DTFT of a rectangular window of the same length, $M-1$? Briefly explain your answer.

(f) *Analysis of sidelobes of* $W(\omega)$: What about the sidelobes other than the peak sidelobe? That is, excluding the mainlobe and the first peak sidelobe on either side of the mainlobe, are the sidelobes of $W(\omega)$ the same, larger, or smaller than the sidelobes of the DTFT of a rectangular window of the same length, $M-1$? Briefly explain.

Problem 3. [30 points]

Let $x[n]$ be a discrete-time random process containing one real-valued sinewave plus noise as described by

$$x[n] = A\cos(\omega_0 n + \Theta) + \nu[n],$$

where the amplitude, A, and frequency, ω_0, of the sinusoid are each deterministic but unknown constants and Θ is a random variable uniformly distributed over a 2π interval. $\nu[n]$ is a white noise process with autocorrelation $r_{\nu\nu}[m] = E\{\nu[n]\nu^*[n-m]\} = \delta[m]$.

You are given the following three values of the true autocorrelation sequence $r_{xx}[m] = E\{x[n]x^*[n-m]\}$:

$$r_{xx}[0] = 3; \qquad r_{xx}[1] = 1; \qquad r_{xx}[2] = -1$$

(a) Knowing that $r_{xx}[m]$ satisfies $r_{xx}[m] = -a_1 r_{xx}[m-1] - a_2 r_{xx}[m-2] + \sigma_w^2\delta[m]$, determine the numerical values of a_1 and a_2.

(b) Determine the numerical value of $r_{xx}[3]$.

(c) Plot the spectral density $S_{xx}(\omega) = \sum_{n=-\infty}^{\infty} r_{xx}[m]e^{-jm\omega}$ over $-\pi \leq \omega \leq \pi$.