EE638 (CC 760-M) Digital Signal Processing I   Exam 3   22 Nov. 1996

Test Duration: 75 minutes. Open Book but Closed Notes.
Do all work in blue books provided. Only return the blue books.

Problem 1. [40 points] Answer each question below in clear and succinct prose, or plot the quantity requested.

(a) *IIR vs. FIR Digital Filter Design.* To achieve a certain passband edge, passband ripple, stopband edge, and stopband ripple via a difference equation, it is well known that an IIR filter can require significantly less multiplies (and sums) per output point than an FIR filter meeting the same specficiations. Name one reason a designer would nonetheless choose to use an FIR filter in a given application. Explain your answer.

(b) Consider a causal FIR filter of length $M = 9$ with impulse response $h(n) = \{1, 2, 3, 4, 5, 4, 3, 2, 1\}$. Let $H(\omega)$ denote the DTFT of $h(n)$. Plot the phase of $H(\omega)$, $\angle H(\omega)$, over $-\pi < \omega < \pi$. Note that $H_r(\omega) = h(\frac{M-1}{2}) + \sum_{n=0}^{(M-3)/2} h(n) \cos[(\frac{M-1}{2} - n)\omega] \geq 0$ for all ω with this particular symmetric impulse response.

(c) Consider designing an equi-ripple symmetric (causal) FIR lowpass filter of length $M = 31$ via the Parks-McClellan algorithm. What is the range for the total number of ripples in the passband ($0 \leq \omega \leq \omega_p$) and stopband ($\omega_s \leq \omega \leq \pi$)? That is, what is the smallest possible number of extremal frequencies and what is the largest possible number of extremal frequencies?

(d) In Section 8.3.2 a digital IIR filter is designed from an analog IIR filter by sampling the impulse response of the analog filter. Describe one shortcoming of this method.

(e) In designing a digital IIR filter from an analog IIR filter via the bilinear transformation $s = c\frac{z-1}{z+1}$,

  (i) what is the importance of the fact that s is mapped to a ratio of two polynomials in z?

  (ii) what is the importance of the fact that the left-half plane of the s-plane is mapped into the interior of the unit circle in the z-plane?

  (iii) what is the importance of the fact that the imaginary axis in the s-plane is mapped onto (one-to-one) the unit circle in the z-plane?

