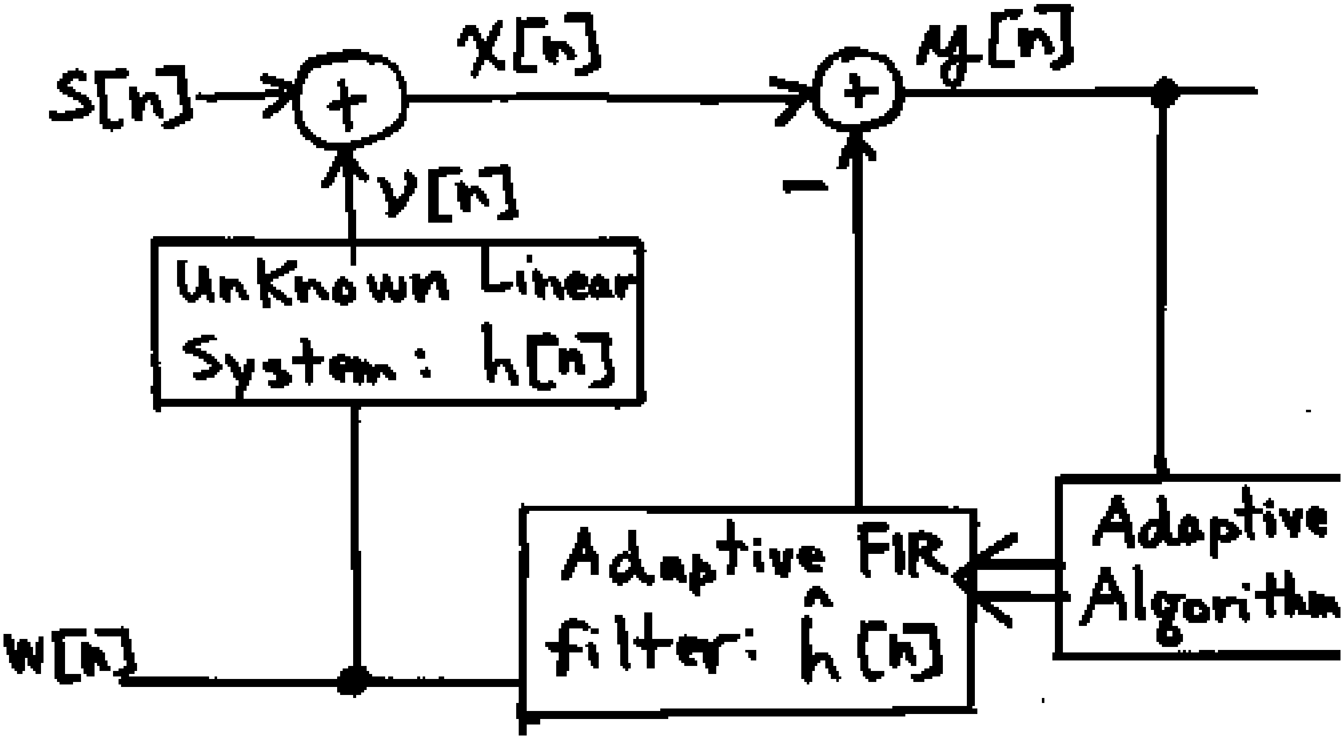
# EE648 (CC761-M) DSP II

## Session 5 (hive: 1/26/99)

Outline:

- Adaptive Noise Cancellation
- Adaptive Suppression of Narrowband Interference in a Wideband Signal
- Sect. 12.1 of 1st Ed. of P+M

- Application: Adaptive Noise Cancellation
- specifically: car phone embedded in steering wheel
- speech masked by "car noise"
- <u>approach</u>: place "mike" at point in car to pick up "car noise" only
- form an estimate of noise contaminating speech and subtract it off


S[n]
x[n]
y[n]
v[n]
−
Unknown Linear
System: h[n]
Adaptive FIR
filter: ĥ[n]
Adaptive
Algorithm
w[n]

- $s[n]$: Speech only (unobservable)
- $w[n]$: noise-only observed at "remote" sensor ("mike")
- $\nu[n]$: filtered version of $w[n]$ that corrupts desired speech (unobservable)

$$\nu[n] = w[n] * h[n]$$

- $x[n] = s[n] + \nu[n] \Rightarrow$ picked up by "mike" in steering wheel

• Choose $\hat{h}[n]$ to minimize

$$E\{y^2[n]\} =$$

$$= E\left\{\left[x[n] - \sum_{k=0}^{M-1} \hat{h}[n]\, w[n-k]\right]^2\right\}$$

$$= E\left\{\left[x[n] - \underline{\hat{h}}_M^T\, \underline{w}[n]\right]^2\right\}$$

