Digital Signal Processing I	Exam 2	Fall 1999
Session 17		Live: 21 Oct. 1999

Cover Sheet

Test Duration: 75 minutes.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **four** problems.
All work should be done in the blue books provided.
Do **not** return this test sheet, just return the blue books.

Prob. No.	**Topic of Problem**	**Points**
1.	Digital Upsampling	35
2.	Digital Subbanding	25
3.	Multi-Stage Upsampling/Interpolation	20
4.	IIR Filter Design Via Bilinear Transform	20

Problem 1. [35 points]

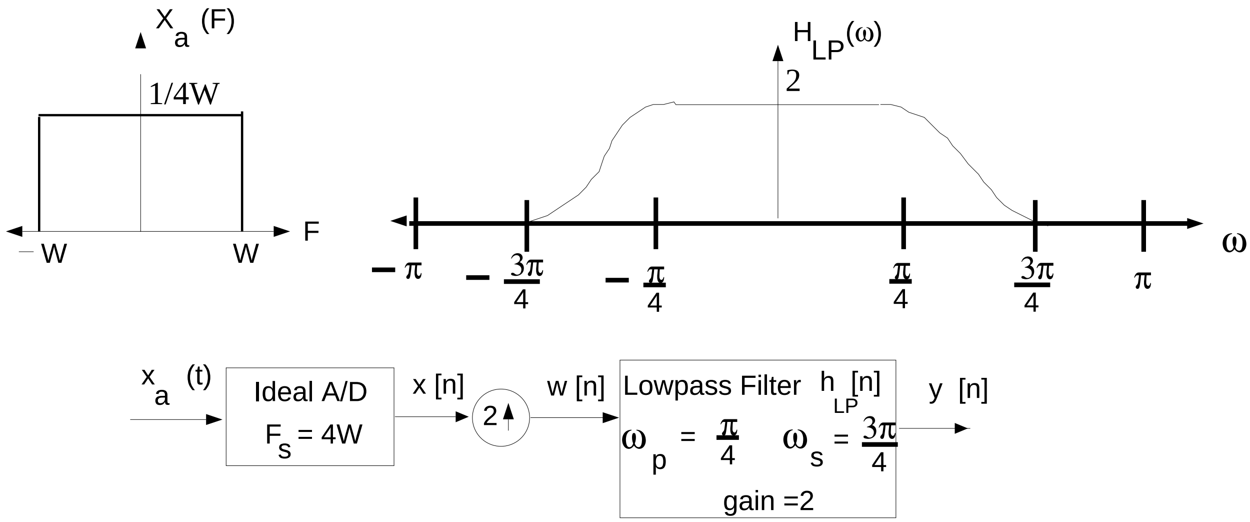


Figure 1.

The analog signal $x_a(t)$ with CTFT $X_a(F)$ shown above is input to the system above, where $x[n] = x_a(n/F_s)$ with $F_s = 4W$, and

$$h_{LP}[n] = \frac{\sin(\frac{\pi}{2}n)}{\frac{\pi}{2}n} \frac{\cos(\frac{\pi}{4}n)}{1 - \frac{n^2}{4}}, \qquad -\infty < n < \infty,$$

such that $H_{LP}(\omega) = 2$ for $|\omega| \leq \frac{\pi}{4}$, $H_{LP}(\omega) = 0$ for $\frac{3\pi}{4} \leq |\omega| \leq \pi$, and $H_{LP}(\omega)$ has a cosine roll-off from 1 at $\omega_p = \frac{\pi}{4}$ to 0 at $\omega_s = \frac{3\pi}{4}$. Finally, the zero inserts may be mathematically described as

$$w[n] = \begin{cases} x(\frac{n}{2}), & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

(a) Plot the magnitude of the DTFT of the output $y[n]$, $Y(\omega)$, over $-\pi < \omega < \pi$.

