EE538 (CC 560-M) Digital Signal Processing I   Exam 1   24 Sept. 1998

Test Duration: 75 minutes.

Open Book but Closed Notes. This test is composed of **four** problems.

All work should be done in the blue books provided.

There is no need to return the test sheet, just send back the blue books.

Problem 1. [25 points]

Consider an FIR LTI system having the following impulse reponse.

$$\begin{aligned} h[n] &= \delta[n] - \delta[n-2] \\ &= \{\underbrace{1}_{\uparrow}, 0, -1\} \end{aligned}$$

(a) Compute $y[n]$ as the linear convolution of $h[n]$ above with the input sequence below. State the numerical value of $y[n] = x[n] * h[n]$ for each n, $-\infty < n < \infty$.

$$\begin{aligned} x[n] &= 0.5\,\{3 - \cos(\pi n)\}\,\{u[n] - u[n-9]\} \\ &= \{\underbrace{1}_{\uparrow}, 2, 1, 2, 1, 2, 1, 2, 1\} \end{aligned}$$

(b) Determine the DTFT of $h[n]$ and plot both the magnitude $|H(\omega)|$ and the phase $\angle H(\omega)$ (separate plots) over the interval $-\pi < \omega < \pi$ showing as much detail as possible. In particular, explicitly point out if there are any values of ω for which $|H(\omega)| = 0$.

(c) Determine and write out a difference equation whose impulse response is $h(n) = \delta[n] - \delta[n-2]$.

Problem 2. [30 points]

Consider the causal LTI system described by the difference equation

$$y[n] = j\; y[n-1] + x[n] - x[n-4] \qquad \text{where: } j = e^{j\frac{\pi}{2}}$$

(a) Determine the transfer function of the system, $H(z)$, equal to the Z Transform of the impulse response $h[n]$.

(b) Plot the pole-zero diagram associated with the transfer function. Note that the N roots of the polynomial equation $z^N - r^N = 0$ are $z_i = re^{j\frac{2\pi}{N}i}, i = 0, 1, ..., N-1$.

(c) State the region of convergence of $H(z)$. State whether $h[n]$ is BIBO stable or not, and explain why.

(d) Let $H(\omega)$ be the frequency response of the system equal to the DTFT of $h[n]$. Plot $|H(\omega)|$ over the interval $-\pi < \omega < \pi$ showing as much detail as possible. In particular, explicitly point out if there are any values of ω for which $|H(\omega)| = 0$.

(e) Find the output $y[n]$ for the input $x[n] = 4 + \sin(\pi n)$, $-\infty < n < \infty$.

(f) Determine the impulse response of the system, $h[n]$. Is the system FIR or IIR?

Problem 3. [20 points]

A speech signal corrupted by a tone at 3 KHz is sampled at a rate of 9 KHz. Design the numerical values of each of the coefficients in the difference equation below to provide a sharp notch for removing the 3 KHz tone without causing much amplitude distortion to the speech signal.

$$y[n] = -a_1\ y[n-1] - a_2\ y[n-2] + x[n] + b_1\ x[n-1] + b_2\ x[n-2]$$

Problem 4. [25 points]

Consider sampling the signal below (which is defined for all time, $-\infty < t < \infty$) .

$$x_a(t) = \frac{\sin(2\pi(3000)t)}{\pi t}\cos(2\pi(1500)t)$$

(a) $x_a(t)$ is sampled at a rate $F_s = 24$ KHz to produce the discrete-time signal $x[n] = x_a[n/F_s)$. Plot the DTFT of $x[n]$, $|X(\omega)|$, over the interval $-\pi < \omega < \pi$ showing as much detail as possible. *Hint:* Consult Tables 4.5 and 4.6 on pp. 304-305.

(b) What is the Nyquist sampling rate for this signal (i.e., the lowest sampling rate for $x_a(t)$ which averts aliasing)?

(c) Let $y[n] = x^2[n]$, where $x[n] = x_a[n/F_s]$ as defined in part (a). Plot the magnitude of the DTFT of $y[n]$, $|Y(\omega)|$, for $-\pi < \omega < \pi$ showing as much detail as possible.