Examination Date: Session 11 (Live: 15 September 2000).
Test Duration: 50 minutes.
Open Book but Closed Notes. This test is composed of **three** problems.
All work should be done in the blue books provided.
There is no need to return the test sheet, just turn in the blue books.

Problem 1. [30 points]

Consider a very simplistic CDMA system with only two users assigned the following length 4 *orthogonal* codes, respectively:

$$\text{User 1's code:} \quad c_1[n] = \{1, -1, 1, -1\}$$

$$\text{User 2's code:} \quad c_2[n] = \{1, -1, -1, 1\}$$

Consider transmitting a block of two bits for each of the two users,

$$\text{User 1's two info. bits:} \quad b_1[n] = \{b_1[0], b_1[1]\}$$

$$\text{User 2's two info. bits:} \quad b_2[n] = \{b_2[0], b_2[1], \}$$

where $b_k[n]$ is either a "+1" representing the binary bit "1" or "-1" representing the binary bit "0" for any value of k or n. The transmitted code-division multiplexed block may be mathematically expressed as

$$x[n] = \sum_{k=1}^{2} \sum_{m=0}^{1} b_k[m] c_k[n - 4m], \qquad n = 0, 1, ..., 7$$

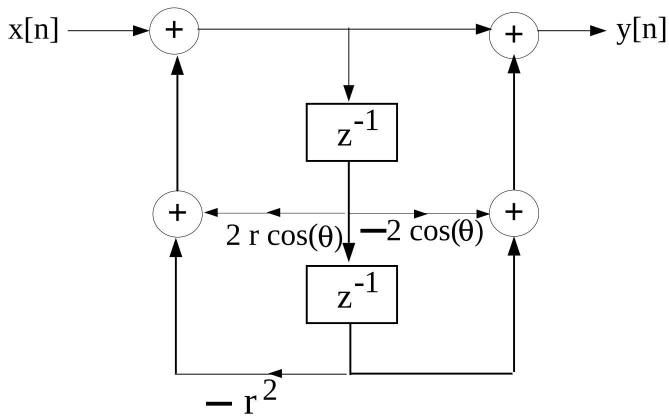
Given that the received block has the following numerical values

$$x[n] = \{\underbrace{0}_{\uparrow}, 0, 2, -2, 2, -2, 0, 0\}$$

where the first entry above is the value of $x[0]$, determine the numerical values of $b_1[n], n = 0, 1$ and $b_2[n], n = 0, 1$. Your answer should consist of 4 numerical values all together. Show all work in arriving at your answer. Note that code 1 is orthogonal to code 2.

Problem 2. [40 points]

A realization of a causal IIR notch filter is the so-called coupled realization pictured below.



(a) Determine the overall transfer function $H(z) = Y(z)/X(z)$ in terms of r and θ. Show all work.

(b) Express the poles of the systems in terms of r and θ. What are the conditions on r and θ in order that the system be BIBO stable?

(c) For this part of the problem, let $r = .95$ and $\theta = 90°$.

(i) Plot the pole-zero diagram.

(ii) State and plot the region of convergence for $H(z)$.

(iii) Determine the DTFT of $h[n]$ and plot the magnitude $|H(\omega)|$ over the interval $-\pi < \omega < \pi$ showing as much detail as possible. In particular, explicitly point out if there are any values of ω for which $|H(\omega)|$ is exactly zero.

Problem 3. [30 points] Consider the transmission of a pulse amplitude-modulated signal

$$x(t) = \sum_{k=-\infty}^{\infty} b[k]p(t - kT_o)$$

where $b[n]$ are the information-bearing symbols being transmitted which may be viewed as a discrete-time sequence. In binary phase-shift keying, $b[n]$ is either "+1" or "-1" for all n. $p(t)$ is the pulse symbol waveform and $1/T_o$ is the bit rate. Sampling $p(t)$ at TWICE the bit rate yields the discrete-time sequence

$$\tilde{p}[n] = p\left(n\frac{T_o}{2}\right) = \frac{\sin(\frac{\pi}{2}n)}{\frac{\pi}{2}n}\frac{\cos(\frac{\pi}{4}n)}{1-\frac{n^2}{4}}, \qquad -\infty < n < \infty,$$

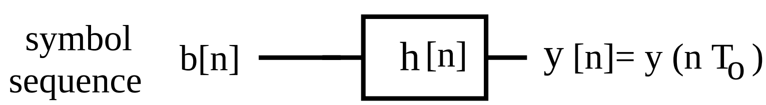
At the receiver, $x(t)$ arrives by both a direct path and a multipath reflection having the same strength as the direct path but at a delay of T_o and phase-shifted by θ. Denoting continuous time convolution as $*$, the received signal, $y(t)$, may be modeled as:

$$y(t) = x(t) * g(t)$$

where $g(t)$ is described below using $\delta(t)$ to denote the Dirac Delta function.

$$g(t) = \delta(t) + e^{j\theta}\delta(t - T_o) \tag{1}$$

Sampling $y(t)$ at the bit rate, $F_s = \frac{1}{T_o}$, it is easily shown that the resulting sequence $y[n] = y(nT_o)$ may be modeled as having been generated by the following discrete-time system.



Determine $h[n]$ and plot the magnitude of its DTFT $H(\omega) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$ over $-\pi \le \omega \le \pi$ for **each** of the following three values of θ:

(i) $\theta = 0$

(ii) $\theta = \frac{\pi}{2}$

(iii) $\theta = \pi$

In each case, $H(\omega)$ will be exactly equal to zero for one specific value of ω. Determine this frequency in each case.