(b) Given that

$$x[n] = \frac{\sin(\frac{\pi}{2}n)}{\pi n} \qquad -\infty < n < \infty,$$

provide an analytical expression for $y[n]$ for $-\infty < n < \infty$ (similar to the expression for either $x[n]$ above, for example.)

THIS PROBLEM IS CONTINUED ON THE NEXT PAGE.

(c) The up-sampling by a factor of 2 in Figure 1 can be efficiently done via the block diagram in Figure 2 below.

(i) Provide an analytical expression for $h_0[n] = h_{LP}[2n]$ for $-\infty < n < \infty$. Simplify as much as possible.

(ii) Plot the magnitude of the DTFT of $h_0[n]$, $|H_0(\omega)|$, over $-\pi < \omega < \pi$.

(iii) Provide an analytical expression for the output $y_0[n]$ for $-\infty < n < \infty$. Is $y_0[n] = x[n]$? Explain why they are the same if you said "YES" or explain why they are not the same if you said "NO."

(iv) Describe an advantage of employing this lowpass filter, $h_{LP}[n]$, in the process of upsampling by a factor of 2.

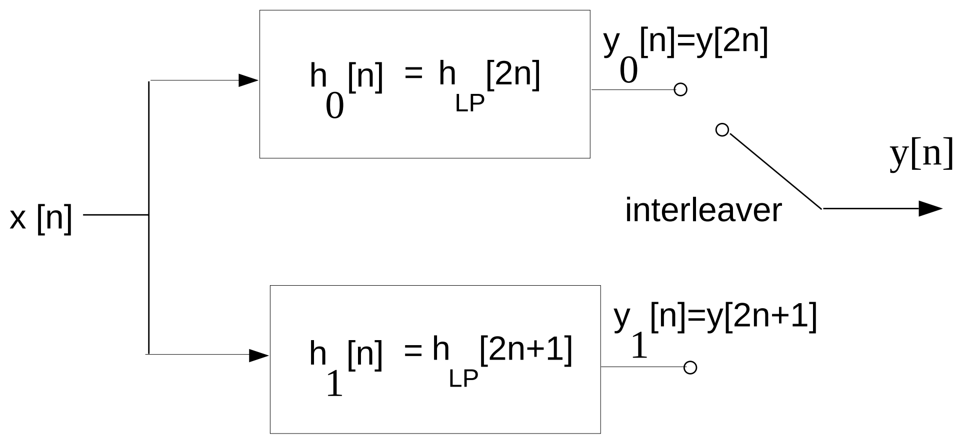
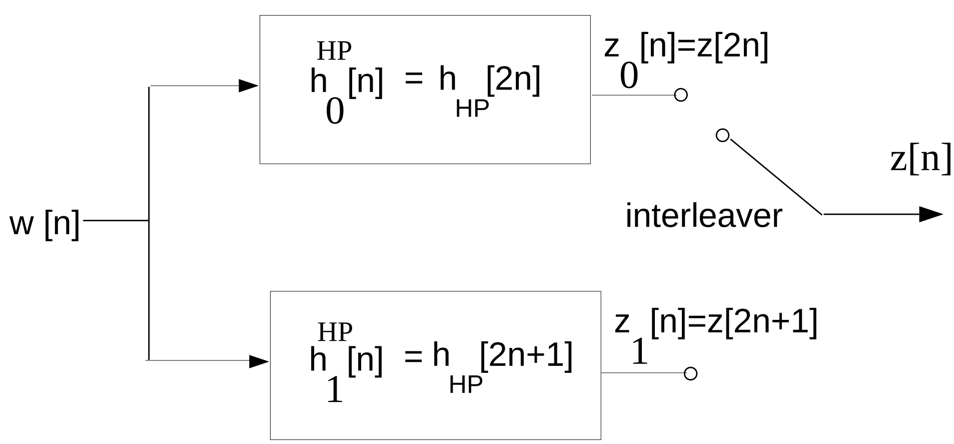


Figure 2.

