EE538 Digital Signal Processing I **Exam 1** **23 Sept. 1999**

Test Duration: 75 minutes.

Open Book but Closed Notes. This test is composed of **four** problems.

All work should be done in the blue books provided.

There is no need to return the test sheet, just turn in the blue books.

Problem 1. [30 points]

Consider a very simplistic CDMA system with only two users assigned the following length 2 codes, respectively:

$$\text{User 1's code:} \quad c_1[n] = \{1, 1\}$$

$$\text{User 2's code:} \quad c_2[n] = \{1, -1\}$$

Consider transmitting a block of four bits for each of the two users,

$$\text{User 1's four info. bits:} \quad b_1[n] = \{b_1[0], b_1[1], b_1[2], b_1[3]\}$$

$$\text{User 2's four info. bits:} \quad b_2[n] = \{b_2[0], b_2[1], b_2[2], b_2[3]\}$$

where $b_k[n]$ is either a "+1" representing the binary bit "1" or "-1" representing the binary bit "0" for any value of k or n. The transmitted code-division multiplexed block may be mathematically expressed as

$$x[n] = \sum_{k=1}^{2} \sum_{m=0}^{3} b_k[m] c_k[n - 2m], \qquad n = 0, 1, ..., 7$$

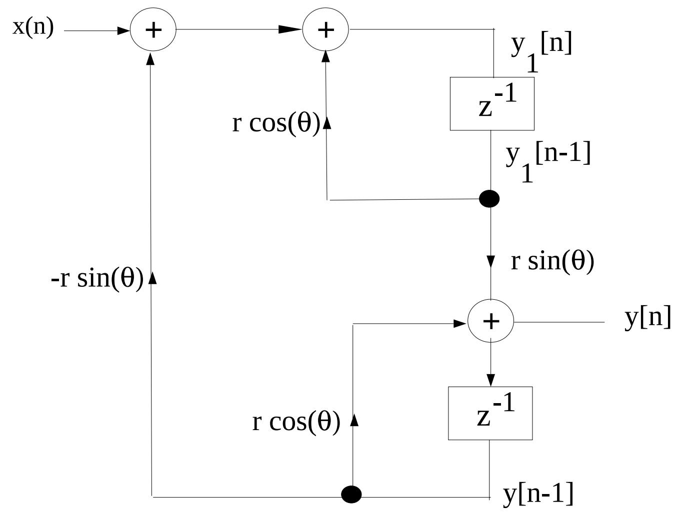
Given that the received block has the following numerical values

$$x[n] = \{\underbrace{0}_{\uparrow}, -2, -2, 0, 2, 0, 0, 2\}$$

where the first entry above is the value of $x[0]$, determine the numerical values of $b_1[n], n = 0, 1, 2, 3$ and $b_2[n], n = 0, 1, 2, 3$. Your answer should consist of 8 numerical values all together. Show all work in arriving at your answer. Note that code 1 is orthogonal to code 2.

Problem 2. [40 points]

An alternative realization of a causal two-pole filter is the so-called coupled realization pictured below.



(a) Determine the overall transfer function $H(z) = Y(z)/X(z)$ in terms of r and θ. Show all work. Note that it may be useful to work with the intermediate output $y_1[n]$ in arriving at your answer.

(b) Express the poles of the systems in terms of r and θ. What are the conditions on r and θ in order that the system be BIBO stable?

(c) For this part of the problem, let $r = 1/\sqrt{2} \approx .707$ and $\theta = 45°$.

(i) Plot the pole-zero diagram.

(ii) State and plot the region of convergence for $H(z)$.

(d) For this part of the problem, let $r = 1$ and $\theta = 90°$.

(i) Determine the DTFT of $h[n]$ and plot the magnitude $|H(\omega)|$ over the interval $-\pi < \omega < \pi$ showing as much detail as possible. In particular, explicitly point out if there are any values of ω for which $|H(\omega)|$ is approaching infinity.

(ii) Determine one input sequence $x[n]$ that leads to unbounded output sequence.

Problem 3. [30 points] Consider the transmission of a pulse amplitude-modulated signal described by

$$x(t) = \sum_{k=-\infty}^{\infty} b[k]p(t - kT_o)$$

where $b[n]$ are the information-bearing symbols being transmitted which be viewed as a discrete-time sequence. In binary phase-shift keying, $b[n]$ is either "+1" or "-1" for all n. $p(t)$ is the pulse symbol waveform and $1/T_o$ is the bit rate. For this problem, sampling $p(t)$ at TWICE the bit rate yields the discrete-time sequence

$$\tilde{p}[n] = p\left(n\frac{T_o}{2}\right) = \{0, 1, 0, -2, \underbrace{4}_{\uparrow}, -2, 0, 1, 0\}$$

At the receiver, $x(t)$ arrives by both a direct path and a multipath reflection that arrives at a delay of τ with the same strength as the direct path. The received signal, $y(t)$, may be modeled as:

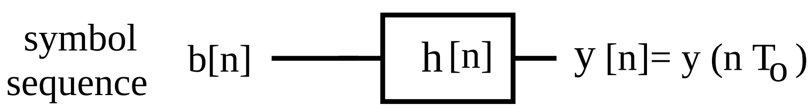
$$y(t) = x(t) * g(t)$$

where $*$ denotes continuous time convolution and

$$g(t) = \delta(t) + \delta(t - \tau) \tag{1}$$

and $\delta(t)$ is the Dirac Delta function.

Sampling $y(t)$ at the bit rate, $F_s = \frac{1}{T_o}$, it is easily shown that the resulting sequence $y[n] = y(nT_o)$ may be modeled as having been generated by the following discrete-time system



(a) For the case of $\tau = T_o$ in $g(t)$ defined in Eqn. (1), determine the impulse response $h[n]$ above for all n so that the output $y[n]$ is $y(nT_o)$ as specified. You answer should specify the numerical values of $h[n]$.

(b) Repeat (a) for the case of $\tau = \frac{T_o}{2}$.