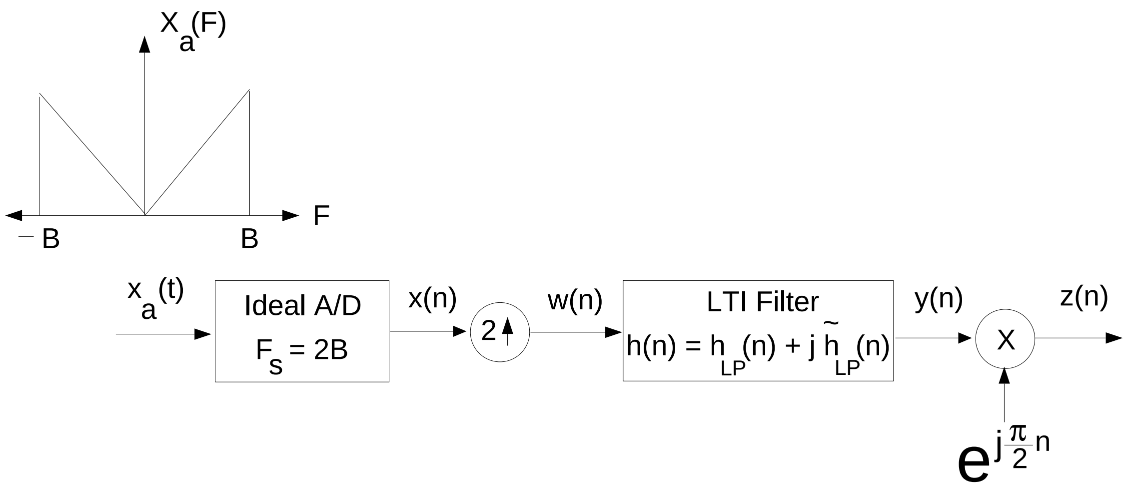
Digital Signal Processing I   Exam 2   30 Oct. 1995

Problem 1. [35 points]



The analog signal $x_a(t)$ with CTFT $X_a(F)$ shown is input to the system above, where $x(n) = x_a(n/F_s)$,

$$w(n) = \begin{cases} x(\frac{n}{2}), & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

$$h_{LP}(n) = \frac{\sin(\frac{\pi}{2}n)}{\pi n}, \qquad -\infty < n < \infty,$$

and $\tilde{h}_{LP}(n)$ is the Discrete-Time Hilbert Transform of $h(n)$. Note that $y(n) = w(n) * h(n)$, where $h(n) = h_{LP}(n) + j\tilde{h}_{LP}(n)$. (If necessary, see page 610 of the text for both a frequency and time-domain description of the DT Hilbert Transform.) Finally, $z(n) = e^{j\frac{\pi}{2}n}y(n)$.

(25 pts) Plot the magnitude of the respective DTFT's of $x(n)$, $w(n)$, $y(n)$, $h(n)$, and $z(n)$ – five plots all together. In each case, plot the magnitude of the DTFT over $-\pi < \omega < \pi$ (NOT just $0 < \omega < \pi$ since most of the sequences are complex-valued.) Show as much detail as possible.

(10 pts) Determine a closed-form expression (NO summation should appear in the final expression) for $\tilde{h}_{LP}(n)$, the Hilbert Transform of $h_{LP}(n)$, as a function of n. Simplify as much as possible.

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Problem 2. [35 points]

Consider the following two $N = 8$ point sequences:

$$x(n) = \cos\left(\frac{\pi}{2}n\right)\{u(n) - u(n-8)\}$$

and

$$h(n) = .5^n\{u(n) - u(n-8)\},$$

where $u(n)$ is the DT unit step function. Let $X_8(k)$ and $H_8(k)$ denote the respective $N = 8$ point DFT's of $x(n)$ and $h(n)$.

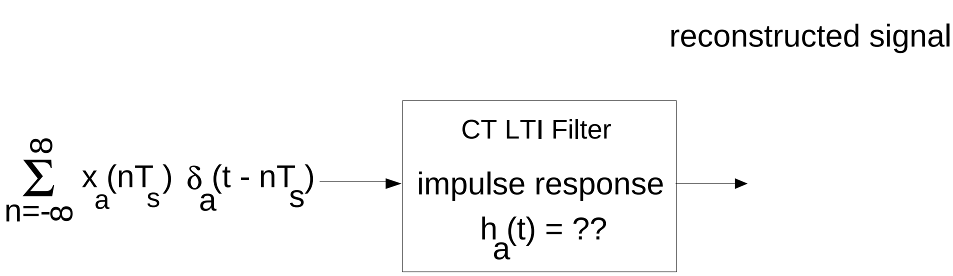
$$x(n) \overset{DFT}{\underset{8}{\longleftrightarrow}} X_8(k) \qquad\qquad h(n) \overset{DFT}{\underset{8}{\longleftrightarrow}} H_8(k)$$

(10 pts) Write a closed-form expression (NO summation in the final answer) for both $X_8(k)$ and $H_8(k)$, respectively (two answers), as a function of k. There's no need to do a lot of work here if you make use of either standard DFT pairs or standard DTFT pairs.

(15 pts) Let $y_8(n)$ denote the $N = 8$ point inverse DFT of the product $Y_8(k) = X_8(k)H_8(k)$. Determine a closed-form expression (NO summation in final answer) for $y_8(n)$. Simplify as much as possible – there should be no $j = \sqrt{-1}$ in the final answer since $y_8(n)$ is real-valued for all n. (You can make the approximation $1 - \frac{1}{2^8} \approx 1$.)

(10 pts) Is there any value of n for which $y_8(n) = y(n)$, where $y(n)$ is the linear convolution of $x(n)$ and $h(n)$ ($y(n) = x(n) * h(n)$)?? If yes, state which values of n does $y_8(n) = y(n)$. Explain your answer.

Problem 3. [10 points]



Consider reconstructing a signal from its samples by linearly interpolating between every two successive sample values (graphically, we draw a line between each pair of successive sample values.) From a theoretical point of view, the reconstruction process may be viewed in terms of the block diagram above. Plot the impulse response of the appropriate continuous time (CT) LTI filter, $h_a(t)$, in the diagram above that achieves this type of reconstruction.

Problem 4. [10 points]

A DT signal is obtained by sampling the CT signal $x_a(t) = e^{-at}u(t)$ as $x(n) = x_a(nT_s) = x_a(n/F_s)$. From class, we know that the DTFT of $x(n)$, $X(\omega)$, and the CTFT of $x_a(t)$, $X_a(F)$, are related as $X(\omega) = F_s \sum_{k=-\infty}^{\infty} X_a\left(\frac{F_s}{2\pi}(\omega + k2\pi)\right)$. Given the CTFT pair

$$e^{-at}u(t) \overset{CTFT}{\longleftrightarrow} \frac{1}{a + j2\pi F}$$

it follows that,

$$X(\omega) = F_s \sum_{k=-\infty}^{\infty} \frac{1}{a + jF_s(\omega + k2\pi)} = ??$$

Determine a closed-form expression (NO summation in final answer) for $X(\omega)$ in terms of a and F_s. (This problem requires very little work and was alluded to in class.)

Problem 5. [10 points]

Suppose we need to compute a $N = 21$ point DFT. The direct approach would require $21^2 = 441$ complex multiplications. What would be the required number of complex multiplications if we employed a divide and conquer approach??