Problem 2. [25 points] Let $w_a(t)$ be a continuous-time signal with a bandwidth of W, the same as the bandwidth of the signal $x_a(t)$ defined in Problem 1. The discrete-time signal $w[n] = w_a(n/F_s)$ is obtained by sampling the signal $w_a(t)$ at a rate $F_s = 4W$. We desire to frequency division multiplex $x[n]$ and $w[n]$. To this end, $z[n]$ is created as shown below.



where

$$h_{HP}[n] = (-1)^n h_{LP}[n] = (-1)^n \frac{\sin(\frac{\pi}{2}n)}{\frac{\pi}{2}n} \frac{\cos(\frac{\pi}{4}n)}{1 - \frac{n^2}{4}}, \qquad -\infty < n < \infty,$$

Finally, we create the sum signal

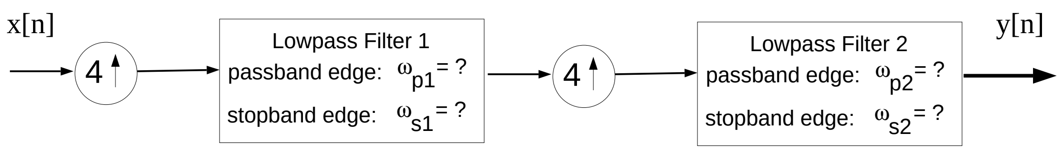
$$s[n] = y[n] + z[n]$$

where $y[n]$ is the DT signal created in Problem 1.

(a) Draw a block diagram of a system to recover $w[n]$ from the sum signal $s[n] = y[n] + z[n]$. The recovery of $w[n]$ must be done in a computationally efficient manner. You CANNOT use any decimators and you CANNOT use any modulators (multiplication by a cosine). Clearly specify all the quantities in your block diagram.

(b) Draw a block diagram of a system to recover $x[n]$, defined in Problem 1, from the sum signal $s[n] = y[n] + z[n]$. The same rules apply as those stated in part (a) above.

Problem 3. [20 points] The signal $x[n] = x_a(n/F_s)$ is obtained by sampling an analog signal $x_a(t)$ having a bandwidth of $W = 20$ KHz at a rate of $F_s = 50$ KHz. It is desired to increase the sampling rate by a factor of $L = 16$ to $F_{s_{new}} = 800$ KHz in two stages via the system below, where ideally $y[n] = x_a(n/16F_s)$



(a) Determine the passband edge, ω_{p1}, and stopband edge, ω_{s1}, of the first lowpass filter.

(b) Determine the passband edge, ω_{p2}, and stopband edge, ω_{s2}, of the second lowpass filter.

PROBLEM 4 IS ON THE NEXT PAGE.

Problem 4. [20 points]

An analog Butterworth filter of order $N = 1$ with a 3-dB cut-off at Ω_c rad/sec has the following transfer function (Laplace Transform):

$$H_a(s) = \frac{\Omega_c}{s + \Omega_c} \tag{1}$$

The goal is to use the bilinear transform technique to design a digital lowpass filter having a 3 dB cut-off at $\omega_c = \frac{\pi}{2} = 0.5\pi$ rads/sec.

(a) Use the bilinear transformation technique where s is replaced by

$$s = c\frac{z-1}{z+1} \tag{2}$$

to create the digital filter. You have freedom to pick the value of c but it must be real-valued and non-negative.

(b) Determine the transfer function, $H(z)$, of the digital filter obtained by applying the bilinear transform in (2) to the analog filter described by (1) with the value of Ω_c determined in part (a).

(c) Is the resulting digital filter stable? Explain your answer.

(d) Let $H(\omega)$ be the DTFT of the impulse response of the resulting digital filter. Plot $|H(\omega)|$ over $-\pi < \omega < \pi$ specifically pointing out the respective values at $\omega = 0$, $\omega = \pi/2$, and $\omega = \pi$.

(e) Determine the difference equation for implementing the resulting digital lowpass filter.