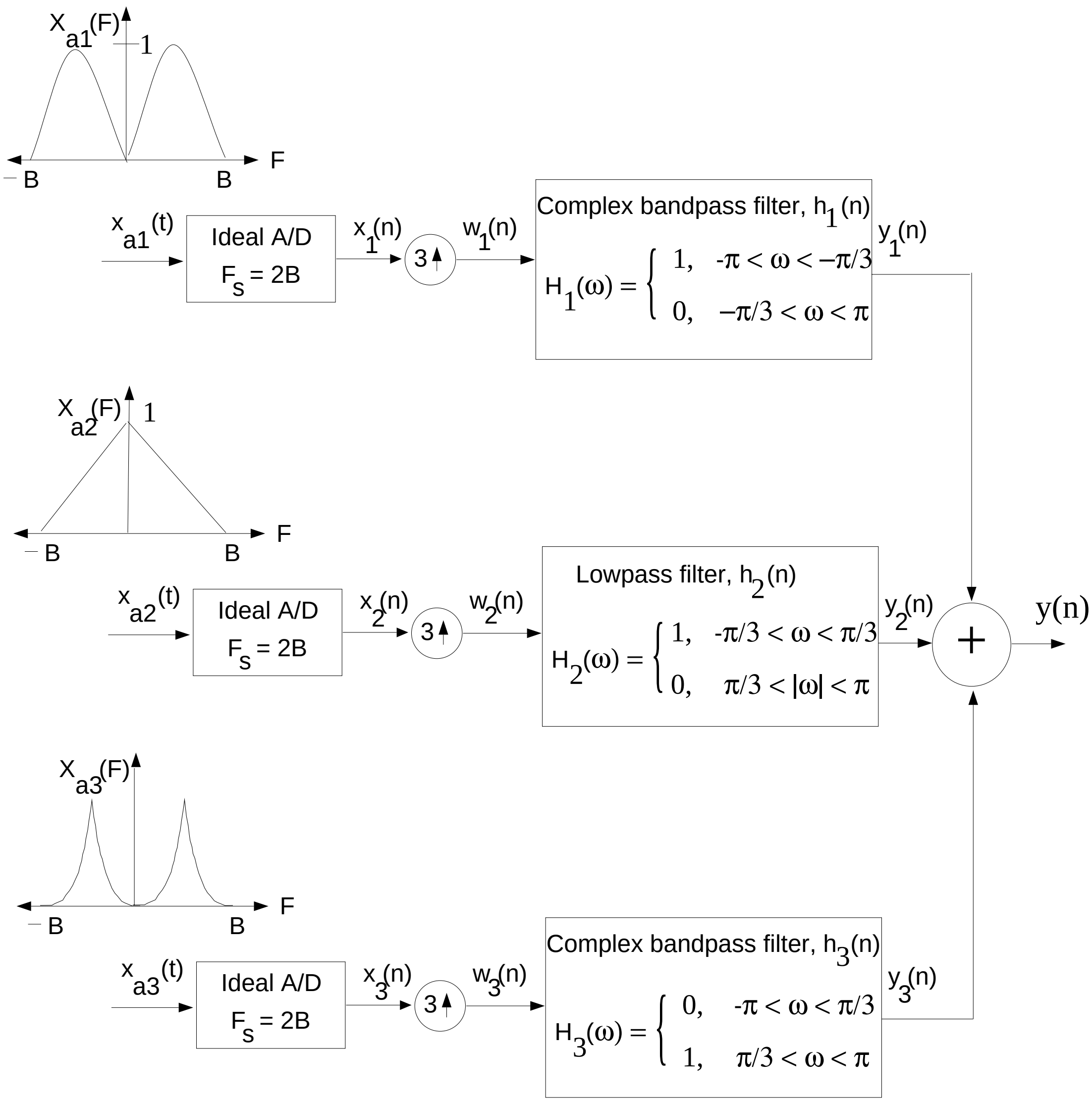

Figure 1: Digital subbanding of three real-valued signals each sampled at Nyquist rate.

Problem 2. [30 points]

Let $x_{a1}(t)$, $x_{a2}(t)$, and $x_{a3}(t)$ be three real-valued (lowpass) signals having the same bandwidth, B, and with corresponding CTFT's $X_{a1}(F)$, $X_{a2}(F)$, and $X_{a3}(F)$ depicted in Figure 1. Each signal is sampled at the Nyquist rate of $F_s = 2B$. The three signals are processed and subsequently summed as shown in Figure 1. Denoting $h_{LP}(n) = \frac{\sin(\frac{\pi}{3}n)}{\pi n}$, the respective impulse responses of each of the three filters are $h_1(n) = h_{LP}(n)e^{-j\frac{2\pi}{3}n}$, $h_2(n) = h_{LP}$, and $h_3(n) = h_{LP}(n)e^{j\frac{2\pi}{3}n}$.

(a) Let $Y(\omega)$ denote the DTFT of the sum signal, $y(n)$, at the output. Plot the magnitude of $Y(\omega)$ over $-\pi < \omega < \pi$. Show as much detail as possible.

(b) Draw a block diagram of a system for recovering each of the three original sampled signals, $x_1(n)$, $x_2(n)$, and $x_3(n)$, from the sum signal, $y(n)$.

Problem 3. [30 points]

An analog Butterworth filter of order $N = 2$ with a 3-dB cut-off at Ω_c has the following transfer function (Laplace Transform):

$$H_a(s) = \frac{\Omega_c}{s^2 + \sqrt{2}\Omega_c s + \Omega_c^2} \tag{1}$$

The goal is to use the bilinear transform technique to design a digital lowpass filter having a 3 dB cut-off at $\omega_c = 2\tan^{-1}\{\sqrt{2}\}$ rads/sec (this cut-off has been chosen so that the numbers work out nicely.)

(a) Pre-warp the frequency $\omega_c = 2\tan^{-1}\{\sqrt{2}\}$ based on the bilinear transformation

$$s = \frac{z-1}{z+1} \tag{2}$$

to determine the analog 3 dB cut-off frequency, Ω_c, required.

(b) Determine the transfer function, $H(z)$, of the digital filter obtained by applying the bilinear transform in (2) to the analog filter described by (1) with the Ω_c determined in part (a). Simplify as much as possible.

(c) Is the resulting digital filter stable?

(d) Determine the difference equation for implementing the resulting digital lowpass filter.