Digital Signal Processing I — Exam 3 — Fall 1998
Session 26 — Live: 24 Nov. 1998

Cover Sheet

Test Duration: 75 minutes.
Open Book but Closed Notes.
Calculators allowed.
This test contains **three** problems.
All work should be done in the blue books provided.
Do **not** return this test sheet, just return the blue books.

Prob. No.	**Topic of Problem**	**Points**
1.	Radix 2 FFT	30
2.	DFT and Time-Domain Aliasing	40
3.	Windows & Symmetric FIR Filters	30

Problem 1. [30 points]

As part of the first stage in a radix 2 FFT, a sequence $x[n]$ of length $N = 8$ is decomposed into two sequences of length 4 as

$$\begin{aligned} f_0[n] &= x[2n], \quad n = 0, 1, 2, 3 \\ f_1[n] &= x[2n+1], \quad n = 0, 1, 2, 3 \end{aligned}$$

We compute a 4 -pt. DFT of each of these two sequences as

$$f_0[n] \overset{\text{DFT}}{\underset{4}{\longleftrightarrow}} F_0[k] \qquad\qquad f_1[n] \overset{\text{DFT}}{\underset{4}{\longleftrightarrow}} F_1[k]$$

The specific values of $F_0[k]$ and $F_1[k]$, $k = 0, 1, 2, 3$, obtained from the length $N = 8$ sequence in question are listed in the Table below.

k	0	1	2	3
$F_0[k]$	2	2	0	2
$F_1[k]$	2	$\sqrt{2}(1+j)$	0	$\sqrt{2}(1-j)$
W_8^k	1	$\frac{1}{\sqrt{2}}(1-j)$	-j	$-\frac{1}{\sqrt{2}}(1+j)$

(20 pts) From the values of $F_0[k]$ and $F_1[k]$, $k = 0, 1, 2, 3$, and the values of $W_8^k = e^{-j\frac{2\pi}{8}k}$, $k = 0, 1, 2, 3$, provided in the Table, determine the numerical values of the actual $N = 8$-pt. DFT of $x[n]$ denoted $X_8[k]$. That is, determine the numerical value of $X_8[k]$ for $k = 0, 1, 2, 3, 4, 5, 6, 7$. **You need to show all work in arriving at your answers to receive full credit.**

(10 pts) The underlying length $N = 8$ sequence $x[n]$ may be expressed as

$$x[n] = 0.5 + \cos(\omega_0 n), \quad n = 0, 1, ..., 7.$$

Given the values of $X_8[k]$ for $k = 0, 1, ..., 7$ determined in part (a), determine the frequency ω_0.

Problem 2. [30 points]

Consider an input sequence, $x[n]$, of length $L = 4$ and an FIR filter with impulse response $h[n]$ of length $M = 4$ as described below.

$$\begin{aligned} x[n] &= u[n] - u[n-4] = \{1, 1, 1, 1\} \\ h[n] &= u[n] - u[n-4] = \{1, 1, 1, 1\} \end{aligned}$$

We compute an $N = 5$-pt. DFT of each of these two sequences as

$$x[n] \overset{\text{DFT}}{\underset{5}{\longleftrightarrow}} X_5[k] \qquad\qquad h[n] \overset{\text{DFT}}{\underset{5}{\longleftrightarrow}} H_5[k]$$

Next, we point-wise multiply the DFT sequences to form $Y_5[k] = X_5[k]H_5[k]$, $k = 0, 1, 2, 3, 4$. Finally, we compute an $N = 5$-pt. inverse DFT of $Y_5[k]$ to obtain $y_P[n]$. Determine the numerical values of $y_P[n]$ for $n = 0, 1, 2, 3, 4$. **You can solve the problem any way you like but briefly explain how you got your answer. Actually computing the DFT's is NOT the way to solve this problem.**

Problem 3. [40 points] Consider the following (normalized) triangular window of length $M = 9$

$$w_T[n] = \frac{1}{25}\{1, 2, 3, 4, 5, 4, 3, 2, 1\}$$

where the first value of the window sequence is associated with the discrete-time $n = 0$. **Note:** $w_T[n] = \frac{1}{25}(u[n] - u[n-5]) * (u[n] - u[n-5])$, where $*$ denotes linear convolution.

(a) Let $W_T(\omega)$ denote the DTFT of $w_T[n]$. Plot both the magnitude $|W_T(\omega)|$ and phase $\angle W_T(\omega)$ (separate plots) over $-\pi < \omega < \pi$ showing as much detail as possible. Explicitly point out the numerical values of the specific frequencies for which $|W_T(\omega)| = 0$.

(b) *Analysis of mainlobe width of* $W_T(\omega)$:

  (i) What is the null-to-null mainlobe width of $W_T(\omega)$?

  (ii) Is the mainlobe width of $W_T(\omega)$ larger than or smaller than the mainlobe width of the DTFT of a rectangular window of the same length, $w[n] = \frac{1}{9}(u[n] - u[n-9])$?

  (iii) Explain the difference in the mainlobe width of the DTFT of the triangular window relative to the mainlobe width of the DTFT of the rectangular window. That is, why is it smaller or why is it larger?

(c) *Analysis of sidelobes of* $W_T(\omega)$:

  (i) Are the sidelobes of $W_T(\omega)$ larger than or smaller than the sidelobes of the DTFT of a rectangular window of the same length, $w[n] = \frac{1}{9}(u[n] - u[n-9])$?

  (ii) Explain the difference in the sidelobe levels of the DTFT of the triangular window relative to the sidelobe levels of the DTFT of the rectangular window. That is, why are they larger or why are they lower?