• where:

$$\underline{\hat{h}}_M = \left[\hat{h}[0], \hat{h}[1], \ldots, \hat{h}[M-1]\right]^T$$

$$\underline{w}[n] = \left[w[n], w[n-1], \ldots, w[n-(M-1)]\right]^T$$

• Show: $\underline{\hat{h}}_M^{opt} = \underline{h}$

• Recall: $x[n] = s[n] + \nu[n]$

$= s[n] + w[n] * h[n]$

$= s[n] + \underline{h}^T \underline{w}[n]$

• Note: Assume $s[n]$ and $w[n]$ are independent random processes and $E\{w[n]\} = 0$

$$E\left\{\left[\left(s[n] + \underline{h}^T \underline{w}[n]\right) - \underline{\hat{h}}_M^T \underline{w}[n]\right]^2\right\}$$

$$= E\left\{\left[s[n] + \left(\underline{h} - \hat{\underline{h}}_M\right)^T \underline{w}[n]\right]^2\right\}$$

$$= E\{s^2[n]\} + 2\left(\underline{h} - \hat{\underline{h}}_M\right)^T \underbrace{E\{s[n]\,\underline{w}[n]\}}_{\rightarrow 0}$$

$$+ \left(\underline{h} - \hat{\underline{h}}_M\right)^T E\{\underline{w}[n]\,\underline{w}^T[n]\}\left(\underline{h} - \hat{\underline{h}}_M\right)$$

$$= E\{s^2[n]\} + \left(\underline{h} - \hat{\underline{h}}_M\right)^T \underline{R}_{ww}\left(\underline{h} - \hat{\underline{h}}_M\right)$$

- thus: $\hat{\underline{h}}_M^{opt} = \underline{h}$ since $\underline{R}_{ww} = E\{\underline{w}[n]\,\underline{w}^T[n]\}$ is positive definite

- in implementing either LMS or RLS;
- $x[n]$: plays role of desired signal $d[n]$
- $y[n]$: plays role of error signal $\varepsilon[n]$
- See summary of LMS/RLS for Adaptive Noise Cancellation delineated in Hmwk. 1 write-up

- Summary of LMS/RLS for Adaptive Noise Cancellation
- for RLS (with $w = \gamma$)

$$y[n, n-1] = x[n] - \underline{h}_M^T[n-1]\,\underline{w}[n]$$

$$\underline{f}[n] = \underline{R}_{ww}^{-1}[n-1]\,\underline{w}[n]$$

$$\mu[n] = \gamma + \underline{w}^T[n]\,\underline{f}[n]$$

$$\underline{K}_M[n] = \underline{f}[n]/\mu[n]$$

$$\underline{h}_M[n] = \underline{h}_n[n-1] + y[n, n-1]\,\underline{K}_M[n]$$

$$\underline{R}_{ww}^{-1}[n] = \frac{1}{\gamma}\left\{\underline{R}_{ww}^{-1}[n-1] + \underline{K}_M[n]\,\underline{f}^T[n]\right\}$$

Go to Step 1

- for LMS: $y[n] = x[n] - \underline{h}_M^T[n]\,\underline{w}[n]$

  $\underline{h}_M[n+1] = \underline{h}_M[n] + \mu\, y[n]\,\underline{w}[n]$

- See demo NoiseCancel.m and WordCancel.m at course web site

- Suppression of Narrowband Interference in a wideband Signal

$$\underbrace{x[n] = s[n]}_{\substack{\text{information signal} \\ \text{(wideband)}}} + \underbrace{i[n]}_{\substack{\text{interference} \\ \text{(narrowband)}}}$$

- Assumption 1: $s[n]$ and $i[n]$ are independent random processes

- $s[n]$: "wideband" signal
- power spectral density, $S_{ss}(\omega)$, is generally non-negligible over a wide range of frequencies
- as a result, the autocorrelation $r_{ss}[m]$ is highly localized near $m=0$
- Extreme case: white noise

$$r_{ss}[m] = \sigma_w^2 \delta[m] \xleftrightarrow{\text{DTFT}} S_{ss}(\omega) = \sigma_w^2 \quad \text{for all } \omega$$

- assumption : $r_{ss}[m] \approx 0$ for $|m| \geq D$
  where $D$ is not too large
- In contrast :
  $i[n]$ : "narrowband" signal
- $S_{ii}(\omega)$ is highly concentrated
  in frequency : $S_{ii}(\omega) \approx 0$ except
  over $\omega_0 - \Delta\omega < \omega < \omega_0 + \Delta\omega$
  where $\Delta\omega$ is small
- as a result: $r_{ii}[m]$ is generally
  nonzero for a large range of $m$

- Extreme case: $i[n] = e^{j(\omega_0 n + \Theta)}$ $\forall n$

$$r_{ii}[m] = e^{j\omega_0 m} \xleftrightarrow{\text{DTFT}} S_{ii}(\omega) = 2\pi\delta(\omega - \omega_0)$$

for $-\pi < \omega < \pi$

- in particular: $r_{ii}[m] \neq 0$ for $D \leq m \leq D+M$
- specifically, assumption is $i[n+D]$ is highly correlated with $i[n], i[n-1], \ldots, i[n-(M-1)]$
- whereas $s[n+D]$ is uncorrelated with $s[n], s[n-1], \ldots, s[n-(M-1)]$

• criterion for suppressing $i[n]$:

$$\min_{h[n]} E\left\{\left[x[n+D] - \sum_{k=0}^{M-1} h[k]\, x[n-k]\right]^2\right\}$$

• thus: $x[n+D]$ plays role of "desired" signal

$$\min_{h[n]} E\left\{\left[x[n+D] - \underline{h}_M^T \underline{x}[n]\right]^2\right\}$$

Define: $$\tilde{h}[n] = \begin{cases} 1, & n=0 \\ 0, & n=1,\ldots,D-1 \\ -h[n-D], & n=D,\ldots,D+M-1 \end{cases}$$

- the objective may be expressed as

$$\min_{\tilde{h}[n]} E\left\{\left[\sum_{k=0}^{D+M-1} \tilde{h}[k]\, x[n+D-k]\right]^2\right\}$$

- See Demo CancelTone.m at web site