Digital Signal Processing I   Exam 3   1 Dec. 1995

Problem 1. [20 points]

A analog speech signal is sampled at a rate of 8 KHz. We would like to zoom in on the frequency range from 1 KHz to 2 KHz evaluating the CTFT of the signal at equi-spaced steps of 10 Hz over this band (including the endpoint 1 KHz but not 2 KHz.) Let $x(n)$ denote the discrete-time signal obtained through sampling. $x(n)$ is of length N equal to the total number of time samples taken during the observation interval. Employing the Chirp Z-Transform, we construct $g(n)$ as

$$g(n) = x(n)(r_0 e^{-j\theta_0})^{-n} \, (R_0 e^{j\phi_0})^{-\frac{n^2}{2}}, \qquad 0 \leq n \leq N-1.$$

With $h(n) = (R_0 e^{j\phi_0})^{\frac{n^2}{2}}$, we form $y(k)$, $k = 0, 1, ..., L-1$, via the linear convolution

$$y(k) = \sum_{n=0}^{N-1} g(n)h(k-n), \qquad k = 0, 1, ..., L-1.$$

Finally, the desired frequency values are computed as $X(z_k) = y(k)/h(k), k = 0, 1, ..., L-1$.

(a) Specify the numerical values of r_0, θ_0, R_0, ϕ_0, and L so that this computation yields the desired frequency values. Assume aliasing effects to be negligible.

(b) If one were to employ DFT based processing to efficiently compute the linear convolution required, what is the minimum length DFT? Your answer should be in terms of N.

Problem 2. [20 points]

We wish to design a linear phase FIR Hilbert Transformer via the Window Method. In this case, the impulse response of the linear phase FIR Hilbert Transformer may be expressed as

$$h(n) = h_d(n)w(n)$$

For $w(n)$, you can only use one of the two windows described below:

$$w_1(n) = \sin\left(\frac{\pi}{M}(n+0.5)\right), \qquad 0 \leq n \leq M-1$$

$$w_2(n) = \sin\left(\frac{2\pi}{M}(n+0.5)\right), \qquad 0 \leq n \leq M-1$$

(a) Write the appropriate function form for $h_d(n)$ as a function of n. (See pages 610-611 of the text for a description of an ideal DT Hilbert Transformer – you don't have to derive anything here.)

(b) Point out which of these two windows you would use; only one is appropriate. Explain why the window you didn't choose would not work.

Problem 3. [20 points]

A Butterworth filter of order $N = 3$ with a 3-dB cut-off at $\Omega_c = 1$ rads/sec has its poles located in the left half of the s-plane at $s_1 = e^{-j\frac{2\pi}{3}}$, $s_2 = -1$, $s_3 = e^{j\frac{2\pi}{3}}$. The transfer function of a digital IIR filter is obtained through a bilinear transformation by taking the $H_a(s)$ for this Butterworth filter and substituting

$$s = \frac{z-1}{z+1}$$

Find the poles of the resulting digital IIR filter and plot them on the complex z-plane by marking an 'x' at their respective locations (you don't need to find the zeroes of the resulting filter or plot them.) Draw the unit circle on this plot as well. Is the filter stable? For this problem, you may use the following approximation:

$$\frac{1}{\tan(\frac{\pi}{3})} = \frac{1}{\sqrt{3}} \approx 0.577 \approx 0.6$$

Problem 4. [20 points]

The transfer function for an ideal analog differentiator is $H_a(s) = s$. The transfer function, $H(z)$, of a digital differentiator is obtained through a bilinear transformation by substituting

$$s = 2\frac{z-1}{z+1}$$

into $H_a(s) = s$.

(a) Determine the frequency response (DTFT), $H(\omega)$, of the resulting digital filter. Simplify as much as possible.

(b) Plot the magnitude of the resulting digital differentiator, $|H(\omega)|$, over $-\pi < \omega < \pi$. Is the resulting digital differentiator a stable system?

(c) Plot the phase of the resulting digital differentiator, $\angle H(\omega)$, over $-\pi < \omega < \pi$. Is the frequency response, $H(\omega)$, purely imaginary?

(d) Show that $H(\omega) \approx j\omega$ for $|\omega| \ll 1$.

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Problem 5. [20 points]

Consider the (causal) window of length M described as

$$w(n) = \begin{cases} \frac{1}{2} - \frac{1}{2}\cos\left(\frac{4\pi}{M}(n+0.5)\right), & 0 \le n \le \frac{M}{4} - 1 \\ 1, & \frac{M}{4} \le n \le \frac{3M}{4} - 1 \\ \frac{1}{2} - \frac{1}{2}\cos\left(\frac{4\pi}{M}(n+0.5)\right), & \frac{3M}{4} \le n \le M - 1 \end{cases}$$

where it is assumed that M is divisible by 4, *i.e.,* that $\frac{M}{4}$ is an integer. Alternatively, $w(n)$ may be expressed as

$$w(n) = \left\{\frac{1}{2} - \frac{1}{2}\cos\left(\frac{4\pi}{M}(n+0.5)\right)\right\}\{u(n) - u(n-M)\}$$
$$+\left\{\frac{1}{2} + \frac{1}{2}\cos\left(\frac{4\pi}{M}(n+0.5)\right)\right\}\left\{u\left(n - \frac{M}{4}\right) - u\left(n - \frac{3M}{4}\right)\right\}$$

(a) Plot a rough sketch of the window as a function of n. Is the window symmetric or anti-symmetric, *i.e.,* $w(n) = w(M-1-n)$ or $w(n) = -w(M-1-n), n = 0, 1, ..., M-1$. Justify your answer.

(b) The DTFT of $w(n)$ may be expressed as

$$W(\omega) = e^{-j\frac{M-1}{2}\omega}\left\{\sum_{i=1}^{6} A_i \frac{\sin\left(\frac{L_i}{2}(\omega - \omega_i)\right)}{\sin\left(\frac{1}{2}(\omega - \omega_i)\right)}\right\}$$

Determine the values of A_i, L_i, and ω_i, $i = 1, ..., 6$. The answers for A_i, $i = 1, ..., 6$, are real-valued numbers while the answers for L_i and ω_i, $i = 1, ..., 6$, are in terms of M. List your answers in the form of a table like:

i	A_i	L_i	ω_i
1			
2			
3			
4			
5			
6			

Hint: choices for L_i: M, $\frac{M}{2}$, $\frac{M}{4}$, and $\frac{M}{8}$ (an answer may be used more than once and not all choices given are actual answers)

Hint: choices for ω_i: 0, $-\frac{\pi}{M}$, $-\frac{2\pi}{M}$, $-\frac{4\pi}{M}$, $\frac{\pi}{M}$, $\frac{2\pi}{M}$, and $\frac{4\pi}{M}$ (an answer may be used more than once and not all choices given are actual answers)

Note: Answers need to be substantiated by some work – no points for pure guesses.